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Boom and Burst in Housing Market with Heterogeneous Agents

Alessandro Spelta
(Università di Pavia)

Guido Ascari
(Università di Pavia)

Nicolò Pecora
(Università Cattolica di Piacenza)

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A. Spelta[*]

University of Pavia, Italy

G. Ascari[†]

University of Pavia, Italy

N. Pecora[‡]

Catholic university, Piacenza, Italy

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Abstract

We study the housing market using a partial "dis"-equilibrium model in which the rational expectations hypothesis is relaxed in favor of an agent-based approach. The chartist-fundamentalist mechanism allows for the behavioral foundation of the expectations, the endogenous development of bubbles and contributes to replicate the recent house price dynamics. We also analyze the role of the interest rate during the boom and, anchoring the interest rate to the change in house price, we investigate the possibility to reduce the volatility and the distortion in the price dynamics.

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[*]alessandro.spelta01@ateneopv.it

[†]guido.ascari@unipv.it

[‡]nicolo.pecora@unicatt.it

1 Introduction

The economies of almost every countries have recently been hit by a turmoil in the financial markets; the financial crisis has demonstrated that developments in financial markets can have impact on real economy. For this reason interdependencies between real and financial markets should be taken into account when doing macroeconomics. The method of Agent-Based computational (ABC) simulation drops the assumption of rational expectations, homogeneous individuals, perfect ex-ante coordination in favor of adaptive learning and simple interaction of heterogeneous agents. One strength of the ABC method is that it allows for the endogenous arising of bubbles, typically observed in housing market.

In this paper we connect a simple ABC model with some typical elements of New Keynesian Dymanic Stochastic General Equilibrium (DSGE) analysis although we are aware that this model cannot be comparable with a pure DSGE model because of some variables are considered exogenous and not microfounded.

Shiller (1981) was the first who emphasized the fact that the movements in the stock prices were much more marked with respect to the relative changes in dividends. From that moment a huge amount of literature was focused on asset price movements and irrational exuberance as well as fundamentals as driving forces of price dynamics (De-Long et al. 1990; Barberis at al. 2001). Furthermore Shiller emphasized the fact that the same forces of human psychology driving financial markets could also have the potential to affect other markets: this seems to be true for housing market.

The recent boom-burst development appears to be anything never seen before, since the late $1990s$ a dramatic increase in housing price has been observed all over the world. For example, London real house price tripled during the period $1996 - 2008$, also in the United State the housing prices increased by 85 percent roughly during the same period. It seems impossible to explain this phenomena merely on a rational point of view because fundamentals such as real rents or construction costs do not match up with this incredible price boom. The speculative thinking and the use of non rational expectations deriving from market psychology are elements that play an important role in determining house prices.

In this paper houses are seen as assets that can be driven by fundamentals and by "animal spirits". Starting from this point, the possibility to predict the future changes in house prices and the deviations between housing prices and fundamentals create opportunity of large gains. We want to stress the importance of the behavioral approach and bounded rationality. Note that ample empirical evidence exists to show that human agents generally act in a bounded rational way (Kahneman et al. 1986, Smith 1991).

Before the 2007 crisis the literature about house price dynamics was poor, except for Iacoviello (2005), who developed a business cycle model with houses as collateral, and Lustig and van Nieuwerburgh (2005), who focused on the role of housing collateral on stocks pricing. The recent fluctuations in housing market have increased the interest of researchers in this field but it is still difficult to explain the large and rapid rise and fall in housing price using a purely rational model.

Some recent papers use models of learning to explain the observed phenomena. Burnside, Eichenbaum and Rebelo (2011) adopted a model in which agents have heterogeneous expectations about long-run fundamentals but are also influenced by infectious optimism, a "social dynamics", that vanishes as soon as people become certain about fundamentals. Adam, Marcet and Kuang (2011) developed a model which is able to replicate quantitatively the house price dynamics from 2001 to 2008 in the $G7$ economies as well as the associated current account, relaxing the rational expectations hypothesis and allowing households to be uncertain about how house prices are related to the economic fundamentals. To reach this goal, they use the concept of "internal rationality", previously developed by Adam and Marcet (2010,2011), where utility maximizer agents do not fully understand how price are formed, so that their subjective probability distribution about prices may not exactly be equal to the true equilibrium distribution.

Following Adam, Marcet and Kuang we do not apply rational expectations and, instead of using a Bayesian learning, we develop a model in which the price formation is created by an Agent-Based[1] mechanism of chartism and fundamentalism, where agents use adaptive learning rules and the continuous evaluation of those strategies according to past performance: this leads to changes in the size of the different groups and finally to the price dynamics.

Our approach is inspired by recent works on Agent-Based financial market models in which the dynamics of financial market depends on the expectation formation of boundedly rational heterogeneous interacting agents. Households are maximizing agents: they can be either chartists, believing the house price trend to continue, or fundamentalists, expecting mispricing will be corrected by the market. When chartists dominate the market, house price can sharply deviate from the underlying fundamental value but, if the "animal spirits" change, the market will be dominated by fundamentalists and the price will converge to the fundamental value.

The reason for this choice is that we want to take into account the households' believes and the psychological variables during the recent housing boom. As stressed in Piazzesi and Schneider (2009), who present evidences from the Michigan Survey of

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[1]To have a survey on the Agent-Based Computational models visit the Tesfatsion website, www.econ.iastate.edu/tesfatsi/ace.htm

Consumers[2], the percentage of the households, believing it was a good time to buy a house because price would be raised further, increased towards the end of the boom. The mechanism of chartism and fundamentalism is one of the simplest method to take into account two different strategies but it also sufficient to create endogenous movement in house price due to the different sizes of this groups.

Adam, Marcet and Kuang (2011) also discuss the role of the interest rate during this crisis: the house price boom would be caused by the persistent reductions in the interest rate. They suggest that for the U.S. economy the boom would have been largely avoided if the interest rate had fallen by less at the beginning of the 2000's. Here we analyze the effect of a policy that takes into account the deviation and the volatility of the house price and the result is similar. It is possible to reduce the price volatility by connecting the interest rate to the house price.

The paper is constructed as follows. The model is developed in section 2 where we explain the Agent-Based price formation and derive the optimality conditions for households and house builders and describe the time of actions and the interactions between demand and supply. The study of the simulations and the impulse response functions is done in section 3. Section 4 shows the capability of the model to replicate the recent house price dynamics taking into consideration the real interest rate dynamics, the credit tightness and a preference shock on house demand. In section 5 we use a particular monetary policy rule to study the impact of a policy that links the interest rate to the house price. Section 6 concludes.

2   The Model

The model essentially reflects the one of Adam, Marcet and Kuang (2011) but it moves away from it in the type of expectations we adopt. Households use backward-looking expectations to infer about future price. We consider house like an asset and, in so doing, it seems suitable to express the expectations starting from the financial Agent-Based literature[3]. Households are utility maximizers belonging to two different groups: agents thinking the trend on house price services will continue in the next periods and others thinking the market will lead the price in the direction of the "perceived" fundamental value. The perceived fundamental value is the long run house price but it can be different from its steady state value. How it is formed will be specified later on.

In the model the size of the two groups is not fixed but it changes across time

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[2]http://www.sca.isr.umich.edu/
[3]See Hommes (2006), LeBaron (2006) or Westerhoff (2008)

according to past strategy performance and this mechanism is able to generate endogenous waves of chartism and fundamentalism that could move the price away from its fundamental value.

Another important difference with Adam, Marcet and Kuang (2011) is the time of actions: demand and supply are not simultaneous and the house price does not emerge from the equality between supply and demand. Indeed households solve their problem daily, which we suppose to be the smallest fraction of time for the real economy. House builders, that reflect the supply side, are slower, and operate on a quarterly basis using as a reference price the average of the past quarter house price. They can also influence future price, changing the perceived fundamental value in the households mind. This particular choice for timing is due to the necessary time that elapses during the construction of a house.

2.1 An Agent-Based approach to House Price

In this section we analyze the price mechanism formation following Lengnick and Wohltmann (2010). The house price Q_t is driven by the different expectations of agents. Chartists (c) and fundamentalists (f) influence the price formation through their demand, which is determined by solving the daily households maximization problem. The total amount of agents using a certain type of expectations is not fixed but it varies over time according to the evaluation of past performances. This mechanism creates an endogenous environment with booms and bursts.

The law of motion for the house price is given by:

$$\hat{Q}_{t+1} = \hat{Q}_t + a(W_t^c \hat{h}_t^{d,c} + W_t^f \hat{h}_t^{d,f}) + \varepsilon_t^Q \tag{1}$$

it can be interpreted as a market maker scenario, where prices are adjusted according to excess demand.

$\hat{Q}_{t+1}$ is the price percentage deviation from its steady state at $t+1$ and it is driven by the past price deviation $\hat{Q}_t$, and respectively by the demand deviation of chartists $\hat{h}_t^{d,c}$ and fundamentalists $\hat{h}_t^{d,f}$ from the steady state. The demand functions are obtained by solving a maximization problem and log-linearizing it around the steady state. The log-linearization allows us to read the real fundamental value as the steady state and equalize it to zero for sake of simplicity. The demand of each group is a function of their respective expectations.

W_t^c and W_t^f are the fractions of agents adopting the two strategies, they vary over time but the total amount of the population is normalized to one; a is a parameter that reflects the impact of the demands on the price formation. The noise term ε_t^Q is i.i.d. normally distributed with standard deviation σ_Q^2 and reflects the fact that the

two strategies are not the only possible strategies that exist into the market. Thanks to the calibration of σ_Q^2, the variance of the simulated quarterly price is the same as the variance of real quarterly price, collected by Federal Housing Finance Agency[4].

Chartists expect the price trend will continue, consequently their expectation is:

$$E_t^c[Q_{t+1}] = Q_t + l^c(Q_t - Q_{t-1}) \tag{2}$$

where the parameter l^c governs the expected trend.

Fundamentalists believe that a fraction of the actual mispricing will be corrected in the future:

$$E_t^f[Q_{t+1}] = Q_t + l^f(Q_t^{fd} - Q_t) \tag{3}$$

the parameter l^f is the amount of the mispricing that fundamentalists expect to be corrected in the next period. Q_t^{fd} is the perceived fundamental value and it will be specified by the firms problem, it represents the long term house price but, following Lengnick and Wohltmann (2010), it can be different from the steady state value.

The amount of agents using a certain type of expectation is not fixed. Households are allowed to learn about the past, changing their believes according to previous performances. Therefore each group evaluates the attractiveness of an action using the following rule:

$$A_t^i = [\exp(Q_t) - \exp(Q_{t-1})]h_{t-2}^{d,i} + \eta A_{t-1}^i \qquad i = c, f \tag{4}$$

the parameter $0 \leq n \leq 1$ is a memory parameter that defines the strength with which agents discount past actions.

The fraction of agents that adopt a particular strategy is not fixed but it is updated thanks to the Gibbs Probability

$$W_t^i = \frac{exp(eA_{t-1}^i)}{\sum_i exp(eA_{t-1})} \qquad i = c, f \tag{5}$$

The more attractive a strategy is, the higher is the fraction of agents using it. The parameter e, called the rationality parameter, reflects the intensity of choice. The higher is e, the greater will be the change in the size of agents that adopt the strategy with the highest attractiveness.

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[4]http://www.fhfa.gov/

2.2 The Households' Problem

Households are maximizing agents that consume and invest. They are allowed to borrow from banks subject to a borrowing constraint as in Kiyotaki and Moore (1997).

The economy is populated by a unit mass of households with identical preferences but different believes, they can be chartists $E_t^c(\cdot)$ or fundamentalists $E_t^f(\cdot)$ concerning expectations about future house price. They take daily decisions (t stands for days) and maximize an inter-temporal utility function:

$$E_t^{c/f} \sum_{t=0}^{\infty} \delta^t (c_t + j_t \log h_t) \tag{6}$$

where $c_t > 0$ is the daily consumption of goods, h_t is the consumption of house services, $\delta \in (0,1)$ is the discount factor, and j_t is a variable that reflects a preference shock for house demand.

The household maximization is subject to the following budget constraint:

$$c_t + [h_t - (1-d)h_{t-1}]Q_t + R_t b_{t-1} + k_t = y_t + b_t + k_{t-1}p_t \tag{7}$$

we denote the house price at time t with Q_t, $d \in [0,1)$ is the rate at which house depreciate, b_t is the households' new loans and R_t is the gross real interest rate maturing on loan b_{t-1}. In the first two section of the paper we consider the steady state value of the interest rate as the daily transformation of the 30-Year Conventional Mortgage Rate in $Q1 - 2004$[5] and R_t as the percentage deviation from it. In section 5 we modify this assumption in order to give some policy suggestions. In addition we use an exogenous process for the income y_t, $k_t \geq 0$ is the capital sold to house builders who use it as an input to produce new houses and this capital fully depreciate in one period. Its remuneration is p_t.

Following Adam, Marcet and Kuang (2011) households have the possibility to borrow from banks and they are subject to a borrowing constraint

$$b_t \leq \theta \frac{Q_t}{R_t} h_t \tag{8}$$

The parameter θ represents the share of assets that can be collateralized, it is fixed and cannot exceed the house value after the depreciation: hence $\theta \in (0, 1-d]$. As in Kiyotaki and Moore (1997) a value of θ lower than one reflects the cost the lenders have in case of default. If the house price tends to grow, the collateral constraint will be relaxed implying that the households will have greater access to credit.

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[5]Data are taken from http://www.stlouisfed.org/

2.2.1 The Solution of Households' Problem

In this section we show the solution of households' maximization problem assuming that the utility from consumption is bounded for high level of c. The first order conditions are necessary and sufficient conditions to achieve a maximum due to the linearity of the constraint in the households' choice variable and the concavity of the objective function.

Households maximize their utility function (6) subject to the budget and borrowing constraints (7-8).

The maximization problem is:

$$\max_{c_t,h_t,b_t,k_t} E_t \sum_{t=0}^{\infty} \delta^t \left\{ \begin{array}{c} (c_t + j_t \ln(h_t)) + \\ -\lambda_t (c_t + (h_t - (1-d)h_{t-1})Q_t + R_t b_{t-1} + k_t - y_t - b_t - k_{t-1}p_t) + \\ +\gamma_t (\theta Q_t h_t - R_t b_t) + \mu_t c_t + \kappa k_t \end{array} \right\}$$

where p_0, k_{-1}, b_{-1} are given initial conditions.

The first order conditions with respect to c_t, h_t, b_t and k_t are:

$$(\partial c_t) : 1 - \lambda_t + \mu_t = 0 \quad (\mu_t \geq 0; \quad \mu_t c_t = 0) \tag{9}$$

$$(\partial h_t) : \frac{j_t}{h_t} - \lambda_t Q_t + (1-d)\delta E_t^{c/f} \lambda_{t+1} Q_{t+1} + \gamma_t \theta Q_t = 0 \tag{10}$$

$$(\partial b_t) : \lambda_t - R_t \delta E_t^{c/f} \lambda_{t+1} - \gamma_t R_t = 0 \quad (\gamma_t \geq 0; \quad \gamma_t (\theta Q_t h_t - R_t b_t) = 0) \tag{11}$$

$$(\partial k_t) : -\lambda_t + \kappa_t + \delta E_t^{c/f} \lambda_{t+1} p_{t+1} = 0 \quad (\kappa_t \geq 0; \quad \kappa_t k_t = 0) \tag{12}$$

Assuming that the non-negativity of consumption holds ($\mu_t = 0$) and $R_t \delta < 1$, households will borrow as much as possible: hence the borrowing constraint is binding ($\gamma_t > 0$). From equation (9) $\lambda_t = 1$; therefore from (11) $\gamma_t = \frac{1}{R_t} - \delta > 0$.

Using these results, from equation (10) it is possible to derive the demand for new houses:

$$h_t^d = j_t \left[\left(1 + \delta\theta - \frac{\theta}{R_t}\right) Q_t - (1-d)\delta E_t^{c/f} Q_{t+1} \right]^{-1} \tag{13}$$

The demand functions for each type of household will be specified in the following section.

The optimal level of borrowing can be derived from the borrowing constraint and is equal to:

$$b_t = \frac{\theta Q_t h_t}{R_t} \tag{14}$$

The capital offered by the consumers to house builders is only restricted to satisfy:

$$(1 - \delta p_{t+1})k_t = 0$$

so that either $p_t = \delta^{-1}$ or $k_t = 0$. This means that if the non-negativity constraint is non-binding, capital and consumption are not uniquely determined and agents are indifferent between increasing slightly the capital sold to firms at time t in exchange for δ^{-1} more units of consumption at $t + 1$. And since firms have a positive demand for k market, market clearing occurs at

$$p_t = \frac{1}{\delta} \tag{15}$$

with capital supply offered by consumers being perfectly elastic, so that k_t is determined by firm's demand.

Finally consumption can be obtained residually plugging (15) into the flow budget

$$c_t = y_t + b_t - \left(h_t - (1 - d)h_{t-1}\right)Q_t - b_{t-1}R_t - k_t - k_{t-1}\delta^{-1} \tag{16}$$

2.3 Housing Supply

House builders operate quarterly (q), the difference in action timing among households and house builders reflects the time that elapses in creating new houses. The house builders employ capital as input received by households in a competitive market and thanks to a decreasing production function

$$(\alpha\delta)^{-1}k_q^\alpha$$

they create new houses. k_q is the sum over a quarter of the daily capital received from household and $\alpha \in (0, 1)$. We also postulate that the market for input is always in equilibrium: therefore the price for capital is $p_t = p_q = \delta^{-1} \quad \forall t, q$.

The maximization problem is:

$$\max_{k_q \geq 0} E_q \left(\tfrac{1}{\alpha\delta}k_q^\alpha Q_{q+1} - \delta k_q \right)$$

The first order condition is:

$$k_q = (E_q Q_{q+1})^{\frac{1}{1-\alpha}} \tag{17}$$

House builders maximize profits but they do not know the future demand. We formulate house builders expectations in a stationary manner $E_q[Q_{q+j}] = Q_q \ \forall j = 1, 2...$ Therefore the profit-maximizing input choice becomes

$$k_q = (Q_q)^{\frac{1}{1-\alpha}}$$

substituting the optimal capital level into the production function we obtain the quarterly house supply

$$h_q^s = (\alpha\delta)^{-1} Q_q^{\frac{\alpha}{1-\alpha}} \tag{18}$$

Since it seems impossible the households are able to identify the true fundamental price, we assume they use a perceived fundamental price. The idea is agents use a function of the current supply as a proxy for this value, such that

$$Q_t^{fd} = (h_q^s)^z \qquad q = floor\left(\frac{t-1}{64}\right), \quad z > 0 \tag{19}$$

The function $floor(\cdot)$ rounds its argument to the nearest integers less than or equal to the argument itself.

The intuition for this choice is that, in this way, the perceived fundamental price becomes a long term variable but it is also biased in the direction of the most recent real economic activity, that is if output is high (low) the fundamental house price is perceived to lie above (below) its true counterpart.

2.4 The Log-Linearized Model

In this part we present the log-linearized model around its steady state, a conditions in which a dynamic system is not influenced by the time. In our case this implies that the timing of actions does not matter. Equalizing demand and supply in a timeless fashion we can find the true fundamental value for the house price and then the percentage deviations of the price from its steady state.

The main steady state equations for our purposes are:

$$h^d = \frac{j}{Q\left(1 + \delta\theta - \frac{\theta}{R} - (1-d)\,\delta\right)} \tag{20}$$

$$h^s = \frac{1}{\alpha\delta}Q^{\frac{\alpha}{1-\alpha}} \tag{21}$$

Equalizing (20) and (21), and solving for Q we obtain the true fundamental value for house price:

$$Q = \left(\frac{j\alpha\delta}{1 + \theta\delta - \frac{\theta}{R} - (1-d)\delta}\right)^{1-\alpha} \tag{22}$$

Now we are able to find the log-linearized equations:

$$\widehat{h}_t^d = \widehat{j}_t + \frac{Qh^d}{j}\left[(1-d)\delta E_t^{c/f}\widehat{Q}_{t+1} - \left(1 + \delta\theta - \frac{\theta}{R}\right)\widehat{Q}_t - \frac{\theta}{R}\widehat{R}_t\right] \tag{23}$$

$$\hat{h}_q^s = \frac{\alpha}{1-\alpha}\hat{Q}_q \tag{24}$$

$$\hat{Q}_t^{fd} = z(\hat{h}_q^s) \tag{25}$$

The demand function in (23) depends positively from the expected price and from the preference for houses but negatively from the current price and from the interest rate. The supply is a positive function of the quarterly price and finally the fundamental perceived price is positively related to the supply.

Inserting (2) and (3) into (23) we can write the two demand functions, for chartists

$$\hat{h}_t^{d,c} = \widehat{j}_t + \frac{Qh^d}{j}\left[(1-d)\delta\left(\hat{Q}_t + l^c(\hat{Q}_t - \hat{Q}_{t-1})\right) - \left(1 + \delta\theta - \frac{\theta}{R}\right)\widehat{Q}_t - \frac{\theta}{R}\widehat{R}_t\right] \tag{26}$$

and for fundamentalists

$$\hat{h}_t^{d,f} = \widehat{j}_t + \frac{Qh^d}{j}\left[(1-d)\delta\left(\hat{Q}_t + l^f(\hat{Q}_t^{fd} - \hat{Q}_t)\right) - \left(1 + \delta\theta - \frac{\theta}{R}\right)\widehat{Q}_t - \frac{\theta}{R}\widehat{R}_t\right] \tag{27}$$

2.5   The Time of Actions

The actions of households and house builders are not synchronized, the former operate daily and the latter quarterly, and this because a house needs time to be built. For this reason demand and supply run on different time scale. We assume that a quarter is composed by 64 days; therefore households perform 64 times, solving their maximization problem and finding the demand for houses, within one increment on house
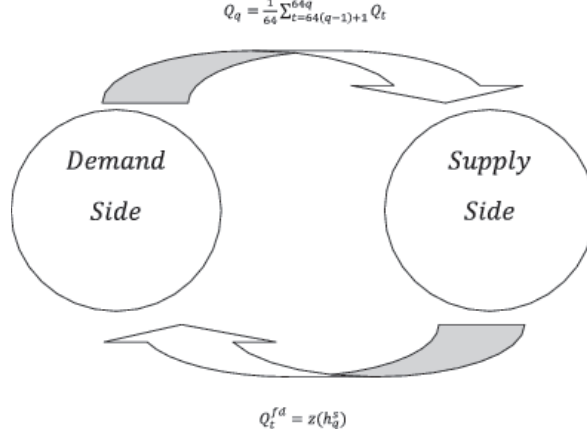
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Figure 1: Channels between demand and supply side

supply's time index q. Hence the model is implemented as follows: we run the daily demand for a quarter, then we take the mean of the daily price over that quarter finding the quarterly price, and we insert it into the supply equation to find the reaction of house builders. Note that the average of house price is determined at the end of each quarter.

A quarter is defined to contain a days $64(q-1) + 1, ..., 64q$.

$$\hat{Q}_q = \frac{1}{64} \sum_{t=64(q-1)+1}^{64q} \hat{Q}_t \tag{28}$$

therefore house supply for the next quarters will be $\hat{h}_q^s = \frac{\alpha}{1-\alpha} \hat{Q}_q$. Note that this variable is the end-of period supply that will remain fixed for the next quarter. From this value the fundamentalists infer the perceived fundamental value using:

$$\hat{Q}_q^{fd} = z \left(\hat{h}_q^s \right) \tag{29}$$

According to this mechanism the demand and supply side influence each other (figure 1): households influence house builders via the daily price formation generated by the chartist/fundamentalist dynamics in (28) and, on the other hand, house builders affect the demand via the perceived fundamental value in (25).

3 Model Simulation

In this part of the paper we analyze the performance of the model. First of all we investigate the capability of the model to generate endogenous waves of chartism and fundamentalism driving the price up and down. In so doing we take an agent-based perspective. The analysis of our model is performed by means of numerical simulation. Afterwards we will try to isolate the impulse response functions of the system to an increase in house preferences.

The parameter calibrations are reported in the table below

Macro parameter	Agent-based parameters
$\alpha = 0.5$	$a = 0.001$
$\delta = 0.96$	$l^c = 0.04$
$\theta = 0.26$	$l^f = 0.04$
$d = 0.03$	$n = 0.975$
$z = 0.5$	$e = 650$

Tab 1: *Calibration of the model*

The parameter values are set according to the baseline calibration used in Adam, Marcet and Kuang (2011) and in Lengnick and Wohltmann (2010) with some differences. In particular the rational parameter e is higher than the one presented in Westerhoff (2008) because of the difference between the financial and housing market. Indeed the former allows greater earnings due to the high frequency of the market, whereas in the latter gains are possible only with greater rationality because of less transactions.

Another difference is in the parameter a that links the demand for houses and the price. This parameter reflects the fact that to have a considerable change in house price the excess demand has to be high. Finally we choose the parameter that relates the house supply to the perceived fundamental value to be $z = 0.5$.

3.1 Waves of Chartism and Fundamentalism

Following Lengnick and Wohltmann (2010), to show the action that the two types of expectations exert on the house price, we simulate a "representative" run for a period consisting in 40 quarters. We have two different sources of shock in the model: the noise term on the house price and a possible increase in the preference for houses. In this

part of the paper we keep $\hat{\jmath}_t$ equal to zero for the whole period, meaning that there is no preference shock. In this way we analyze only the response of the system to a repeated draw realization of the noise term ε_t^Q. The reason driving this exercise is that usually the Agent-Based simulation is doing with one draw for each period from a pseudo random number generator to discover an hidden order that emerges spontaneously from the model. The implementation of a DSGE model, on the contrary, studies the response of the system to a single exogenously imposed realization of a noise term.

Figure 2 shows the dynamics of the relevant variables: the top left panel displays the quarterly house prices; the top right shows the quarterly house supply. The daily house price along with the perceived fundamental value are exhibited in the middle of the plot whereas waves of chartism (green) and fundamentalism (yellow), called *animal spirits,* are presented below. Finally the bottom left panel displays the optimal borrowing and the bottom right panel exposes the optimal demand (the last two variable are founded as a weighted sum of the daily chartists and fundamentalists borrowing and demand).
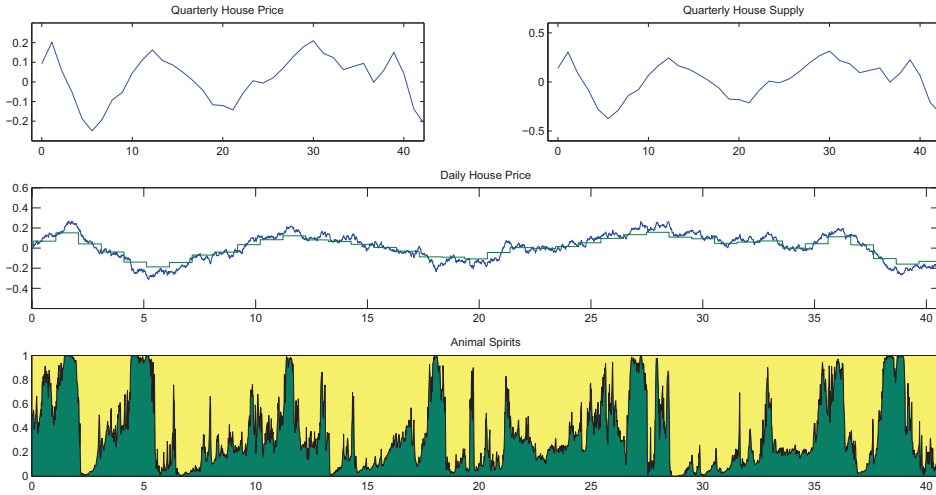

Figure 2: Representative run of the relevant variables

The two strategies dominate the market from time to time, but the continuous evaluation of past results and the endogenous competition among them assure that none dominates forever. It is evident that fundamentalists dominate for most of the time but in some particular periods the optimal strategy becomes chartism. When chartists prevail, the house price departs from its perceived fundamental value, in

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particular this movement is strong for $q = 0-2$ or $q = 26-27$ with a positive increment in price (a boom) or for $q = 4 - 6$ or $q = 17 - 18$. In the latter case chartists expect a negative trend and hence they create a burst. In phases dominated by fundamentalists, on the contrary, the house price tends to go back to its fundamental value, which is evident for $q = 7 - 9$. The quarterly house price is smoother than the daily one and the supply follows the path of the quarterly house price because there are not other shocks affecting the preference for houses.

3.2   Impulse Response Functions

In this section we analyze how house price reacts to a positive exogenous preference shock via impulse response function. We try to isolate this impact to better study its effects in the following way:

1. Generate the model dynamic with $\varepsilon_t^j = 0 \ \forall t$

2. Generate the same dynamic with the identical realizations of ε_t^Q and with $\varepsilon_t^j = 1$

3. Calculate the difference between the trajectories of step 1 and 2 which gives the isolated effect of the preference shock.

   In this experiment we are interested in the impact of an unanticipated transitory preference shock in $t = 128$, $\varepsilon_{128}^j = 1$, with a daily persistence $\hat{j}_t = \rho \ \hat{j}_{t-1} + \varepsilon_t^j$ with $\rho = 1$ in $t = 128$ and $\rho = 0$ otherwise.
   Figure 3(a) shows the resulting responses of the daily and the quarterly house price to an exogenous shock: a daily shock on house preferences increases the daily house price on impact and the effect persists about 20 quarters although the size is very small. Looking at the quarterly variable the rise in price is not instantaneous, and this is due to the way we have defined this variable, which is an average of daily prices. What we have noticed with this exercise is the quarterly variable shows more persistence than the daily one: indeed, the impact of a positive preference shock in $t = 128$ (i.e. the last day of the second quarter) increases the quarterly price but the maximum level is reached in the next quarters. This process can be explained by the endogenous mechanism of the Agent-Based model, and in particular with the backward-looking expectations inside the model: the positive effect of the shock does not vanish immediately but influences agents' behavior for more periods. The second experiment extends the length of the shock to a quarter, because a daily shock seems not to have a particular economic meaning. In this case the exogenous process for houses' preferences $\widehat{j}_t = \rho\widehat{j}_{t-1} + \varepsilon_t^j$ has been modified with $\rho = 1$ from $t = 128$ to $t = 192$ and $\rho = 0$ otherwise, and $\varepsilon_{128}^j = 1$.
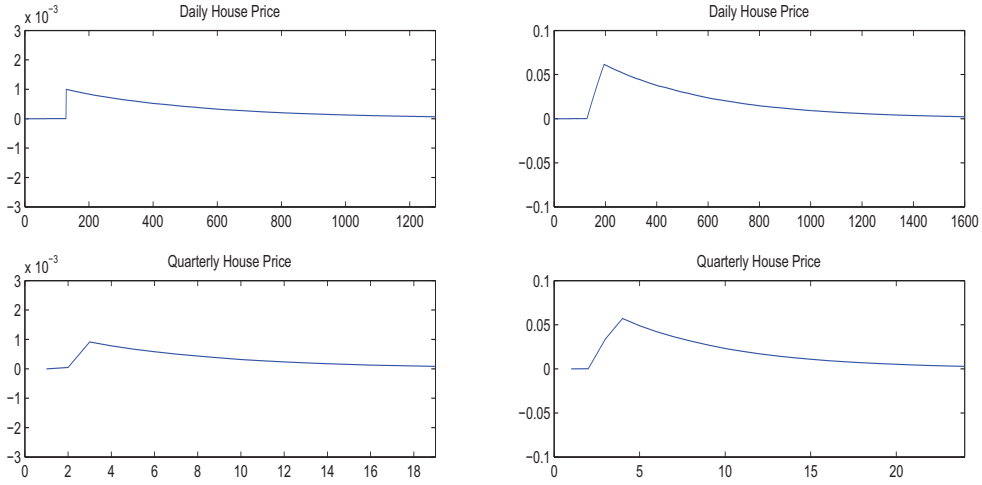   In figure 3(b), three main differences emerge with respect to the previous one:

Figure 3: *(a)* Daily shock to house preferences and *(b)* Quarterly shock to house preferences

- The daily variable increases but not at impact: the maximum is reached at $t = 192$ when the shock vanishes.

- The size of the shock is much more important, roughly from 0.001 to 0.05: this movement is due to the backward-looking expectations hypothesized in the model.

- The movement of the variable toward the steady state is much more prolonged (it takes more than 20 quarters).

This second exercise gives greater emphasis on the role of expectations: since they are backward-looking the persistence of the shock is amplified due to a learning mechanism.

4 Matching Real Data

The aim of this section consists in explaining what are the main driving forces acting on house price dynamics. It is known from literature that house price are volatile relatively to fundamentals such as interest rate. Moreover this dynamics cannot be explained by

means of pure rational model. Other contributions, based on non-rational expectation, are able to match the data quite well: see, for example, Adam, Kuang, Marcet (2011).

House prices are usually connected to interest rates or credit availability, as in Favilukis, Ludvigson and Van Nieuwerburgh (2010). They developed an overlapping generation model in which heterogeneous households face limited risk-sharing opportunities as a result of incomplete financial markets; they focus on the macroeconomic consequences of three systemic changes in housing finance, with an emphasis on how these factors affect risk premia in housing markets, and how risk premia, in turn, affect house prices. Their results show that credit tightness can be a driving force of boom and bursts on house price because it has a huge impact on risk premia whereas the interest rate does not have such an influence.

Moving from these results, we take into account also behavioral features because we think that changing in agents' believes may play a significative role on housing dynamics. To be more precise we consider shocks on preferences. We look at the Michigan Consumers Surveys, more specifically at the quarterly table showing the Buying Condition for Houses as a proxy to calibrate the demand shock for houses.

Our attempt is to match quarterly house price for the period going from $Q1 - 2004$ to $Q1 - 2009$. We use the technique adopted in constructing the impulse response function, that is we look at the difference between the response of the system to the various driving forces and its value in $Q1 - 2004$. The data, Seasonally Adjusted Purchase-only Index, are taken from the Federal Housing Finance Agency and to make our approach consistent, we compute the percentage deviation of the real house price with respect to its value in $Q1 - 2004$.

First of all we consider separately the three different elements that can be driving factors of the recent boom and burst: these are the percentage change in 30-Year Conventional Mortgage Rate, a decrease in the value of parameter θ, that can be seen as a proxy of credit tightness and an exogenous preference shock based on the dynamics of the Michigan Consumers Surveys. Then we look at these three possible causes unitedly.

In so doing we define the steady state value of the interest rate as the daily transformation of the 30-Year Conventional Mortgage Rate in $Q1 - 2004$ and we look at the percentage deviation from it, as figure 4 shows.

The percentage deviation is quite high from 2004 to 2006 but it moves down in the following years, and it sharply increases in 2009. Noteworthy that the percentage change in this variable is small. We fit this series into the demand function to check how the price reacts. Results are shown in figure 5: the reaction (red) is very small compared with real data (blue); moreover in the small box, where the fitted prices are shown more clearly, it is possible to note that the series is always increasing.
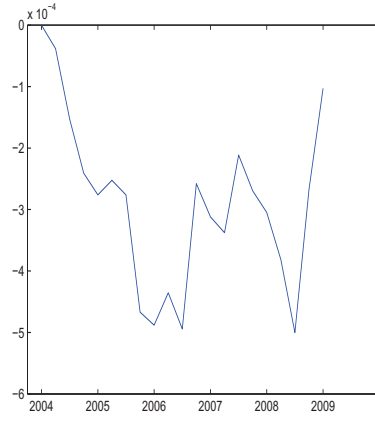
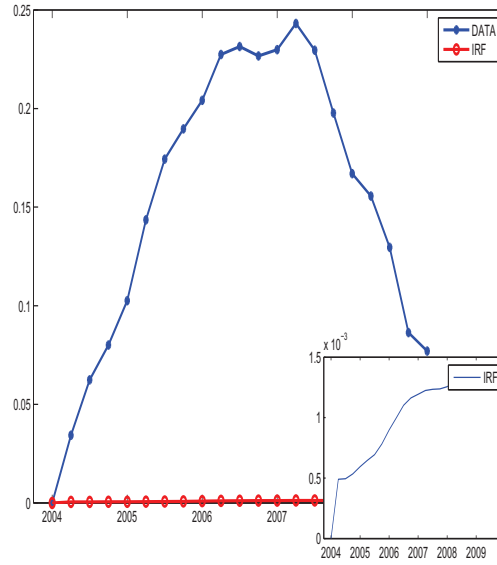Figure 4: Percentage deviation of 30-Years Conventional Mortgage Rate from

Figure 5: Model reaction to interest rate change

Concerning the tightness of credit, we take the parameter θ in the borrowing constraint as a proxy for it. We calibrate this parameter to be equal to Iacoviello (2005): $\theta = 0.55$. Then we consider "The January 2012 Senior Loan Officer Opinion Survey on

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Bank Lending Practices" [6], especially the Net percentage of banks reporting tightening credit standards for US. (see figure 6) From 2004 to the third quarter of 2006 access to credit has remained stable, whereas from that date on credit tightness increased sharply.
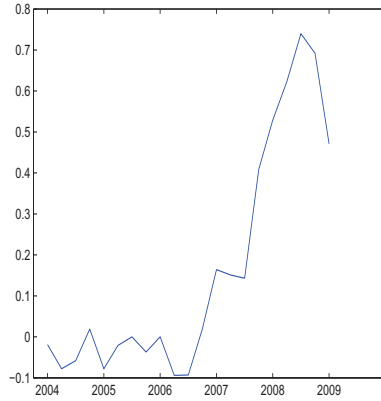

Figure 6: Net percentage of banks reporting tightening credit standard

We define θ from $Q1 - 2004$ to $Q1 - 2009$ as follows: $\theta = 0.55 - 0.55 * tight\ credit$, because the credit availability is an increasing function of θ. Results (see figure 7) do not match the data because the size of change in fitted price is much smaller than real house price. The small box reproduces this dynamics which is very high up to the second quarter of 2004 and then continuously decreases. The rationale behind this behavior comes from the impact of the changing θ into the demand equation. Indeed when θ decreases there are two different effects generated by the impact of house price and interest rate on the demand, the former has a negative effect whereas the latter a positive one. The second effect is offset by the first one: therefore demand decreases. This mechanism generates a reduction on house price as well. Anyway in our model we can't consider this element as the driving force of price dynamics.

Finally we consider the shock on house preferences $\widehat{j}_t$. We look at the Michigan Consumers Surveys, more specifically at the quarterly table showing the Buying Condition for Houses as a proxy to calibrate the demand shock for houses. Figure 8 summarizes the answers at the following question: "Generally speaking, do you think now is a good time or a bad time to buy a house?" We focus on the percentage of positive answers,

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[6]http://www.federalreserve.gov/boarddocs/snloansurvey/201201/default.htm
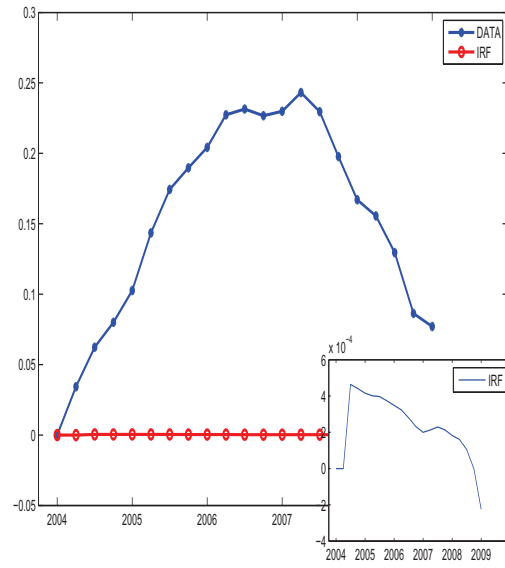
Figure 7: Model reaction to credit tightening change

transforming the series in a way to have figures in the subset $(-1, 1)$, with the steady state value of the parameter j being equal to 0.3063 which is the mean over the whole sample.
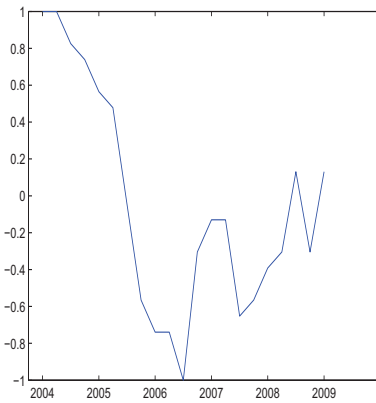

Figure 8: Preference shock

As we can see from figure 8, values of the series higher than 0.3063 mean a positive preference shock, whereas we observe the contrary for lower values. In particular the positive trend goes from 2004 to the first half of 2006 and the negative trend starts in the second half of 2006 and it lasts until 2009.
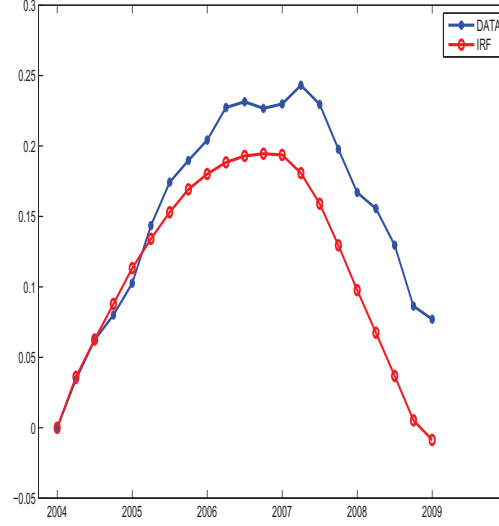

Figure 9: Model reaction to preference shock

Figure 9 shows the response of the system to an exogenous preference shock calibrated using the Michigan Consumers Surveys. Our model economy is able to replicate quite well the real price dynamics. The two series follow the same path during the first year, the maximum percentage deviation of house price is reached in 2007, nevertheless after this year the fitted series has a steeper decrease than the Seasonally Adjusted Purchase only Index.

The heterogeneous framework gives the right persistence in the house price dynamics, the hump shape of the series is given by the self fulfilled mechanism induced by the backward looking expectation. Indeed in a rational expectations model the inertia is the result of a lag transmission of exogenous shock. In contrast, our behavioral model is capable to reproduce the inertia in the price series without imposing lags in the transmission process. This type of inertia is called endogenous inertia and it is given by an informational problem the agents experience. This is typical of boundedly rational model in which agents do not fully understand the nature of the shock or its

transmission mechanism and therefore they apply a trial and error learning rules.
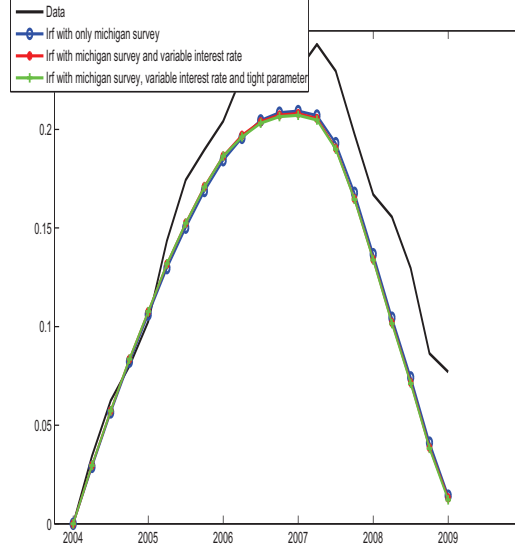

Figure 10: Model reaction to the three effects

The final exercise consists in taking together the three effects (see figure 10): as the blue line shows, the most of the dynamics is generated by the preference shock. Adding the interest rate and the credit tightness effects, we observe only a minimum anticipation of the price movement.

Our analysis emphasizes the importance of the behavioral approach and the selection mechanism among different expectation rules as determinant factors of the boom and bursts cycle in the housing market.

The model matches the data quite well, in particular it captures the moment of maximum percentage increase of the house price starting from psychological studies and empirical surveys. A rational household would have anticipated and discounted any movement in price. We have to stress the importance of incorporating in economic models behavioral features: this is still rather simple in our model but a perfect rational agent, able to forecast all the relevant variables for his actions, seems to be too unrealistic, notably after (and during) this recent crisis[7].

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[7]For recent critiques to rational expectation hypothesis see [5], [7] and [9]

5 Policy suggestions

In this section we try to give an answer to the question: "Could the boom in house price have been avoided if the interest rate had been increased?" The question arises for a simple reason: the growth in U.S. house price coincides with a fall in the ex-ante real interest rate. Adam, Kuang and Marcet (2011) are the first that recognized this problem, and their model predicts the recent house price dynamics would have been avoided and the current account deficit would have been considerably smaller, if the interest rate had fallen by less at the beginning of the 2000's[8].

To answer at the question we bind the real interest rate to the house price and we use it to minimize two different measures that represent the fluctuations of the house price. These are the volatility and the distortions of the house price.

The volatility represents the rate of change in the value of the simulated time series.

$$vol(Q) = \frac{1}{T-1} \sum_{t=2}^{T} |Q_{t-1} - Q_t| \tag{30}$$

The distortion measures the difference between the variable and its true steady state (implicitly set to zero).

$$dis(Q) = \frac{1}{T} \sum_{t=1}^{T} |Q_t| \tag{31}$$

Noteworthy we do not use the variance of the simulated series because this measure interprets the volatility via the average squared distance from the mean, and our time series shows long deviations from the mean (which we interpret as boom and bursts). To avoid confusion we do not use the variance which is usually computed when one is interested in calculating the mean squared distortion. Moreover, for this exercise, we adopt two policy rules, first of all we link the real interest rate to the quarterly house price.

$$R_q = r^q Q_q \tag{32}$$

In the second rule we modify the target making the interest rate to respond to the difference in house price between two subsequent periods.

$$R_q = r^q (Q_q - Q_{q-1}) \tag{33}$$

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[8]See also Himmelberg et al. (2005)

Then we set the preference shock in house demands equal to zero and run the model for 100 quarters, with different values of r^q as well 1000 different realizations of the pseudo random generator for each r^q.
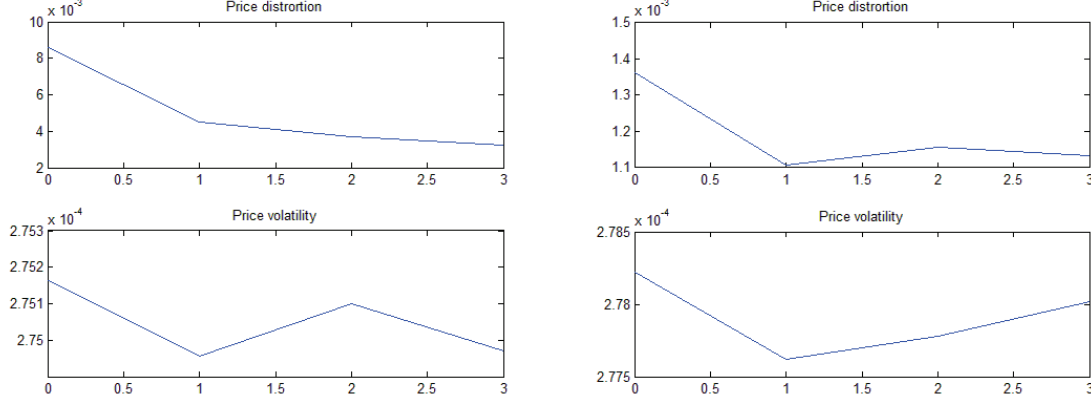

Figure 11: *(a)* House price distortion and volatility with the interest rate reacting at house price and *(b)* House price distortion and volatility with the interest rate reacting at difference in house price

Figure $11(a)$ upper panel summarizes the results on price distortion using (32): on the x-axis we have the value of the r^q parameter and on the y-axis the value of the distortion. Note that the distortion is small and this is due to the Monte Carlo approach that considers the mean of repeated simulations.

We can see that the distortion is minimized for $r^q = 3$ meaning that the interest rate has to react triple on a percentage point deviation of the house price. Other experiments, implying higher values of r^q, show that the distortion is minimized for the maximum value of this parameter and this means a negative correlation between the distortion and the interest rate. Our exercise shows also that most of the distortion is deleted only taking into account a one-to-one reaction of the interest rate to a movement in the house price. Figure $11(a)$ upper, in fact, shows a broken line with the highest slope in the segment between zero and one meaning that a one-to-one response of the interest rate is sufficient to have a huge decrease in price distortion.

Figure $11(a)$ lower panel shows the change in the volatility of prices to different values of the reaction parameter in the interest rate rule (32).

The values representing the volatility on the y-axis are lower than the ones of the distortion and this is due to the meaning of this two measures, the first gives

the discrepancy between two subsequent house prices whereas the second depicts the distance between the house price and its steady state.

The result is quite difficult to read: again we have a broken line but composed by three different segments with different slopes. The volatility decreases in the segment between zero and one reaches its minimum exactly in one then increases until two and then decreases again until three. The minimum is reached for $r^q = 1$ meaning that, taking into account a one-to-one response of the interest rate to a change in price, the government should be able to minimize at the same time volatility and cancel out most of the distortion in house price.

Figure 11(b) shows the results about distortion and volatility using (33). Now the volatility results minimized for $r^q = 1$ and the distortion exhibits the same behavior of the volatility in figure 11(a). Intuitively this happens because of the different target in the interest rate rule. Noteworthy that the distortion in figure 11(b) is lower that the one in figure 11(a) whereas the volatility results to be approximately of the same size. Using the interest rate to respond one-to-one to the difference in house price the government could minimize both volatility and distortion on house price.

Although we know the limits of our results a policy suggestion emerges clearly from the experiments. Taking into account the possibility to use the interest rate to influence the house prices, the government could avoid some dangerous movements at the heart of booms and bursts.

6 Conclusion

We have developed a model to study the housing market starting from an Agent-Based perspective. Relaxing the rational expectation hypothesis and allowing households to have a backward-looking behavior we have shown that it is possible an endogenous creation of bubbles leads the price to long-last deviate from its fundamental steady state. The chartist-fundamentalist mechanism matches real data quite well. The exogenous preference shock, calibrated using the Michigan Consumers Surveys, is the main force driving the system. Adding the interest rate and the credit tightness effects, we can observe only a minimum anticipation of the price dynamics.

The heterogeneous framework gives the right persistence in the house price dynamics, the hump shape of the series is given by the self fulfilled mechanism induced by the backward looking expectation. Indeed in a rational expectation model the inertia is the result of a lag transmission of exogenous shock; in contrast, our behavioral model is capable to reproduce the inertia in the price series without imposing lags in the transmission process.

The model has also some space for policy investigation anchoring the interest rate to house price. The distortion and the volatility of prices can be reduced using an appropriate degree of reaction.

We know this model is still rather simple in incorporating a really psychological foundation of expectations but the mechanism of chartism and fundamentalism is sufficient to create endogenous movement in house price due to the different sizes of this groups and this simple interactive dynamics has a huge influence on the economic system.

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