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An Agent-Based Model of Schumpeterian Competition

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Abstract

The paper presents an Agent-Based extension of Nelson-Winter model of schumpeterian competition. The original version did not provide any insight about the direction of firms' innovative activities and of technological change as a whole. As a result, it lacked an explicit structure governing firms interaction and the shape of externalities. We address these criticisms by taking explicitly into account the structure of technology in use in the industry, that we shape as a directed network of nodes and links: nodes represent technological skills to be learnt by firms looking for 'new combinations' and links represent their reciprocal interdependencies. The network is created in order to reflect the defining properties of Technological Paradigms and Technological Trajectories, as they emerge by evolutive-neoschumpeterian literature. Firms' ability to learn technological skills through imitation of competitors generates spillover effects related to the process of diffusion of innovation. The basic model presented here focuses on a particular aspect of schumpeterian competition: the relationship between industry initial concentration and its overall innovative performance and, *vice-versa*, between innovation process and the evolution of industry structure over time. In this same perspective we also analyze how firms' interactions and the structure of technology concur in determining the success or failure of an innovative strategy. Finally we argue that the model presented here might constitute a flexible framework worthy of further applications in the study of innovation process and technological progress.

1 Introduction

Innovation and Technological progress is increasingly seen as one of the driving forces of economic growth in industrialized economies ([Maddison 1991]). Based on a fast growing empirical literature on different aspect of technological progress process the neoschumpeterian school of thought has

highlighted the fact that technological change takes place along ordered and selective patterns shaped by technological and scientific principles as well as by economic and other societal factors (Verspagen [2007]). Concepts like incremental and radical innovations, technological paradigms and technological trajectories ([Dosi 1982]), natural trajectories ([Nelson And Winter 1982]), techno-economic paradigms ([Perez 2009]) have been developed to capture patterns holding across sectors ([Breschi, Malerba and Orsenigo 2000]). While recognizing the existence of an high degree of heterogeneity in technologies employed and firm sizes among different industries, typical patterns of industry evolution and the general importance and structure of knowledge accumulation processes have been established ([Dawid 2006]).
Many attempts have been done to explore the 'black box' of innovation focusing primarily on the dynamic relationship between the process of generation of new knowledge and its diffusion in the socio-economic system. Concepts like learning by doing ([Arrow 1962], [Romer 1986], [Romer 1990], [Forey 2000])and learning by using ([Rosenberg 1982]) has become very popular also in mainstream theory of growth highlighting the positive feedback effects and the increasing returns associated with innovation process. Many scholars, at the same time, has stressed how firms' learning processes cannot be reduced to a simple by-product of 'doing' since they represent a conscious and expressly focus activity, relying on a variety of source of knowledge both internal and external. Learning by firms is one of the most dynamic processes taking place within industries, implying both a quantitative and a qualitative change in the production process: indeed every learning process yield enhancements in the stock of knowledge and technological capabilities of firms, which in turn generate a whole range of trajectories of technological advance ([Malerba 1992]). A growing attention was also put to the role played by institutional variables and the socio-economic context in which innovation takes place. Indeed notions such as learning by communicating ([Lundvall 1992]), collective learning ([Cassiers and Forey 2002]), national systems of innovation ([Lundvall 1988], [Freeman 1995], [Lundvall 2010]) were mainly addressed to the analysis of learning processes resulting from the interaction of different agents operating in a particular environment. At the same time the study of spillover effects and network externalities have been a central issue in the field of technological/innovation diffusion models[1]
Despite the huge variety of issues addressed, it is still possible to identify a common thread among all the previous fields of research. Indeed they all share a similar interpretation of the nature of technological change and

[1] Following [Geroski 2000] we can classify this set of models in epidemic models, Probit models, legitimation and competition models and information cascaded models. However this classification is not exhaustive. In recent years other types of model have been increasingly used to analyze innovation and technological diffusion such as percolation models and models implying social networks analysis ([Silverberg and Verspagen 2005], [Hohnisch, Pittnauer and Stauffer 2006], [Cantono and Silverberg 2008], [Goyal 2003])).

of innovation process, that we can summarize as follows ([Dawid 2006]): i) Technological change is a dynamic process intrinsically cumulative and path-dependent. The success of innovative activities of a firm depends to a large extent on the size and structure of the knowledge base the firm has accumulated in the past. Innovation process cannot be understood without taking into account the entire process of knowledge accumulation over time. ii) Innovation is affected by fundamental uncertainty ([Dosi and Egidi 1991]). Obviously different degrees of uncertainty are associated to innovation processes depending both on the nature of innovation (for example radical or incremental) and on the industry in which it is carried out. Nevertheless uncertainty is always unavoidable. The success or failure of an innovative process depends on a large extent on aspects that cannot be managed or even foreseen by firms. These sources of uncertainty can be related both to technical aspects regarding the implementation of an innovation and to difficulties in foreseeing the reaction of markets. In turn this implies that it is never possible to assess *ex ante* the superiority of a technological path over another one with certainty. iii) Heterogeneity among agents performing innovative activity is a key feature of technological advance. From an empirical point of view we observe that firms may have different strategies towards innovation even in the same industry. On one hand heterogeneity and complementarity of firms' knowledge base trigger the generation of new knowledge and facilitate the exploration of alternative technological lines of development; on the other heterogeneity actually emerges as a consequence of technological change. iv) Finally the effects of technological change are not simply confined to the production process. Indeed innovation significantly affects the evolution of industry and market structure through selection processes. But this causal link also works in the opposite direction since firms' innovative performance is considerably affected by a number of circumstances related to industry structure, such as firm size and firm's market power. This dynamic relationship between economic and technological evolution has been central in particular inside the neoschumpeterian-evolutive analysis.

The complexity of the arguments just proposed has proved to be difficult to address using traditional analytical tools. This explains why economists, in order to tackle this kind of issues, has largely exploited the opportunities offered by simulation approaches. The use of computer simulation is well established within the neoschumpeterian tradition where phenomena of qualitative change and development are at the front of the research program ([Windrum 1999], [Pyka and Fagiolo 2005]). Actually the very first computational exercises in economics dates back to the seminal work of Nelson and Winter. The inspiring idea of Nelson and Winter's models - developed in the 1970s and then collected in their famous book *An Evolutionary Theory of Economic Change* - was that the intrinsic dynamic process underlying schumpeterian competition, never formalized by Schumpeter in an coherent analytical model, could naturally be translated into a computational process in which firms not only make short-term production and investment decisions but also performs a search for new technologies ([Andersen 1996]). Their joint work led to the emergence of an evolutionary synthesis combining Schumpeter's theory of economic development and business cycles with the work of Simon ([Simon 1947]) on rules and satisficing behavior and the work of Nelson and Alchian ([Alchian 1950])on natural selection. The core of the emerging evolutionary approach was represented by the idea that the properties of an economic systems could not be deduced by simply looking at the properties of its constituent parts, taken alone. The whole was more than the simple sum of its parts. Therefore not surprising that the evolutionary theory has developed over the years a fruitful dialogue with complexity economics that, from a methodological point of view, resulted in increasing adoption of the modeling approaches developed in this research field: based on the concepts of adaptivity, local interaction in well-structured space, procedural rationality and heterogeneity (see [Epstein 1996] and [Epstein 2006]) the so-called Agent Based Models (ABM) have proven to be well suited to the study of innovation processes[2].

The aim of the paper is to contribute to this line of research by rearranging the Nelson and Winter's depiction of Schumpeterian competition in the context of agent-based models. Nelson and Winter wished to examine the simultaneous existence of multiple firms and different innovative behaviors by assuming that initial heterogeneities between firms with respect to their innovation strategies. In particular, they considered an industry which was a mix of imitators (investing only in imitative R&D) and innovators (investing in imitative and innovative R&D). According to their innovative strategy choice firms experimented different innovative performance. Therefore the different paces of capital accumulation led to selection effects of behavior on the industry level. However, the computational model developed by Nelson and Winter only provides a quantitative discussion of the effects of innovation and does not consider the direction of technological advancement of enterprises operating in the industry and the repercussions that this determines on the innovation process itself. Our goal is to show how even companies that follow the same strategy towards innovation could experience different economic performances as a result of their decision to specialize along certain technological paths among all possible ones. In fact such a decision, taken on the basis of simple procedural rules and dependent on their past technological paths (i.e., on their previous accumulation of knowledge),

[2]ABM try to depict economies as complex systems whose aggregate properties (or rather 'emergent' properties) have to be inferred - in a bottom-up perspective - from interactions and behaviors of decentralized micro entities. In turn the macro level can affect significantly the behavior and the structure of the dynamic interactions of agents at a micro level through feed-back effects. For this reason the ABM can rightly be considered as a 'third way' in the debate between micro and macro-foundation of economic theory.

affects firms' ability to perform successfully the imitative activity. In order to learn and to implement in their production processes certain pieces of technological knowledge, firms must first have acquired other pieces of knowledge. This idea is implemented in the model by assuming that the structure of the technology is given at the beginning of the simulation and it can be represented by a network structure defining all the possible paths of technological development that firms can explore.
The paper is organized as follows: Section 2 we present and discuss Nelson and Winter's model of Schumpeterian Competition [Nelson and Winter 1982] which provides the basic structure for the model developed in the present work. Particular attention will be given to the notions of Technological Paradigms and Technological Trajectories in order to highlight the major limits of Nelson and Winter's computational approach. In section 3 the main features of the technology network structure and a description of the stochastic algorithm used to generate it are presented. Then, in the following 2 sections the behavioral rules of agents (with particular attention for those referring to firms' innovative strategies) are specified and the formal structure of the model is defined. The set-up used to calibrate the model under each simulation scenario and the results of the simulations are shown in sections 6 and 7. Finally a brief discussion about some possible further application of the present work is presented. In particular, even if the aim of the present model is purely theoretical, the recent empirical work made by some scholars on patent citation networks, could provide the opportunity to apply the model for the study of specific technology sectors.

2 The Model

The model developed in the present paper is inspired by the work of Nelson and Winter on schumpeterian competition. The authors began to work on this issue by using a computational approach since 1977 [Nelson and Winter 1977 (1)]. Then this job came out into the model presented in Part V, Ch.12 of their 1982 book [Nelson and Winter 1982]. This is the formalization to which we refer in order to develop the basic structure of our model[3].
The model presents a single homogeneous product industry that faces a

[3] Indeed this version of model can be interpreted as representing the Nelson and Winter's fundamental framework for analyzing schumpeterian competition. Starting from this point the author then provided further extensions: in ch.13 of their 1982 book, for example, they modified the model to analyze the so-called schumpeterian trade-off between static and dynamic efficiency. Particularly important is the extension provided by Winter in his later work [Winter 1984]. In this paper the author analyzes the process of schumpeterian competition under different technological regimes. This version of the model, which includes a mechanism of entry and exit form the industry and introduces a degree of adaptivity of the strategies followed by firms on the basis of the results achieved, it is certainly closer to the logic of agent-based modeling.

downward sloping demand curve with constant[4] elasticity.
Following Nelson and Winter we begin by making some simplifying assumptions about the characteristics of technology and of the production processes carried out by companies. Every firm is characterized by the use of a single techniques, the best it knows. All techniques are characterized by constant returns to scale and fixed input coefficients. Each firm uses in each period its whole capacity to produce output. Firms' capacity is constrained by their stock of capital. Given the stock of capital, each firm then purchases the quantity of complementary inputs required. For simplicity reason, the model assumes that all factors supplies are perfectly elastic. Hence all factors prices constant.
Furthermore, since each techniques requires the same complementary inputs per unit of capital, costs of production per unit of capital are constant both across firms and over time. What differs is the output per unit of capital implied by each technique that, given the stock of capital, determines the level of production costs per unit of output (i.e., the unitary costs of production).
Under these assumptions the state of the firm i in a particular period can be characterized by the binary $\{K_i, A_i\}$, where K_i is the stock of capital and A_i represents the productivity of capital. Productivity will vary across firms and over time as 'new combinations' are carried out, allowing firms to implement better techniques with lower unit costs. These new combinations can be interpreted both as organizational and technological innovations. In any case, every innovation, regardless of type, require the generation of new knowledge to be achieved by firms.

2.1 Technological paradigms and technological trajectories

The generation of knowledge is characterized by specific attributes: knowledge is at the same time the output of a specific activity and the input for the generation of new knowledge. In other words knowledge creation and innovation are characterized by high cumulativity and path-dependency. The existing literature has identified at least two ways in which firms can enrich their knowledge base: through the use of the internal resources as well as through the use of resources located externally[5]. The first way is traditionally identified, on one hand with the activity carried out by specific units of the enterprise explicitly addressed to innovation, such as R&D laboratories and, on the other hand with 'internal' learning economies such as *learning by do-*

[4]Actually we assume that the elasticity of prices to quantities is unitary. A more detailed description fo the model will be presented in section 5.

[5]Indeed many scholars has stressed the fact that, more properly, the innovative activity carried out by firms cannot be seen as a process relying only on internal resources. New knowledge is always the product both private and public, codified and tacit knowledge already accumulated in the past (see [Forey 2000]).

ing ([Arrow 1962], [Romer 1986]) and *learning by using* ([Rosenberg 1976], [Rosenberg 1982]). The latter implies the mobilization of external resources, derived from other economic actors. Examples are learning by communicating, learning by interacting ([Lundvall 1992], [Lundvall 1996]) and network externalities. We do not go into details. What is relevant, for the aim of this work, is that innovation is indeed the collective result of the interdependent and interactive action of economic agents, involving the reciprocal —sometimes unintentional— exchange of pieces of knowledge [Cassiers and Forey 2002]. These kinds of interactions are fundamental in shaping the direction of technological advance.

Following Dosi [Dosi 1982], we can interpret technology as *'a set of pieces of knowledge, both directly practical (related to concrete problems and devices) and "theoretical"(but practically applicable although not necessary already applied), know-how, methods, procedures, experience of successes and failures and also, of course, physical devices and equipment'.* Pushing on a parallel with Khun's *scientific paradigms* Dosi defined a *technological paradigm* as *"model" and "pattern" of solution of selected technological problems, based on selected principles derived from natural sciences and on selected material technologies.* Roughly speaking a technological paradigm is defined by the generic task to which it is applied, the material it selects, the chemical-physical properties it exploits and the technological and economic dimensions of the trade-offs it focuses upon. Hence technological paradigms shape the basic structure of the technology at stake, delimiting the boundaries in which the different possible *technological trajectories* will develop. Therefore technological trajectories are clusters of possible technological directions inside the boundaries of a technological paradigm.

The notions of *technological paradigms* and *technological trajectories* play a central role in our model of schumpeterian competition. The original model by Nelson-Winter did not really provide any insight of the way through which an innovation is carried out. Firms could increase their productivity by investing either on innovative R&D activity, targeted to the development of new techniques, or on imitative R&D activity, the latter allowing to copy the 'best practice' (i.e. the one characterized by the highest level of capital productivity) of the industry. Both processes were modelized as a two-stage random process. In every period firms could obtain with a certain probability (depending on their expenditure on the related R&D activity) an innovative and/or imitative draw. Then, in the case of a successful innovative draw, they just sampled from a probability distribution of technological alternatives (i.e. allowing different levels of productivity gains). A successful imitative draw, on the other hand, enable a firm to automatically implement the best practice of the industry.

This approach however relies on a number of implicit assumptions. First of all it assumes that each firm has got perfect knowledge about the techniques in use among all other firms of the industry. Then they are always able to rank them according to the productivity levels they allowed and to identify the best practice of the industry. Finally firms are always allowed - in case of a successful imitative draw - to copy the best practice of the industry, regardless the existent gap between their own level of productivity and the level implied by the best practice and, even more important, regardless their past technological path. In reality firms are not able to know exactly all the possible production techniques in use in the industry. For sure they can improve their knowledge of the feasible techniques by looking at what other firms are doing but this 'scanning' activity generally imply a cost both in terms of financial resource and time. In any case it's difficult to imagine that such efforts can lead them to acquire a perfect knowledge of all industry production techniques.

But, even if this were the case, it does not seem reasonable to assume that *knowing which is the best practice* is sufficient to ensure that they *will be always and immediately able to implement it.* Indeed the technological evolution of a firm is always and deeply characterized by path-dependency. Today innovative choices of a firm not only affect their performance but also 'constrain' their future innovative possibilities. Indeed even if firms' technologies, within a particular industry, were rooted in the same technological paradigm, each one of them would probably experience different technological trajectories according to their original endowment of technological competences, their past technological paths, the technological possibilities offered by their environment, the set of opportunities and constraints defined by the legal, institutional and social context. These trajectories have a powerful *exclusion effect* in the sense that they tend to restrict the innovative efforts in rather precise directions while they appear blind to other technological possibilities. The generality and the strength of trajectories can vary and it seem reasonable to think that there could be complementarities between the different trajectories. However, switching from one trajectory to an alternative one is usually difficult and costly. Hence the ability of an imitative firm to adopt the best practice of the industry is not automatically guaranteed since it depends crucially on the technological distance — or rather, technological consistency — between its current technological skills — determined by its past experience — and the ones required to implement the best practice.

Following the same line of reasoning we can go further in considering the way Nelson-Winter model approach innovative activity. As mentioned earlier, once a firm get an innovative draw, it can sample from a probability distribution of technological alternatives[6], each one allowing a different level of productivity gain. This distribution admits two different specifications,

[6]A log-normal distribution of values of the average productivity of capital. Remember that this is the only relevant productivity since all other production factors are proportional to capital in all feasible techniques.

being its mean allowed to increase over time exogenously at a fixed rate — situation they refer to as 'the science-based regime' — or being centered on the 'prevailing productivity of a firm (the 'cumulative technology' case). In both cases, the final result of this process is simply to provide the firm a 'number' that adds up to its current level of productivity. Again, no insight of the *direction* of technological change is given. Technology is depicted as something neutral, a flat space in which all that matters is the absolute value of the productivity gains that an innovation generates. In such a way we are excluding *a priori* from the model the possibility to analyze particular aspects of the activity dedicated by firms to innovation that can be rich of implications in the context of schumpeterian competition.
These arguments seem to be consistent with the critical review of Dawid [Dawid 2006] who, while recognizing the pioneering role played by Nelson and Winter's 1982 work, points up three major limits affecting their model:

1 The assumption that firms never adapt their decision rule[7]

2 The lack of any explicit structure governing interactions between firms and the shape of externalities

3 Innovation possibilities only rely on current R&D spending. There is no accumulation of R&D and no role for a firm's accumulation of knowledge. The mechanistic nature of the innovation process leaves no room for considering the directions both of individual firm's innovative activity and of technological change as a whole.

The definition of an adaptive mechanism depends fundamentally on the kind and degree of rationality with whom agents are endowed. In order to be 'activated' this mechanism also required the prior definition of a 'satisficing' level of some target variable that the agents use as benchmark to evaluate the opportunity of changing their behavior. Adaptivity of behavioral rules is for sure a key feature of many Agent-Based models. A variety of techniques has been developed in the literature to modelize this phenomena. These techniques range from simple fixed algorithm to more complex evolutive tools such as Artificial Neural Networks, Genetic Algorithms and Classifier Systems. However, in the model presented here, we decided — mainly for simplicity reason — to leave apart this kind of criticism while focusing on limits 2 and 3. What we argue is that not only the direct improvement of productivity but also the direction of technological advance is relevant to determine the future performance of a firm. While the former affects directly the profitability of the firm, the latter puts the basis for further innovative developments in the future. In Nelson-Winter model the results

[7] Actually this point is approached in Winter's later work [Winter 1984]. Here innovation strategies are adaptive and firms change the composition of their R&D expenditure between imitation and innovation according their past performance.

of innovative activity can affect future innovative developments only by the measure in which they affect — today — the profitability of the firm (and then the future resources devoted to R&D). In our model, on the contrary, it is important to consider — beside the current improvement of productivity allowed by innovation — also the direction, inside a technological paradigm, in which the firm is moving, since this circumscribes the field for future innovative and imitative activity.

In our simulations every firm tends to specialize along a particular technological trajectory through a search process which goes from less to more specialized technological skills (an *in-depth* search process). The choice of the trajectory tends to privilege the cumulative, path-dependent aspect of technological innovation process, being fundamentally based on the the set of technological competences already owned by each individual firm. Indeed, it is impossible, *a priori*, to compare and asses the superiority of one technological path over another[8] [Dosi 1982]. What we are arguing is that technological progress and innovation process are affected by *strong substantive uncertainty* [Dosi and Egidi 1991]. Hence firms are characterized by *bounded rationality* [Simon 1947]. That is firms are not able to evaluate in a precise way the complete set of technological opportunities (i.e. the complete sequence of productivity gains) enabled by each possible trajectory. Since firms are not able to identify the best feasible trajectory they tend to follow some simple behavioral rules (i.e., *heuristics*) to face uncertainty. This is the reason why, in order to choose the direction of their technological efforts, they first look at their past technological experience. This behavior characterizing enterprises' search process is an example of *procedural rationality* in Herbert Simon's sense ([Simon 1955] and [Simon 1976]). This fact has relevant consequences that affect the innovative process. Given the impossibility of defining *ex ante* — in a static perspective — an optimal technological path, the success (or failure) of an innovative strategy — leading a particular firm to specialize in-depth along a definite trajectory — depend not only on the intrinsic 'goodness' of the trajectory chosen but also on the dynamic of the whole system. In particular, the possibility of acquiring technological skills (i.e. adopting new and more productive techniques), through imitation of competitors, generates *spillover effects*[9]. The dimension of these knowledge spillovers is related to the way firms in the industry specialize along the different technological trajectories. Then the ability of each firm to exploit this spillover effects through imitation of competitors is affected not only by its own decisions but also by other firms innovative strategies. The corollary of this observation is that the sole superiority — from a pure technological point of view— of a trajectory does not necessary imply that this will be the most widespread in the industry nor that firms

[8] Yet it is possible to find some objective criteria to compare them, but only *ex post*.

[9]

specializing in it will outperform competitors specialized in 'less efficient' trajectories[10]. Interesting, for its consistency with our arguments, is the comment of Howitt [Howitt 2006, p.1615] about Brian Arthur's work on the nature of technological progress:

> *"(...) because of this fundamental uncertainty, the pace and direction of innovation are necessarily guided by short-term considerations, even though they can lead society down irreversible paths whose long-run consequences are of great import, especially when there are 'network externalities' involved. That is, the course of technological progress, rather than reflecting the intentions of those individuals that create it, is a social process driven by the largely unforseen consequences of individual decisions."*

3 Modelling a technological paradigm as a network structure

Trying to translate in a formal model the complex structure of the previous arguments is one of most challenging aims of the present work. The way we found to address this issue starts taking explicitly into account the structure of the technology in use in the industry. We assume that the industry is initially endowed with a *Technological Paradigm* (TP) containing all the possible innovations that can be achieved by firms. Each innovation refers to the discovery of a new and more productive technique, allowing a certain gain in terms of capital productivity. The TP is represented by a network of nodes and links generated in the initialization phase of the simulation: nodes can be thought of as the set of knowledge and technological skills required to implement a particular innovation (i.e., to adopt a new technique of production that rise productivity) and links between nodes define the requirements for implementing each node. That is, links define the way the acquisition of the knowledge-competences required to carry out an innovation (i.e. to 'learn a node') depends on the prior acquisition of other knowledge-competences (i.e. of other nodes, we call them its 'parents nodes').
The TP is generated at the initialization phase of the simulation through a complex stochastic algorithm. This algorithm is inspired by Morone and Taylor's agent-based model of knowledge diffusion and innovation ([Morone

[10] This issue has already been studied in a number of works. See for example ([Arthur 1988 (1)], [Arthur 1988 (2)] and [Arthur 1989]) where the process of adoption of competing technologies is described as a *self-organizing*, non-ergodic process [Silverberg, Dosi and Orsenigo 1988] leading to multiple equilibria (due to the presence of increasing return to adoption), generating *lock-in* phenomena and characterized by potential inefficiencies (an inherently better technology with 'bad luck' in gaining early adherents can be driven out by less efficient one)

and Taylor 2005] and [Morone and Taylor 2010]), conveniently modified in order to create a network capable of reflecting some general features of *technological paradigms* and *technological trajectories*.
We first set the dimension of the network equal to an exogenously given number N. Thus the network will be constituted of N nodes: $N_0, N_1, ..., N_{N-1}$. Then the TP generation process start by setting a list of n initial nodes N_1 to N_n (n is exogenously given too) that are themselves directly linked to the root node N_0. The root node represents the origin of the network and can be thought of as the set of technological knowledge and skills that constitute the basis of the TP [11]. Accordingly we can look at the set of the n initial nodes — the ones directly connected to the root node — as the first set of technological applications to productive process of the knowledge contained in the root node. The remaining nodes (N_{n+1} to N_{N-1}) are embedded into the TP network by the following procedure:

Point 1 The number of parents for each node is randomly determined as an integer between 1 and the dimension of its *List of Potential Parents* (LPP). The LPP for a node is chosen randomly among all the possible ones. We will show in Point 2 how the Lists of Potential Parents are constructed. Once chosen (randomly) a particular LPP and extracted the number of parents each node sample its actual parents from its PPL. A directed link going from each parent to the 'child' node is drawn. The parents are collected in the *Parents List* of the node. Once inserted in the network each node is also characterized by a *Genealogy List* (GL), the list of nodes representing the entire genealogy of the node under consideration, from the root nodes to its 'direct' parents (i.e., the nodes it has just sampled from the LPP). The GL of a given node provides the complete list of nodes (we can call them the 'ancestors') that is necessary to learn before being able to access the node under consideration.

Point 2 The first set of Potential Parent Lists is obtained by splitting the list of the n initial nodes in j parts. The number j is determined as a draw from a Binomial Distribution with parameters $(n, P_{Split)}$, with P_{split} exogenously given. As explained in Point 1 each node N_{n+1} to N_{N-1} will first sample a LPP and then sample its parents. However, as soon as it is embedded in the network, it is also added to the List of Potential Parents from which it has sampled, thus becoming itself a potential parent for the following nodes (still to embed in the network). So each List of Potential Parents changes during the network generating procedure as new 'child' nodes are added to it. However

[11] Following Dosi's definition [Dosi 1982] we can interpret these basis as ' "model" and a "pattern" of solution of *selected* technological problems, based on *selected* principles derived from natural sciences and on *selected* material technologies.'

the possibility of splitting the Potential Parents Lists is not confined to the initial stage of the network construction. In fact, before adding each node to the network, a split in a randomly chosen List of Potential Parents may occur with probability P_{split} [12].

Point 3 Finally for each node just embedded in the network we make a further procedure just to avoid having redundancies in its genealogy list: if it comes out that a node in its Parents List is already an ancestor of another parent (i.e. it already appears in the GL of another parent), we delete it form the Parents List[13].

Through the splitting procedure of the Lists of Potential Parents explained in Point 2 we are introducing the possibility of branching the network representing the TP. This is maybe the most important feature of the TP network generating algorithm. In our interpretation the different branches of the network represent the possible technological trajectories that firms can try to explore through their search activity. In other words we are introducing the possibility of having — inside the same technological paradigm — different and relatively independent technological trajectories.

Figure 1 shows an example of PT network generated using the just described procedure. The network presented is the first one used for our simulation experiments. The set-up used for the algorithm will be explained in section 6. By looking at the figure we can easily identify the root node (marked by index one), with the initial nodes arranged radially around it. Branching of the network divide it into different independent area, representing different technological clusters of innovations.

4 Firms' innovative behavior

4.1 Firms' Skill Profile and the direction of innovation

Once the network of nodes and links representing the PT is defined, each nodes is endowed with a certain level of productivity gain, i.e. the gain in terms of productivity that firms can obtain by learning the technological

[12]Roughly speaking this means that, before adding each one of the $n+1$-N nodes, we make a bernoullian trial with probability of success equal to P_{split}. In case of success, a List of Potential Parents is drawn at random and divided randomly in two parts. In this way the number of List of Potential Parents will generally increase as the nodes are progressively embedded in the network. Hence the total number of splits during the network generating procedure can be interpreted as a draw from a Binomial Distribution with parameters $(N-1, P_{Split})$.

[13]Indeed, the fact that a node N_i appears in the genealogy of the node N_j (with $j > i$) means that all the requirements in terms of technological skills and knowledge that are needed to implement N_i are already implied by the requirements of the N_j node. Consequently the addition of N_i to a PL that already include N_j turns out to be useless and logically redundant.

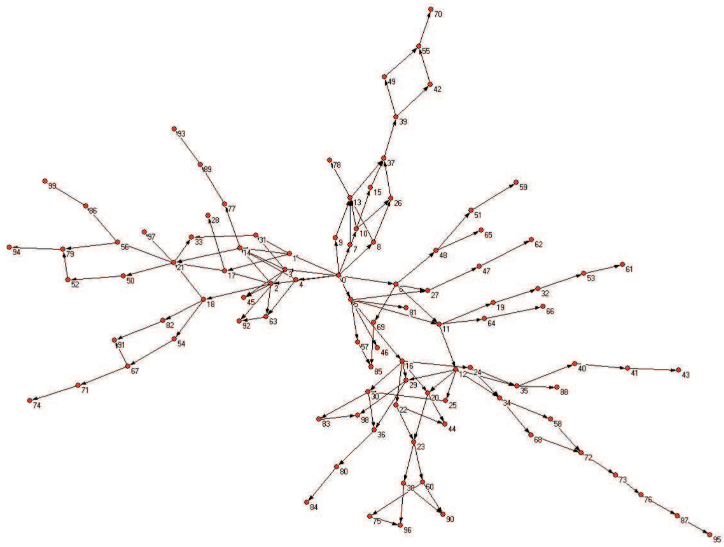

Figure 1: PT Network: $N^{nodes} = 100$, $InitialNodes = 10$, $P^{split} = 25\%$

skills implied by the node. In a similar way each firm, in the initialization phase, is endowed with an initial *Skill Profile*(SP), representing the set of nodes the firm already owns at the beginning of the simulation. These 'skill-nodes' are randomly chosen among the n initial nodes of the TP network, their number N_{skill} being exogenously determined. Therefore the initial Skill Profile of each firm constitutes its initial base of technological knowledge and skills. Also these aspects will be treated in more detail in section 6, dedicated to the set-up of the simulations runs[14].

Firms in the industry compete with each other by trying to lower their unitary production costs, i.e. by increasing their capital productivity. This means that firms try to enhance their technological base by implementing new innovation-nodes. Once a node is learned, it is added to the firm's Skill Profile. However firms can learn a node only after having already acquired all the nodes that constitute its genealogy. We assume that, due to substantive uncertainty affecting technological progress, firms cannot know — *ex ante* — the gain in productivity enabled by each innovation node[15]. Therefore

[14]Indeed it should be quite intuitive that both the logic followed to distribute productivity gains among the nodes of the TP network and the way initial skills are distributed among depends in a certain measure on the nature of the phenomena under study and the objectives of the research work.

[15]This is obviously a strong assumption, motivated by our will to focus on the path dependent nature of technological progress and on the role of technology structure in shaping network externalities. Nevertheless it would be possible to modify the model by assuming that firms are able, at least, to have some expectation about the increase of productivity allowed by 'next-to-them' nodes. Although this assumption appears more

the distribution of productivity gains across nodes does not affect the way firms choose *which nodes* trying to discover and *in which order*. Hence, what are the determinants of the decision by firms of specializing in a particular technological area? To manage substantive uncertainty firms tend to follow some heuristics, based on their past experience. That is, firms follow simple behavioral rules whose output depend on their own past technological path. generally speaking each firm will choose — in each period — the direction of its technological advance by choosing a 'target' node (i.e. the node towards which it will try to move). The choice of the target will be done by looking at its current Skill Profile: each firm then compares the set of nodes already in his possession with the list of nodes required to adopt each reachable node.
More formally, in each period firms decide the target nodes according to the following method:

Step 1 Firms first look at the children of each nodes in their SP. These nodes are the 'candidates' to be chosen as 'target'[16].

Step 2 To choose the target each firm considers, for each candidate, the list of nodes required to implement it (expressed by the candidate Genealogy List) and then check how many of them are already in its possession (i.e. in the firm's current Skill Profile). The difference between the two list provides the set of nodes that a firm should learn before being able to implement the candidate node. Hence it can be interpreted as a rough measure of the distance between the candidate node and the current technological endowment of the firm, represented by its SP.

Step 3 Then they rank the candidate nodes according to this distance, following a 'first the nearest nodes' rule, In this way they restrict the set of candidates to the nodes with the minimum distance from their current SP.

Step 4 If the previous step is not sufficient to identify a unique target node — i.e. there are more than one nodes with the same minimum distance — they rank the survived candidates looking at the subtrahend of the previous difference. In other words they look again, for each

realistic we do not think this is going to undermine the fundamental logic of our arguments. Indeed, being able to know the gains allowed by nearest innovation-nodes is not sufficient to evaluate, ex ante, the 'efficiency' of the whole trajectory to which nodes belong. See [Arthur 1988 (2)]. Furthermore for reasons explained in section 6 and related to the aim of the present work, we set the productivity gains implied by each each node equal to a constant parameter. Hence, even knowing exactly the productivity gains allowed by the nearest nodes, firms would not be able to determine *ex ante* the superiority of a feasible trajectory over another one.

[16] This means that firms can set as target a node only if they already possess at least one of the nodes required for its implementation (the root node being not taken into account).

survived candidate, to the number of nodes in its Genealogy List that appear also in the current Skill Profile of the firm. Then they choose the candidate for which this number is higher. The intuition is that firms tend to privilege nodes for which they have already accumulated more knowledge. This behavioral specification helps to increase the consistency of the innovative choices made by firms step by step, assuring that — in general — once the firms have chosen a target, they will pursue it until they will have successfully implement it. [17]

Step 5 If still there are more than one node on a par, firms choose randomly among them.

Step 6 Then firms, once chosen a target, must choose the order through which they will try to learn nodes required to implement it. This implies the choice of a sub-target. For simplicity reasons we assume that firms start from the nodes with the lower index number[18].

Note that, since we assumed that each nodes provide a productivity gain, both the targets and the sub-targets represent 'innovations'. However, while the sub-target is the node that the company will actually try to achieve in the current period, the target provides the overall direction towards which the company will focus its innovation efforts. The procedure by which the target is chosen allows the firm to move forward in the exploration of the network structure that represents the Technological Paradigm. Given these simple behavioral rules, developed in order to highlight the path-dependent nature of the search process carried out by firms, we expect that each firm will tend to specialize along a particular trajectory. Which one depends fundamentally on firm's initial set of technological competences but, if more than one possibility are still open, it may be determined also by chance. This implies that even firms with similar initial technological competences and identical innovative strategies [19] may experiment different technological paths, different sizes of network economies and, therefore, different economic performances.

4.2 Innovative and imitative activities

Like in Nelson-Winter model firms can discover more productive techniques by either two methods: by doing R&D aimed at developing an innovation in-

[17] Actually, a replacement of the target node originally chosen is possible only if, while firms are moving towards their target (i.e. while they are learning the nodes in its genealogy), new and more convenient — according to the method of evaluation described in the second and third steps above— technological opportunities (i.e. candidate nodes) are discovered.

[18] This simple rule automatically assures that firms will always choose nodes of the genealogy that they are immediately able to learn, given their current SP.

[19] Following [Nelson and Winter 1982] we distinguish between 'pure imitative' and 'both imitative and innovative'.

ternally ('local search' or 'purely innovative' strategy) or by imitating other firms. Firms may differ in their policies towards innovation and imitation. Following Nelson and Winter's original model we distinguish between firms pursuing a pure imitative policy and firms carrying on both imitative and innovative R&D policies. Each policy is defined in terms of spending per unit of capital, devoted to the respective innovative or imitative R&D activities. Therefore the expenditure on innovation and imitation grows or declines according to the size of the firm: large firms spend more on R&D than do small firms. At the same time this greater spending provides them a greater chance to obtain in each period an innovative or imitative draw. Furthermore, like in Nelson-Winter model there are also 'appropriability advantages' of large size firms, since everyone is always able to immediately apply an innovation to its entire stock of capital without further costs.

Innovative activity is shaped so that, in each period, the probability of success by a firm (i.e. the probability of being actually able to learn the desired innovation node) is proportional to its current spending on innovative R&D. This formulation is very closed to the first-stage innovative activity in Nelson-Winter 1982 model (already described in section 3). However there is a fundamental difference between the two approach. In our model, every innovation produces two effects: on one hand it raises firm's productivity[20]. This was the only effect taken into account in Nelson-Winter specification. On the other every innovation concurs in defining the direction in which the firm is moving within the technological network, prompting the firm to specialize along a particular technological path, thereby constraining also the direction of future technological advance by the firm.
Alternatively a firm can try to learn a node through imitation by observing the technology in use among its competitors. In this case it first selects a number of firms to 'imitate' and then looks at their Skill Profiles. If the node it is looking for belongs to the SP of anyone among them, it can successfully implement it. Otherwise, in the next period, it tries again, looking at a different group of competitors. In each period, the number of competitors that the company tries to imitate is defined as a draw from a Binomial Distribution in which the parameter defining the probability of success of the underlying bernoullian experiment is proportional to firm's current spending on imitative R&D activity. Needless to say, the higher this number the greater should be the chance that the imitator finds the desired node. How to choose the competitors to imitate? Imitators, at the initial period of each simulation run, choose the firms to imitate randomly. In fact they don't have any information about other firms' technological skills. However let's consider what happens when a firm succeeds in imitating a competitor. This

[20]The increased productivity, in turn, can increases future R&D outlays via profit, investment and capital accumulation.

fact should be interpreted by the firm as a clue about the possibility that the imitated competitor is specializing in a similar technological area. In this case, the imitative firm should be induced to believe that already successfully imitated firm can provide further imitative draws also in the future. Hence it is reasonable to introduce an additional behavioral specification for firms according to which, when a firm gets a successful imitative draw by looking at the SP of some competitors, it assigns them a *priority* (to be imitated) for the next periods. This means that, in its subsequent attempts to acquire new nodes through imitation, the firm will first look at the competitors that have already yielded a successful imitative draw in the past[21]. If imitation does not lead to desired results (that is, none of the imitated firms had got in their Skill Profiles what the imitator was seeking) the firm, in the next period, will choose randomly a different set of competitors to look at. The priority recognized to a firm by an imitator is lost if the imitated firm does not yield any further imitative draw for a number of subsequent attempts. We must stress the fact that the presence of firms that follow an imitative policy introduces in the model *spillover effects* related to the *process of diffusion of technological innovation* that were totally absent in Nelson and Winter's original model. Indeed the probability of being successful in obtaining an imitative draw, while looking for a node located on a particular trajectory of the PT network, is positively affected by the number of firms that are specializing on that same trajectory. Roughly speaking for an imitator, its probability of finding a competitor in possess of the node It is looking for is higher the higher is the number of firms that specialize in the technological area in which the desired node is located. This means that firms that follow an imitative strategy and are specializing along a densely populated trajectory may have a relative advantage with respect to firms specializing along less densely populated trajectories. This aspect explicitly shows how the success or failure of firms innovative efforts relies not only on their own strategies towards innovation but also on what other firms do. This gives us the opportunity to highlight an important feature of the model: even firms with identical strategies towards innovation, identical initial endowments of capital, identical initial level of productivity, but different initial sets of technological skills, may experiment different technological paths. Given the fact that spillover effects are characterized by different strength along the different feasible technological trajectories — depending on the number of firms specializing on each one of them — this implies that similar firms may experiments radically different economic performances. Evaluating ex ante the goodness of an innovative strategy is then something problematic.

[21]If the total number of firms it can look at (that is a function of its expenditure on imitative R&D) is higher than the number of firms 'with priority', it will sample the remaining firms to imitate by randomly choosing among other competitors. In the opposite case the firm will have to extract randomly the firms to imitate among the ones with a priority.

5 The formal model

In this section we translate into a formal structure the model already present in the previous part of the paper. Except for what concerns the representation of the search process by firms and the depiction of technology through a network structure, the model is kept relatively close to the original version by Nelson and Winter.
In each period, given the capital stocks and the productivities levels of each firm, both the production of each firm and the total output are determined:

$$Q_{it} = A_{it} \cdot K_{it} \tag{1}$$

$$Q_t = \sum_i Q_{it} = \sum_i A_{it} \cdot K_{it} \tag{2}$$

The price is then determined through the product demand-price function:

$$P_t = D\left(Q_t\right) \tag{3}$$

The profit on capital of firm i equals product price multiplied by output per unit of capital (i.e., the productivity of capital, A_{it}), minus production costs per unit of capital (c)[22], minus R&D costs. For a firm following both an imitative and innovative strategy R&D costs per unit of capital are given by (r_{in},r_{im}) and its rate of profit is given by:

$$\pi_{it} = P_t \cdot A_{it} - c - r_{in} - r_{im} \tag{4}$$

Similarly a firm spending only on imitative R&D, will have a profit function:

$$\pi_{it} = P_t \cdot A_{it} - c - r_{im} \tag{5}$$

Innovative research activity is characterized by a random variable d_{in} which takes values 0 or 1 according to whether firm does or does not get an innovative draw in period t. The probability of success is defined according the following function:

$$Pr\left\{d_{in} = 1\right\} = \alpha \cdot r_{in} \cdot K_{it} \tag{6}$$

The number of competitors that an imitative firm can look at in each period is a random variable defined as the maximum value between one and the result of a draw from a binomial distribution with parameter (n, p) where n is equal to the total number of firms in the current period and p (the probability of success of the Bernoulli experiment) is an increasing function

[22]Remember that, since the price of inputs are constant and all inputs coefficients are fixed c is constant both across firms and over periods.

of firm's current expenditure on imitative search activity. Formally N_{im} is a draw from:

$$Binomial\ \left(N_t^{firms} - 1\ ,\ \beta \cdot r_{im} \cdot K_{it}\right) \tag{7}$$

For each firm the 'price-cost' ratio is determined as $P_t/(c/A_{it})$. We assume that a firm's desired expansion or contraction is a function of its price-cost ratio and its market share and that its ability to finance investment is constrained by its profitability. In other words the greater the price-cost ratio the greater the firm's ability to persuade capital market to provide the required finance. But then the amount of R&D outlays, reduce the funds received to finance investment. Since firms produce an homogeneous product it is reasonable to have firms that follow a 'quantity policy' rather than a 'price policy'. This is implemented through their investment decision[23]. Market share enters in the investment function since firms may have some degree of awareness of the effect that an increase in their own output will have on industry price. Hence firms may recognize that by increasing over a certain level their production they risk to 'spoil' their own market. Hence, the higher their current market share, the higher the price-cost ratio shall be in order to induce a certain level of expansion. Different investment behaviors could be studied by reshaping the investment function in order to account for different level of wariness about spoiling the market [see again [Nelson and Winter 1982] in the last chapters of part V). However, for simplicity reason, we assume that firms have a correct perception of the industry demand curve elasticity. More formally the first order differential equation describing the dynamic of a firm's capital stock can be expressed as:

$$K_{i(t+1)} = I\left(\frac{P_{t \cdot A_{it}}}{c}, \frac{Q_{it}}{Qt}, \pi_{it}\right) \cdot K_{it} + (1-\delta) \cdot K_{it} \tag{8}$$

with δ being the physical depreciation rate, and with $I(\cdot)$ nonnegative given by:

$$I\left(\rho, s, \pi\right) = Max\left\{0, Min\left[(1+\delta) - \frac{2-s}{\rho \cdot (2 - 2 \cdot s)}, f\left(\pi\right)\right]\right\} \tag{9}$$

where $\rho = \frac{P_t \cdot A_{it}}{c}$ and $f\left(\pi\right)$ is the finance constraint given by:

$$f\left(\pi\right) = \begin{cases} \left(\delta + \pi\right), & \text{if } \pi \leq 0 \\ \left(\delta + B^{regime} \cdot \pi\right), & \text{if } \pi > 0 \end{cases} \tag{10}$$

with $B^{regime} > 1$ being a parameter defining the financial regime. A more detailed explanation of the logic underlying the definition of the investment function can be found in Appendix A1.

[23] Remember that in our model firms exploit their whole productive capacity. Thus, given their productivity, the only way they can determine the level of production is through their capital stock, i.e. by expanding or reducing their capital stock through investment.

6 The setting

In the present work we use the model to analyze the connections between market structure and innovation within an industry characterized by the presence of firms with differentiated strategies towards innovation. This issue was central in Schumpeter's analysis, in particular the one presented in *Capitalism, Socialism, and Democracy* [Schumpeter 1950] where the author stressed the importance of large firms in pushing the dynamic process of innovation. As Nelson-Winter [Nelson and Winter 1982] noted market structure is endogenous to an analysis of schumpeterian competition, with the causal link going both directions. Large firms may have innovative advantages with respect to small firms due, for example, to managerial and R&D economies of scale and to higher capabilities in obtaining finance. In our model large firms spend more on R&D and hence have an higher probability of carrying out an innovation. Another important source of a relative advantage with respect to small firms is represented by appropriability advantages related to the possibility of exploiting an innovation on a greater scale of production. In a context — like the one describe here — where there are no Intellectual Property Rights and imitative firms can easily copy an innovation, the payoff of the innovator depends crucially on its ability to exploit innovation in a short period of time and on the highest possible scale of production. In the model we assume that firms can immediately apply an innovation to their whole stock of capital. Actually, as Nelson and Winter point out, these advantages are related more to firms' dimension than to market structure. However market structure may influence the capability of innovators to exploit an innovation by affecting the speed at which imitators are able to erode innovators' advantage. If there are a few competitors it is more likely that an innovator can keep its advantage for a longer period than it would be in a market characterized a great number of firms. At the same time successful innovators and imitators may invest their higher profits in order to increase their dimension and thereby dominate the market.

In a schumpeterian perspective, the study of the connections between market structure and innovative performance of an industry is primarily concerned with the analysis of evolutionary struggle between innovative and imitative strategies. Firms compete in the market and, according to the success of their policy towards innovation, they grow or decline pushing the market towards a more or less concentrated structure. In particular the probability for a firm following an imitative strategy to result successful depend on its ability to exploit spillover effects. In turn, the dimension of these spillovers is influenced by the number of firms specializing in each feasible trajectory, and hence by initial market structure. hence, while the original version of the model, with a science-based technological regime, depicted a regime in which innovative R&D activities were always somewhat unprofitable (on average), here different initial market structure may determine a context more

or less favorable for either innovative or imitative policies.
For this reason we run several experiments with different initial industry structures: four structures are examined, with respectively four, eight, sixteen and thirty-two firms at the initial period. In order to provide the clearest possible explanation of the mechanisms underlying schumpeterian competition and industry evolution we choose initial conditions and parameters looking for some kind of symmetry among firms. For the same reason we also rule out entry and we assume that in each simulation half firms follow a pure imitative strategy while the other half spend both on innovative and imitative R&D activities. In order to give an account of the role played by the spillover effects generated along each branch of the TP network, we assign to every innovation node the same productivity gain[24]. The initial stock of capital is the same for all firms. Each firm is also given the same number of initial skills, chosen randomly among the initial nodes of the PT network. Furthermore we assume that each firm has got the same initial level of capital productivity. Hence, at the first period of the simulation, the levels of production, the market shares, the mark-up ratios and the desired net investment will be the same for all firms.
The initial firm's capital stock is chosen so that, for each initial structure, the firm's desired net-investment for the first period is equal to zero[25]. Then r_{in} and r_{im} are adjusted to compensate the differences in initial levels of capital so that the initial total expenditure on imitative and innovative R&D is the same in all runs. In particular, following Nelson and Winter r_{in} is chosen in order to maintain constant the ratio between R&D spending and sales at a level of $0,12$[26]. Then the coefficient α is set so that the probability of obtaining a successful innovative draw gives on average two innovative findings for the whole system every four periods (i.e. a year), at initial condition. Then in Nelson-Winter model the coefficient r_{im} and the parameter β, defining imitative R&D policy were set in order to give an expected number of draws equal to the innovative ones. In order to highlight the cost of doing innovative R&D they chose a value for r_{im} approximately equal to $\frac{1}{20}r_{in}$ and a value of β equal to 10α[27]. In this way they ensured that the model shows the same 'initial progressiveness' among all simulation runs. However in our model things are quite different. In Nelson-Winter model r_{im} and β were parameters defining the probability of obtaining an imitative

[24]In this way we are avoiding any disturbing elements that an asymmetric distribution of gains among the different branches could generate.

[25]This implies that the initial stock of capital is lower the higher is the initial number of firms.

[26]

[27]Remember that while all spend on imitative R&D only an half spend on innovative R&D. In this way the total expenditure on imitation is 1/10 of the total expenditure on internal innovation. However given the above specified relation between the coefficient α and β the expected number of imitative draws per period are equal to the expected number of innovative draws, at initial conditions.

draw, that allowed a firm to automatically adopt the best techniques of the industry. Here, on the contrary, they simply concur in defining a parameter of the Binomial distribution from which firms sample the *number* of firms they will try to imitate in order to get the desired node. Hence, while leaving the same characterization for r_{in} we choose α so that the probability of obtaining a successful innovative draw gives, on average, only one innovative findings for the whole system every four periods. At the same time, in order to ensure the symmetry between initial conditions for all simulation runs, we set $r_{im} = \frac{1}{10} r_{in}$ and then we choose β so that, in the first period of each run, the expected value of the Binomial distribution for each imitative firm (i.e. the expected number of competitors they can look at) is equal to one in all the different scenarios. The coefficient β is then a function of the number of firms: $\beta = 1/(r_{im} \cdot K_{i0})$. This specification provides the model the same degree of 'progressiveness' under each scenario[28]. Finally demand elasticity is set equal to one, formally: $P_t = 67/Q_t$, unitary costs per unit of capital c are set equal to initial capital productivity and the parameter b_{regime} defining the finance regime is set equal to 2,5.

The basic set-up for the PT network generating algorithm is the following: the network is made up of 100 nodes, with 10 initial nodes beside the root one. In each stage of network creation the probability of splitting one of the original Potential Parent Lists is set at 25%.

The complete quantitative statement of the model under different scenarios is shown in Table 1.

Each run lasts 100 periods, each period representing a quarter of year, hence for a total of 25 years. Given a particular PT network structure, we run 100 simulations for each scenario. The average results of this simulations, with their standard deviations are presented in the following section. Then we repeat all the previous simulations under four different PT network structures — all generated with the same characterization of the network generating stochastic algorithm — in order to check the robustness of our results under different realizations of the same network generating stochastic algorithm. The average results and the standard deviations for these simulations are shown in the first table in appendix 2. Finally, in order to analyze the effects on simulations results of a change in the parameters used to build the network, we re-execute the simulations using 2 different set-ups of the network generating algorithm, by assuming a probability P_{split} respectively equal to 10% and 40%.

[28]Indeed, once given the number of competitors the imitative firm can try to imitate, its probability of success, in the first period, is not a function of the number of firms. Actually this probability depends on the structure of the network and on of the number of skills distributed to each firm in the initialization phase.

Table 1: Initial values under four different initial market structure

	Number of Firms			
	4	8	16	32
K	89,73214	48,85417	25, 32762	12,87822
r_{in}	0,0224	0,0206	0,01984	0,01951
r_{im}	0,00224	0,00206	0,001984	0, 001951
α	0,06219	”	”	”
β	1,6584	1,4216	1,3267	1,2839
c	0,16	”	”	”
δ	0,03	”	”	”

7 Results of the simulations

As explained in the previous section our analysis starts by considering 5 Technological Paradigm networks with ten initial nodes (beside the root one) obtained by setting $P_{split} = 25\%$. However, to clarify our statement of the results, we will focus only on one of them (specifically the one marked by the index 1 in the first Table in Appendix 2). Indeed it is easy to show the fundamental properties of the model seem to holds also under the other 4 network structures used in this first stage of our discussion [29].
In order to analyze how industry initial concentration affects industry innovative performance we compare the results of the simulation under the four different experimental scenarios, each one being characterized by a certain number of firms initially in business. The data used to plot the graphs are the averages and the standard deviations of the final results of the 100 simulations run under each scenario. Figure 2 shows the means of best-practice and average productivity in the last period of the simulations. Both best practice and average productivity decrease as we move from a more concentrated to a less concentrated industry. This means that, given the fact that firms' initial productivity is the same under all initial conditions, both best practice and average productivity rise more rapidly under the small-number case. This result is consistent with the famous *Schumpeter Mark II* argument according to which a more concentrated market structure is 'the price we have to pay' for a better innovative performance.

Best practice productivity depends crucially on innovative R%D activity. Indeed imitation actually allows to implement only nodes that have been already dicovered by some firm in the industry. So imitation cannot push ahead the knowledge frontier of the technology structure. Only inno-

[29]Table 1 in appendix 2 shows that the results are robust under the different realizations of the network generating algorithm obtained with the previous setting

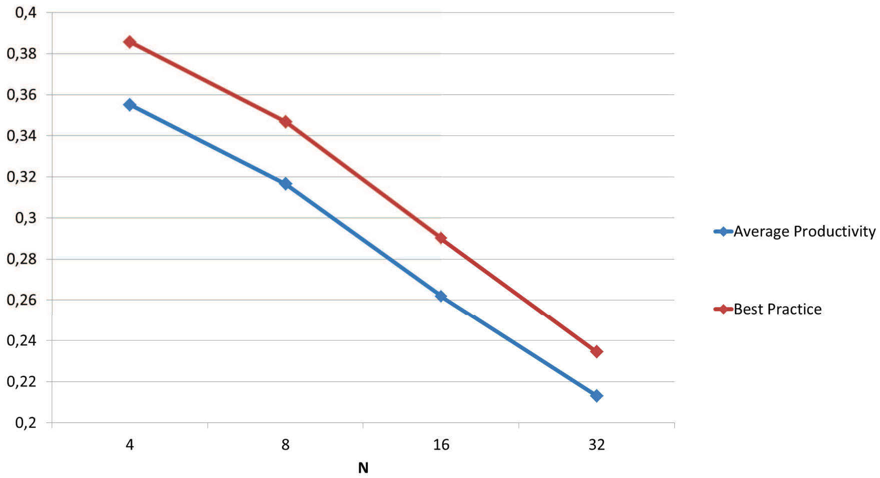

Figure 2: Average and best practice productivity

vative activity allows to move towards technological paths not yet explored, thus pushing on also the boundary of what can be learnt through imitation. Hence, when the network presents a significant number of independent trajectories (this number depending on the value of P_{split}), the role of imitation in determining the level of best practice productivity is narrow[30]. It is the success of innovative activity of firms specializing along different trajectories to determine the boundary up to which the exploration of each possibile technological path of the TP network will come. Therefore the best practice productivity level is strongly affected by the number of innovations draws obtained by innovative firms. Not surprising that the decline in best practice productivity — while moving towards less concentreted industry initial structures — is accompanied by a fall both in the average number of innovations obtained by innovative firms as a whole and in the mean of the maximum number of innovations obtained by a single firm in each simulation (see Figure 3). This is primarily due to the fact that the coefficient r_{in} and α governing firms' innovative activity were set so that, at initial conditions, the industry as a whole obtains — on average — the same number of innovative draws in each periods under the four scenarios. But this implies that, as the number of firms initially in the industry increases, their individual probability of obtaining an innovative draw declines. This determines the drop of the number of innovative draws achieved by each innovative firm — taken individually — which in turn determines the observed fall of best practice productivity. However, given the previous assumption, we would also expect the average number of innovative draws of the industry not to change

[30]On the contrary we'll wee that imitation plays a central role when the network is poorly branched and its nodes are more interrelated

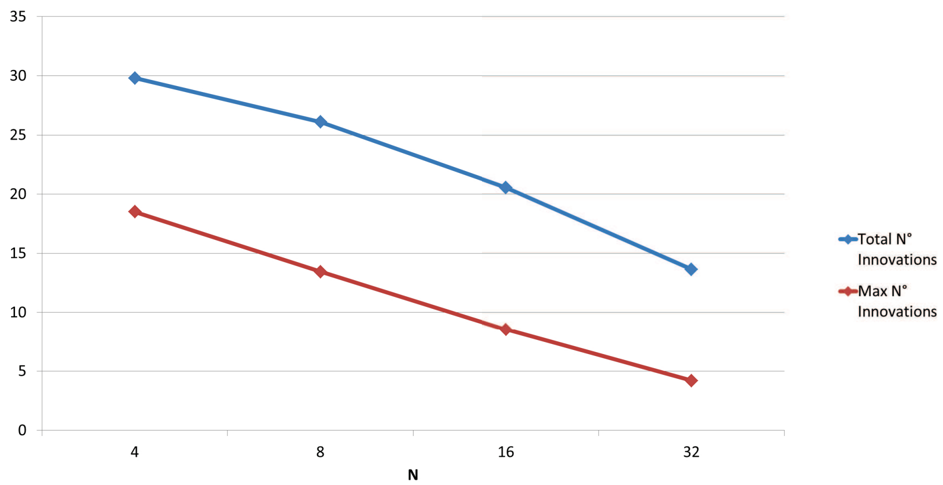

Figure 3: Total number of innovations achieved in the industry and maximum number of innovative draws achieved by a single firm

significantly among the different scenarios. On the contrary, the data show that industry average number of innovation slightly decreases as the number of initial firms increases, suggesting that something more is happening. The reason explaining this observation lies in the role played by spillover effects in our model. Moving towards less concentrated industry structures, innovative firms suffer more the competition of imitative firms and this, in turn, causes a slowdown of their performances: for reason that we are going to explain the imitators, under less concentrated initial structures, can imitate innovators more easily. This in turn reduces, on average, the length of the periods for which innovators can take advantage of the quasi-rents deriving from achieved innovations (thus recovering the higher costs incurred for doing innovative R&D). The lower profitability of innovative firms has a negative effect on their capital accumulation, thus reducing the resources directed to innovation and, consequently, the number of innovation actually realized.

In order to better understand what is going on we must put our attention to the the evolution of firms' performances among the different scenarios, by comparing innovative and pure imitative firms' performances. In Figure 4 we plot the mean ratios between imitative and innovative firms' average productivity at the end of the simulations. The graph shows that moving towards less concentrated industries imitative firms are able 'to track' innovative firms' productivity more and more closely. As explained in section 4 a firm's probability to obtain an imitative draw while looking for a node located on a certain trajectory depends crucially on the number of firms specializing on it: the more 'densely populated' a technological trajectory, the higher the probability for a firm specializing along that trajectory to find

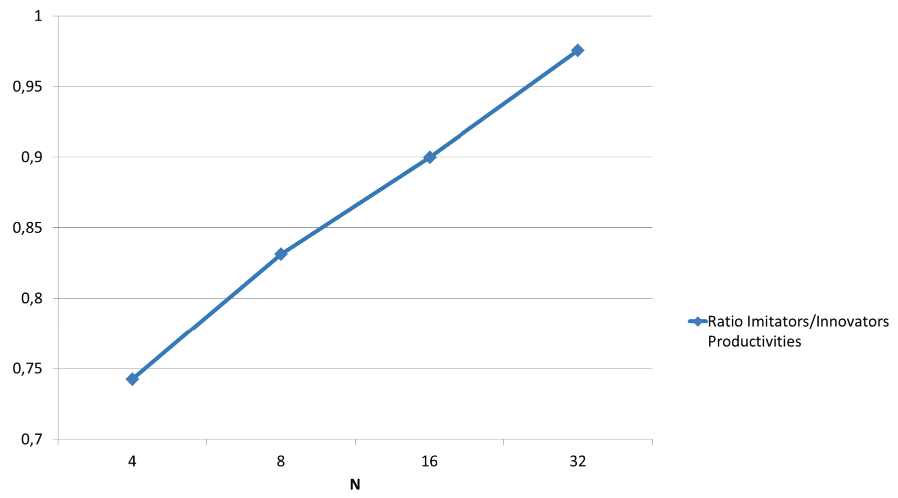

Figure 4: Ratio between imitators' and innovators' average productivities

a competitor to imitate successfully. In each period of the simulation firms choose the direction of their technological advance (i.e. the target node) on the base of the technological skills already in their possession. Hence the decision of a firm to specialize along a particular trajectory of the TP network is fundamentally related to its initial skills, randomly assigned in the initialization phase. The higher the number of firms in the industry at the beginning, the higher the probability that the initial skills will be distributed in a way such that each area of the TP network will find some firm that, given their initial Skill Profile, have decided to specialize on it[31]. Hence the probability of success for an imitative firm is positively affected by the number of firms initially in business. Roughly speaking this means that, when the number of firms is low, it is less likely that an imitator will find someone specializing along its same trajectory. Consequently, on average, imitative firms will find more problematic to carry out imitation draws and, in turn, they will not be able to track closely the dynamic of innovative firm's productivity. On the contrary when the number of firms is high, it should be easier for imitators to find someone to copy. Thus, on average, the dynamic of their productivity — compared to that of innovative firms — will improve. Figure 4 shows that in the 32 firms case the ratio between imitative and innovative average productivity is actually near to one.
The fall in average productivity can be explained primarily as a result of the fall in the level of best practices. However this result is also a conse-

[31]In particular the higher the number of firms, the higher the probability that each trajectory will be crossed by at least one innovative firm, whose R%D activity defines — as already said — the frontier of what can be discovered through imitation along each trajectory of the TP network.

quence of the particular view of firms' nature underlying the model. Indeed we assumed that, within the boundaries of a firm, technological knowledge is equally and costlessly applicable to all units of capital. That is, the increased level of productivity implied by an innovation applies automatically to the whole stock of capital. Hence the larger the capital share of the firm which enhances its technological base by learning a node of the TP network (either through innovation or imitation), the larger will be the effect on industry average productivity. So, even leaving aside any consideration about the shape and the role played by the dynamic competition between innovators and imitators under the different scenarios, we should expect average productivity to decrease as a consequence of the smaller amount of capital affected by each innovation when the number of firm is higher[32].
The improved performance of imitative firms is also reflected by the observed inverse relationship between their average capital share and industry initial concentration. Figure 5 shows that imitative firms' capital share considerably increases as the number of initial firms increases. In the latter

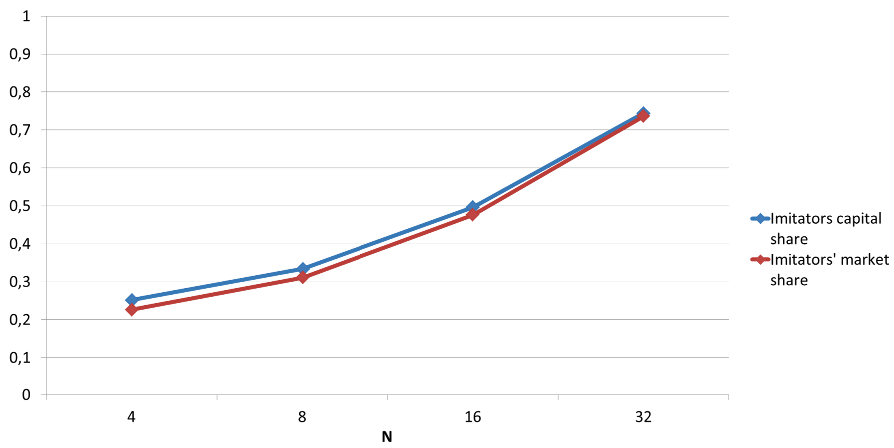

Figure 5: Capital and market shares of imitative firms

two scenarios, with 16 and 32 firms, it is respectively almost equal and even higher that innovative firm's capital share, despite the fact that imitative firm's productivity is still lower than innovators' one. According to the desired investment function defined in the previous section, for two firms with the same market share, the firm with the lower production costs (i.e. higher productivity) will have a higher target output and, in turn, an higher target investment. Nevertheless we must also consider the different structure of

[32]Actually, in Nelson and Winter's model, this was the only way by which the average productivity drop arose. On the contrary best practice dynamic was unrelated to industry's structure due to the adoption of an exogenous 'science-based' technological regime governing the growth of 'latent productivity'.

costs affecting imitative and innovative firms. While the former spend only on imitative activity, the latter have to face both innovative and imitative R&D costs[33]. If we take two firms, one imitative and one innovative, with the same market share, the same productivity level and hence the same price-to-unitary costs ratio, the firm following a pure imitative strategy will result more profitable. When the level of profit is sufficiently high the financial constraint is not binding, and the firm with lower unitary costs of production will invest more than the firm with higher costs, even though by including R%D expenditure the former may have higher total costs per unit of output than the latter. This seems to be the situation depicted in the first two case of our simulations (4 and 8 firms). The gap in productivity levels between innovative and imitative firms seems sufficiently remarkable to compensate for the higher R&D costs associated to an innovative strategy. But moving towards less concentrated initial structure the performance of imitative firms improves and they appear more and more capable of plugging the gap in productivity. An increased portion of innovative firms is run out the business (i.e. their market share becomes negligible) and even for those who remain in the market, the rate of profit significantly decreases due to the increased competition so that their financial constraint becomes more and more binding. On the contrary imitative firms unitary costs of production are now closer to innovators' regardelss the fact that they spend less on total R&D. Therefore their profitability constraint to finance investment, expressed by equation (9), will be less binding and the resulted investment on capital accumulation higher. This reversed situation is depicted by the two latter scenarios: when the ratio between imitative and innovative firms' productivity is close to 90% or higher imitative firms' capital share balances or even exceeds innovatives' on average.

Finally it should not be surprising that also average market shares evolution across the different scenarios shows the same tendency of the previous graphs. As moving towards less concentrated initial structures, the relative position of imitative and innovative firms changes in favor of imitators. In the fourth scenario, on average, they actually outperform innovative ones. This result is obviously a direct implication of the evolution of firms' productivities and capital shares.

The results presented in the above discussion were aimed to highlights the relationship between industry initial concentration and innovative performance. But what can we say about the reverse effect of schumpeterian competition on the evolution of market structure? Figure 6 displays the averages of the end-of-simulation values of the Herfindahl Numbers Equivalent[34]. This constitutes a common measure of output concentration in an

[33]Moreover remember that, following Nelson and Winter, we set $r_{im} = 1/10 r_{im}$ in order to highlight the cost of doing innovative R&D.

[34]The Herfindahl Numbers Equivalent is formally defined as the inverse of the Herfindahl-Hirschman Index: $HHI = \sum_i s_i^2$

industry. When all firms have equal market shares, the Herfindahl Numbers Equivalent is simply equal to the number of firms in the industry. Instead, when firms have unequal shares, it gives the number of equal-sized firm in an industry characterized by the same degree of concentration as the actual industry, according to the Herfindahl-Hirschman Index. In all cases we see a clear tendency towards higher concentration throughout the simulations. This tendency is even more clear if we look at the average % decrease of the

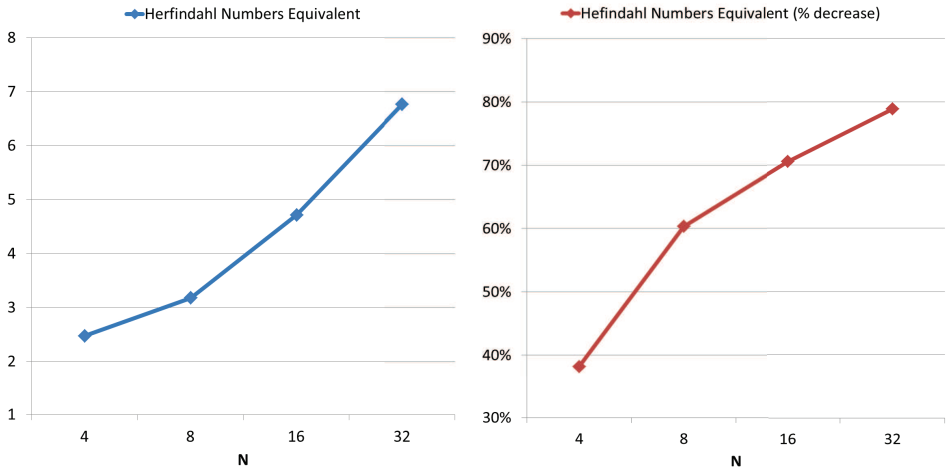

Figure 6: Herfindahl Numbers Equivalent end-of-simulations values and % reductions

Numbers Equivalent from the beginning to the end of the runs[35]. Figure 6 clearly shows that the tendency towards higher concentration is strong under each scenario but becomes particularly pronounced when the initial structure of firms is less concentrated. In the last two cases, with 16 and 32 firms, the process of schumpeterian competition among firms results in an end-of-simulation market structure characterized, on average, by a degree of concentration comparable with a market in which more than 70% of the firms have disappeared and the remainder are equally sized.
Finally we must deserve particular attention to the standard deviations of the results, displayed in the tables in appendix 2. Although the analysis based on average results is sufficient to highlight a clear tendency, moving from more to less concentrated initial industries, for each considered variable, we also observe significantly high values of the standard deviations of the results in all the scenarios[36]. This particular feature of the model

[35]Note that at the beginning of the simulation total output is equally distributed among firms. Hence the Herfindahl Numbers Equivalent in the first period of each run simply equals the number of firms initially in business.

[36]These values, displayed between brackets in tables in appendix 2, are calculated as the standard deviations of the final values of the 100 simulations run under each scenario, for each TP network

is again related to the role played by spillover effects. What the standard deviations tell us is that different distributions of initial skills among firms may significantly affect the final results of the simulations. Hence, even if on average different initial market structures determine a situation more or less favorable for the two types of firms, the final results of each simulation run under each scenario are not obvious. Even in the less favorable case for imitative firms (the one with the lower initial degree of concentration), if the initial distribution is such that imitative firms are induced to specialize on the same technological trajectories of innovative ones, they may have the chance to follow innovators technological advance very closely if they are sufficiently rapid in recognizing which is the right competitor to imitate. In this cases, the final values of best practice productivity and average productivity will be relatively low, for the reasons explained above. On the other hand the ratio between the final values of imitative and innovative firms' averages productivities will be higher. Consequently imitators' capital and market shares will tend to rise and the final structure of the industry will tends to remain less concentrated[37]. On the contrary, even in the large number cases (generally more favorable for imitators), if the initial distribution of skills among firms is such that imitative and innovative firms decide to specialize in technological areas significantly different, imitators will face more difficulties in obtaining imitative draws and a greater number of them will be pushed out of the market. The resulting lessening of the competitive pressure coming from imitators will boost innovators' growth. A greater amount of resources will be dedicated to innovative R&D, leading to a relative improvement in best practice and average productivity[38]. This aspect of the model seem to suggest that, despite the fact — highlighted by our previous arguments based on average results — that different initial structures create more or less favorable conditions for either innovative or imitative strategies, the inherent cumulative nature of firms' technological advance makes it difficult to determine *ex ante* an optimal strategy from the perspective of individual firms. Even firms with identical initial conditions regarding capital stock and its productivity, and the same strategy towards innovation, may experiment radically different technological paths (as a result of the characteristics of the technological knowledge base inherited from their past, embedded in their initial Skill Profiles) and then, radically different economic performances.

[37]It must be noted that even slight differences in the ratio between imitators' and innovators' averages productivities may have a great impact on their capital and market shares. If we compare the mean results of the two extreme scenarios (4 and 32 firms), we notice that an increase of about 0.2 points of the previous ratio roughly triples both the capital share and the market share of imitators. This explains the amplified magnitude of the standard deviations of the latter two variables, compared to those refferring to productivities measures.

[38]As the capital share of imitators becomes more and more negligible their impact on average productivity becomes narrow.

As already mentioned, the data used to plot the previous graphs refer to the network marked by index 1 in table 2 of appendix 2. The same set of simulations have been run under 4 other different TP networks, all generated with a probability of branching the network equal to 25% in each stage of their generation procedure. Also these data are shown in table 2. Although some minor differences exist, as a consequence of the stochastic character of the algorithm used to build the networks, both the average results and their standard deviations are substantially in line with the results shown for network 1. Therefore, this seems to confirm that the trends highlighted for all the variables of interest are robust under different realizations of the same network generating algorithm.
In the last part of our computational experiment, we will briefly consider how different set-ups of the algorithm used to construct the networks representing Technological Paradigms affect the results of the simulations. To do this we re-executed the simulations using 10 different networks, 5 obtained by setting the probability of branching P_{split} at 10% and 5 obtained by setting the probability of branching at 40%. The outputs of the simulations are shown in table 3 and 4 in appendix 2. Data again seem to confirm the same trends discussed earlier in this section. The reduction in the number of firms initially in business generates, on average, a drop of average and best practice productivity at the end of the simulations. At the same time, the situation in the industry gradually changes in favor of imitative firms, as shown by the rise in the ratio between the average productivities of the two types of firms and by the dynamic of the capital and output shares held by imitators. Then we take the averages of the results obtained with the 5 networks considered for each of the 3 set-ups used for the network-generating algorithm. In this way we can make a comparison between the outputs obtained under the 3 different types of network. Note that an higher (lower) value of P_{split} determines a more (less) branched structure of the network representing the technological paradigm. Thus the higher P_{split}, the greater will be the number of possible technological trajectories on which firms can specialize. In turn the less branched the network, the greater will be the probability that firms will specialize in similar technological areas (and vice-versa). So it is not surprising that networks built with $P_{split} = 10\%$ depict a situation relatively more favorable for imitators while the opposite happens for networks obtained by setting $P_{split} = 25\%$. This appears evident by looking again at the graphs on the ratio between the average productivities of imitators and innovators and those refferring to the capital shares and the market shares. More complicated is the analysis of the dynamic of best practice and average productivities across the different types of network. Indeed the results concerning these two variables are not obvious and seem to be at odds with our previous arguments. In fact, following our previous arguments we should expect both best practice and average productivity to decrease as the ease to imitate increases. In order to understand what

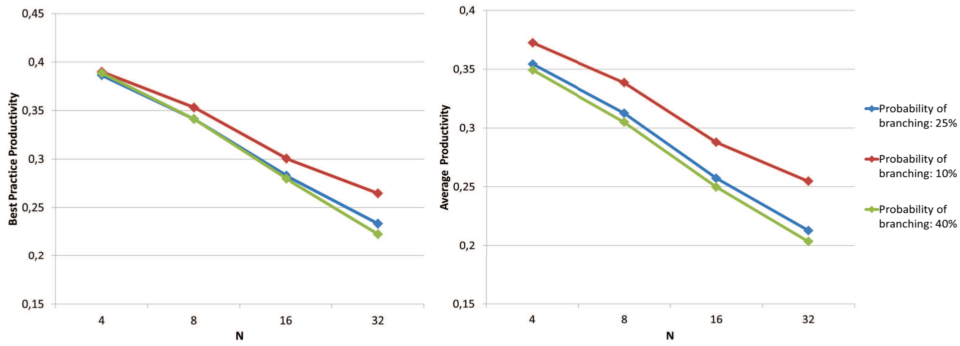

Figure 7: Best practice and average productivity with different TP network structures

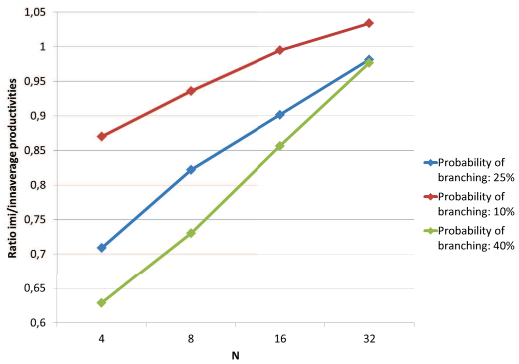

Figure 8: Ratio between imitators' and innovators' average productivities with different TP network structures

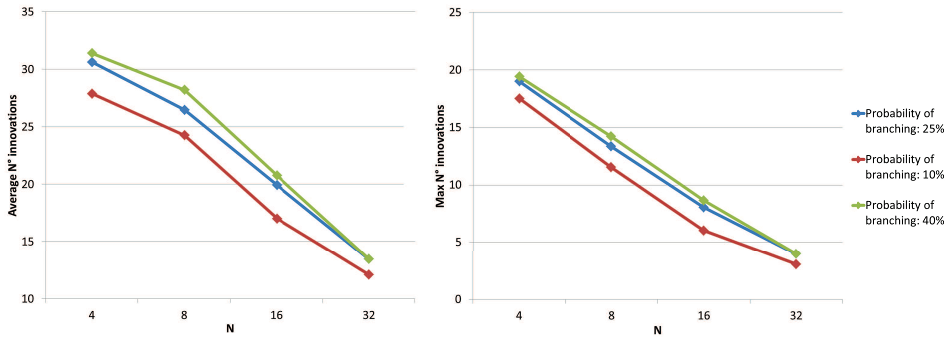

Figure 9: Total number of innovations achieved in the industry and maximum number of innovative draws achieved by a single firm with different network structures

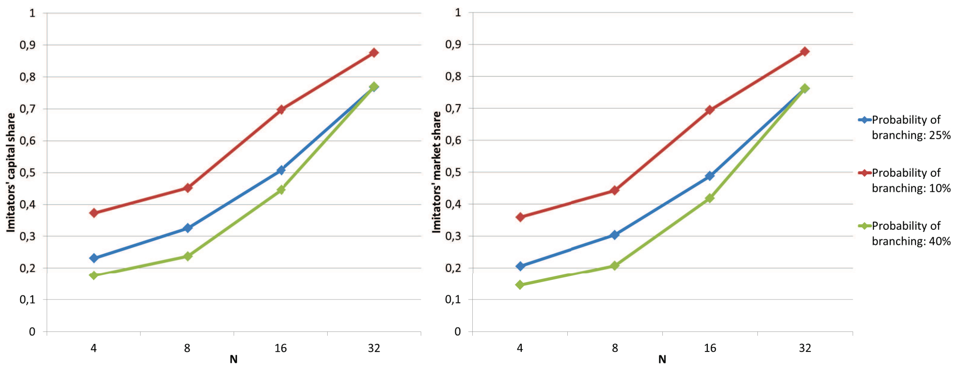

Figure 10: Capital and market shares with different network structures

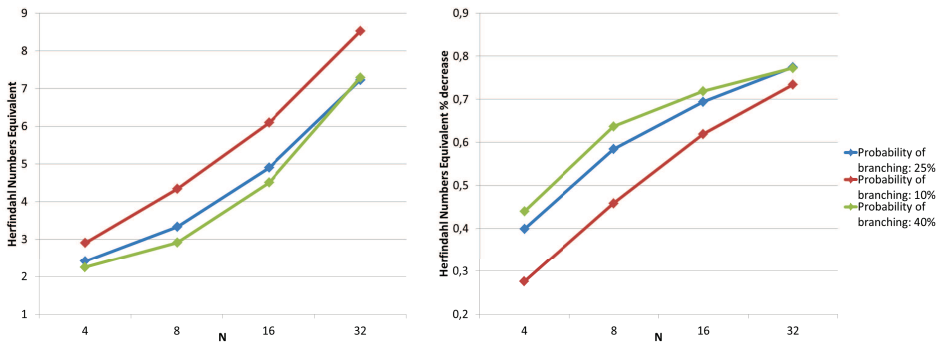

Figure 11: Herfindahl Numbers Equivalent end-of-simulations values and % reductions with different network structures

is going on we have to focus again on the determinants of best practice productivity. As already told, when the network structure used to represent the technological paradigm is highly branched, the dynamic of best practice fundamentally relies on innovative R&D activity. In this cases the structure of the network will show an high number of feasible technological trajectories. Each firm will tend to specialize upon one of them. Since each trajectory is essentially independent from the others, innovators, in order to explore deeper and deeper the trajectory they have chosen, must rely on their innovative activity. Indeed, as firms are moving in depth along different trajectories their skill profiles becomes more and more heterogeneous and this prevents innovative firms from exploiting imitation in order to achieve the sought nodes. This situation significantly changes when the network structure considered is poorly branched. In this case the structure of the network appears much more interrelated and the number of independent areas significantly reduces. You can get an impression of this fact by comparing Figure 12 which displays an example of the networks obtained by setting $P_{split} = 10\%$, with Figure 1 and Figure 13 where the networks diplayed have been obtained with P_{split} equal to 25% and 40% respectively.

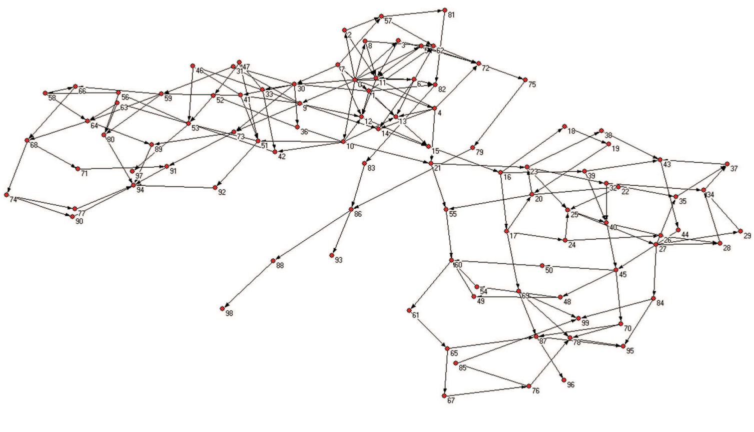

Figure 12: PT Network: $N^{nodes} = 100$, $InitialNodes = 10$, $P^{split} = 10\%$

In this case the degree of specialization of firms is significantly lower and the skill profiles among firms will be characterized by greater homogeneity. As already seen this facilitates imitators and generates a reduction in innovators' capital share and innovative R%D spending (thus leading to a reduction, on average, of both the mean and the maximum number of innovations carried out during each simulation, as shown by Figure 9). But this is not the only effect: indeed, the greater complementarity between firms' skill profiles (a consequence of the fact that the nodes are more interrelated) implies that innovators will not have to rely only on their innovative activity

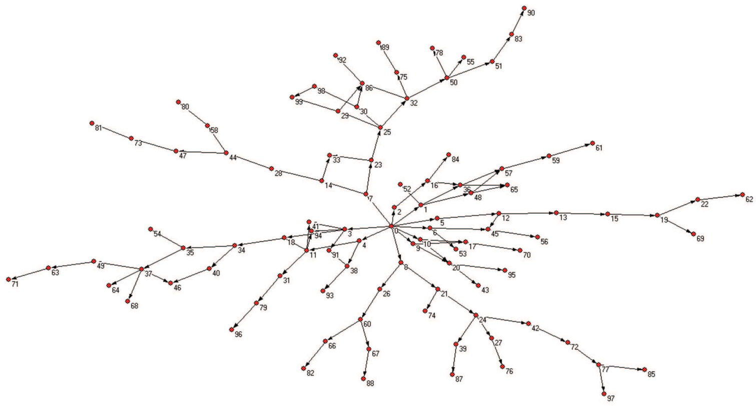

Figure 13: PT Network: $N^{nodes} = 100$, $InitialNodes = 10$, $P^{split} = 40\%$

in order to achieve each subsequent target node. Innovators can now exploit also imitation in order to enhance the overall number of nodes achieved. The results of the simulations regarding best practice and average productivity, clearly show that this possibility overcompensates the slowdown of innovator's innovative R&D performance.

The explanation for the $P_{split} = 40\%$ case results is almost specular. However, taking as benchmark the $P_{split} = 25\%$ case, we shall stress the fact that, although the direction of the changes in results in the other two cases is symmetric, the dimension of these changes is not equal. Actually the difference in the mean results of the simulations — compared to the benchamark — is much slighter in the 'more branched network' than in the 'less branched' case, so suggesting —as a first approximation— that the impact of using different network structures — on the analyzed process of schumpeterian competition — tends to decrease as the degree of branching of the network increases.

Hence, to summarize, the role played by spillovers in our model is twofold: on one side, given the technological structure, they concur to determine the transition dynamic from more to less concentrated initial industry structures; on the other side, for each industry initial structure, they affects the overall industry performance in different ways according to the shape of the network representing the Technological Paradigm. More generally, the results of the simulated model confirm that — if one takes explicitly into account the inherent cumulative nature characterizing technological progress — the analysis of the structure of technology, and in particular the analysis of the directions of technological advances undertaken by firms, are crucial for the understanding of industries evolution. Obviously in real economies, the paths of technology development can only be observed *ex post*. From an empirical point of view, much has been done in recent years. In the last section we will briefly show how the work done in this field of research can stimulate further developments of the present work.

8 Concluding remarks

Although in this paper we chose to focus on the relationship between market structure and innovation in the context of schumpeterian competition, we argue that the framework developed here might be worthy of applications to a number of other issues related to innovation and technological process. Some of them have been already handled through the original model by Nelson and Winter. In chapter fourteen of their book, for example, the authors re-elaborate the basic model in order to address what has become known in the literature as the *Schumpeterian Trade-off* between static and dynamic efficiency in innovative activity. This theme, nowadays, has become central in the debate about the opportunity and the optimal regulation of Intellectual Property Rights. On one hand the possibility of protecting an innovation through patents[39] increases the quasi-rents of innovative firms by stretching the period of time through which they can exploit an innovation. In this sense it creates an appropriability incentive that spurs innovation. On the other hand patents protection, since it ensures a monopoly on the use of a new technology, weakens the incentive of competition and reduces the strength of network economies (by constraining knowledge diffusion)[40]. Our model, since it provides an explicit structure to network externalities, seems to be particularly adequate to study this kind of trade-off.

Another interesting application of the present framework is represented by the study of competition between alternative technologies. This issue has been largely studied by Brian Arthur (see again [Arthur 1988(2)] and [Arthur 1989]) who has shown how the competitive process underlying this struggle for a market of adopters may lead to multiple equilibria, i.e. to a variety of possible outcomes. Where increasing returns and network externalities are involved, the process under consideration is intrinsically non-ergodic: different patterns of small events, determined by chance, can lead to a variety of different outcomes (eventually non-predictable). This, in turn, constitutes a source of 'potential inefficiencies' in the sense that there is no automatic mechanism implying that the technology which comes to dominate the market is the one with the longer-term higher efficiency. Inflexibility or lock-in effects may arise as well due to the fact that the *'left-behind technology*

[39]However also with spotty patent protection, there are still other 'natural' mechanism for protecting innovation, such as 'imitation lags', 'reputational advantages' and 'cost advantages' (see [Levine, Klevorick, Nelson and Winter 1987] on the role of these mechanisms in preserving monopolistic quasi-rents of innovators).

[40]Actually it also implies a variety of indirect social costs, such as increasing duplicative R&D efforts (which, in turn, reduce the efficiency of innovative research for a given amount of cumulative R&D expenditure).

would need to bridge a widening gap if it is to be chosen by adopters at all' (cit. [Arthur 1988 (2)], p.593). The basic version of the model described in the previous sections rules out the possibility that such a kind of inefficiencies arise, by assuming that each nodes provides the same level of productivity gain. Hence it is simply meaningless to make a comparison between more and less efficient trajectories (from a purely technical point of view). In order to study, in the perspective of the generative approach to simulation defined by Epstein [Epstein 2006], the conditions under which such inefficiencies arise, we should abandon this assumption. In particular we need to change the logic used in defining the set-up of the simulation by relaxing the imposed symmetry in the initial conditions characterizing firms. At the same time we would also need to differentiate the productivity gains allowed by each trajectory in order to allow a comparison between more and less efficient technological trajectories and to identify under which conditions firms will either tend to specialize along the most efficient ones or become locked in the less efficient technological paths. Finally we would like to stress a suggestive opportunity provided by recent empirical works on patent citation networks. In recent years some scholars (see for example [Verspagen 2007], [Nomaler and Verspagen 2007], [Fontana, Nuovolari and Verspagen 2009] and [Jaffe, Trajtenberg and Henderson 1993]) have used patent citation data to put the idea of technological trajectories to the test. Furthermore, by using the methodology of 'citation networks', they were able to identify the main paths of development of technology and hence to map technological trajectories in different sectors. This might provide the chance to push our analysis from a pure theoretical perspective towards more realism: by replacing our theoretical PT network — created *ad hoc* to reflect the properties of technological trajectories, as they emerge from neoschumpeterian literature — with an empirically based one, the model could become susceptible of empirical application to the investigation of innovation processes determinants, of knowledge spillovers, and industry structure evolution within different real economy sectors.

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A Appendix

A.1 Investment

As explained in the text the function defining desired investment reflects the idea that firm show some degree of awareness about the risk of spoiling the market by increasing their output simply through capital accumulation, at a certain level of productivity. In fact by increasing output they tend to lower price and hence their realized mark-up. The magnitude of this effect obviously depends on two variable: firm's market share and demand elasticity. hence the desired investment of the firm is based on a comparison between firm's current mark-up and a target markup that depend on market share. As shown in [Winter 1984] the first order condition for profit maximization implies that each firm will chose, under the conjecture that its competitors will not modify their level of output, a quantity such that:

$$\rho = \frac{\eta}{\eta - s} \tag{11}$$

where η represents demand elasticity (constant) If firm's conjecture is that the rest of the industry consists of price-takers that respond along a supply curve with constant elasticity ψ we obtain a more general results. The profit maximizing equilibrium ratio of price to marginal cost is given by:

$$\rho = \frac{\eta + (1 - s) \cdot \psi}{\eta + (1 - s) \cdot \psi - s} \tag{12}$$

Note that when $\Psi = 0$ we obtain again the previous equilibrium condition. The right-hand side of the above equation can then be interpreted as the target mark-up of a firm $\rho^T(s)$, expressed as a function of firm's market share. Hence we can expressed desired investment as:

$$I_D = \delta + 1 - \rho^T \cdot \frac{c}{P_t \cdot A_{it}} \tag{13}$$

When the realized mark-up $P_t \cdot A_{it}/c$ is exactly equal to the target mark-up $\rho^T(s)$ the firm considers itself to be in a profit maximizing equilibrium at the current level of production. So its desired investment is simply equal to the amount required to replace depreciated capital. On the contrary, when the realized mark-up is greater (smaller) than the targeted, desired net investment will be positive (negative). Equation (8) was obtained by setting $\eta = 1$ and $\psi = 1$.

A.2 Tables of results

Table 2: Results of the simulations - 5 networks generated with $P_{Split} =$ 25% & *Initial parents number=10*

		Number of Firms			
	Network	4	8	16	32
Best Practice	1	0,386 (0,055)	0,347 (0,054)	0,290 (0,057)	0,235 (0,035)
	2	0,395 (0,059)	0,336 (0,057)	0,273 (0,049)	0,229 (0,034)
	3	0,385 (0,062)	0,337 (0,052)	0,276 (0,053)	0,233 (0,040)
	4	0,381 (0,050)	0,349 (0,044)	0,295 (0,043)	0,247 (0,031)
	5	0,386 (0,055)	0,338 (0,054)	0,281 (0,050)	0,224 (0,040)
Average Productivity	1	0,355 (0,056)	0,317 (0,049)	0,262 (0,050)	0,213 (0,026)
	2	0,362 (0,058)	0,307 (0,052)	0,249 (0,043)	0,212 (0,029)
	3	0,349 (0,055)	0,309 (0,048)	0,252 (0,045)	0,214 (0,033)
	4	0,356 (0,052)	0,325 (0,038)	0,274 (0,039)	0,221 (0,027)
	5	0,350 (0,052)	0,306 (0,052)	0,249 (0,044)	0,203 0,031)
Ratio Average Productivity: Imitators/Innovators	1	0,742 (0,218)	0,891 (0,181)	0,899 (0,132)	0,976 (0,068)
	2	0,682 (0,234)	0,840 (0,168)	0,918 (0,120)	0,981 (0,087)
	3	0,677 (0,215)	0,829 (0,186)	0,903 (0,133)	0,975 (0,085)
	4	0,798 (0,224)	0,884 (0,158)	0,967 (0,080)	1,007 (0,058)
	5	0,646 (0,209)	0,723 (0,199)	0,829 (0,148)	0,966 (0,085)
Industry Total Innovations N#	1	29,80 (6,42)	26,10 (6,71)	20,55 (7,75)	13,65 (4,38)
	2	31,80 (6,83)	25,56 (6,67)	19,10 (6,65)	13,57 (5,48)
	3	31,12 (7,07)	26,52 (6,53)	19,11 (6,55)	13,48 (6,38)
	4	29,00 (6,33)	26,35 (7,26)	19,23 (6,20)	12,88 (4,86)
	5	31,33 (5,96)	27,78 (6,90)	21,60 (6,57)	13,61 (5,18)

Table 2: Continued

Max N# Innovation by a single firm	1	18,51	13,44	8,53	4,19
		(3,45)	(4,03)	(4,06)	(2,61)
	2	19,72	13,01	7,54	3,89
		(4,07)	(3,94)	(3,40)	(2,51)
	3	19,03	13,42	7,84	4,16
		(4,03)	(3,64)	(3,61)	(3,28)
	4	18,54	13,02	7,33	3,40
		(3,84)	(3,99)	(3,48)	(1,94)
	5	19,25	13,88	9,00	4,17
		(3,66)	(3,74)	(3,92)	(2,97)
Imitators' Capital Share	1	0,251	0,334	0,496	0,744
		(0,172)	(0,171)	(0,206)	(0,152)
	2	0,217	0,339	0,549	0,783
		(0,175)	(0,165)	(0,203)	(0,163)
	3	0,206	0,323	0,515	0,728
		(0,172)	(0,174)	(0,208)	(0,199)
	4	0,304	0,387	0,590	0,839
		(0,180)	(0,191)	(0,187)	(0,139)
	5	0,178	0,246	0,385	0,743
		(0,168)	(0,182)	(0,188)	(0,177)
Imitators' Market Share	1	0,226	0,311	0,476	0,736
		(0,176)	(0,182)	(0,223)	(0,165)
	2	0,190	0,316	0,532	0,777
		(0,179)	(0,174)	(0,217)	(0,176)
	3	0,177	0,301	0,496	0,720
		(0,177)	(0,184)	(0,224)	(0,212)
	4	0,284	0,372	0,582	0,839
		(0,188)	(0,202)	(0,197)	(0,134)
	5	0,150	0,218	0,352	0,734
		(0,171)	(0,189)	(0,202)	(0,196)
Herfindahl Number Equivalents	1	2,473	3,175	4,715	6,765
		(0,427)	(0,787)	(1,445)	(2,087)
	2	2,326	3,353	5,194	7,261
		(0,368)	(0,872)	(1,634)	(2,145)
	3	2,360	3,320	4,779	7,032
		(0,453)	(0,862)	(1,455)	(2,156)
	4	2,615	3,810	5,446	7,434
		(0,534)	(0,933)	(1,175)	(1,49)
	5	2,250	2,978	4,353	7,634
		(0,336)	(0,743)	(1,558)	(2,901)
Herfindahl % Decrease	1	38,2	60,3	70,5	78,9
	2	41,9	58,1	67,5	77,3
	3	40,1	58,5	70,1	78,0
	4	34,6	52,4	66,0	76,8
	5	43,8	62,8	72,8	76,1

Table 3: Results of the simulations - 5 networks generated with $P_{Split} =$ 10% & *Initial parents number=10*

		Number of Firms			
	Network	4	8	16	32
Best Practice	1	0,403 (0,043)	0,377 (0,040)	0,320 (0,035)	0,285 (0,027)
	2	0,390 (0,056)	0,349 (0,047)	0,306 (0,035)	0,277 (0,023)
	3	0,397 (0,051)	0,360 (0,052)	0,309 (0,046)	0,250 (0,035)
	4	0,380 (0,056)	0,329 (0,041)	0,273 (0,029)	0,247 (0,012)
	5	0,380 (0,049)	0,351 (0,042)	0,295 (0,036)	0,265 (0,022)
Average Productivity	1	0,392 (0,044)	0,366 (0,038)	0,308 (0,036)	0,275 (0,027)
	2	0,373 (0,057)	0,335 (0,044)	0,294 (0,033)	0,266 (0,023)
	3	0,370 (0,049)	0,338 (0,051)	0,290 (0,041)	0,237 (0,031)
	4	0,367 (0,056)	0,320 (0,040)	0,267 (0,027)	0,242 (0,014)
	5	0,360 (0,052)	0,334 (0,043)	0,280 (0,034)	0,254 0,022)
Ratio Average Productivity: Imitators/Innovators	1	0,941 (0,116)	0,963 (0,073)	1,010 (0,037)	1,037 (0,034)
	2	0,878 (0,200)	0,964 (0,070)	1,003 (0,035)	1,033 (0,036)
	3	0,805 (0,222)	0,878 (0,166)	0,972 (0,119)	1,047 (0,064)
	4	0,909 (0,189)	0,969 (0,068)	1,005 (0,018)	1,021 (0,021)
	5	0,817 (0,225)	0,905 (0,155)	0,984 (0,054)	1,031 (0,044)
Industry Total Innovations N#	1	27,08 (6,23)	24,75 (6,70)	16,35 (5,12)	12,76 (3,52)
	2	27,02 (6,73)	22,30 (6,49)	16,36 (5,52)	11,88 (3,43)
	3	28,87 (6,39)	25,44 (7,15)	19,07 (7,24)	11,96 (4,18)
	4	28,05 (6,69)	23,40 (6,40)	16,11 (4,20)	11,41 (3,36)
	5	28,34 (6,21)	25,35 (6,81)	17,11 (5,77)	12,45 (4,12)

Table 3: Continued

Max N# Innovation by a single firm	1	17,21	11,90	5,55	3,18
		(3,91)	(3,61)	(2,61)	(1,25)
	2	17,04	10,78	6,16	3,14
		(3,94)	(3,54)	(3,16)	(1,30)
	3	18,03	12,43	6,90	3,13
		(3,83)	(3,68)	(3,43)	(1,66)
	4	17,43	10,54	5,06	2,63
		(3,85)	(3,47)	(2,02)	(0,89)
	5	17,90	12,02	6,42	3,16
		(3,78)	(3,09)	(2,83)	(1,50)
Imitators' Capital Share	1	0,420	0,481	0,745	0,887
		(0,151)	(0,168)	(0,135)	(0,054)
	2	0,391	0,497	0,718	0,870
		(0,173)	(0,158)	(0,134)	(0,078)
	3	0,329	0,383	0,607	0,862
		(0,195)	(0,191)	(0,231)	(0,108)
	4	0,405	0,484	0,764	0,900
		(0,155)	(0,165)	(0,129)	(0,037)
	5	0,321	0,414	0,650	0,858
		(0,186)	(0,180)	(0,166)	(0,118)
Imitators' Market Share	1	0,412	0,475	0,746	0,890
		(0,160)	(0,173)	(0,140)	(0,055)
	2	0,377	0,490	0,717	0,873
		(0,182)	(0,164)	(0,138)	(0,081)
	3	0,309	0,367	0,600	0,865
		(0,204)	(0,201)	(0,245)	(0,113)
	4	0,395	0,479	0,764	0,902
		(0,164)	(0,170)	(0,131)	(0,037)
	5	0,304	0,402	0,645	0,859
		(0,196)	(0,189)	(0,174)	(0,123)
Herfindahl Number Equivalents	1	3,105	4,625	6,535	8,610
		(0,554)	(0,903)	(0,999)	(1,595)
	2	2,998	4,577	6,131	8,134
		(0,646)	(0,972)	(1,106)	(1,690)
	3	2,678	3,674	5,251	7,587
		(0,571)	(0,899)	(1,193)	(1,935)
	4	3,001	4,686	6,881	10,171
		(0,638)	(1,009)	(1,233)	(2,342)
	5	2,705	4,112	5,668	8,097
		(0,570)	(0,900)	(1,089)	(1,703)
Herfindahl % Decrease	1	22,4	42,2	59,2	73,1
	2	25,0	42,1	61,7	74,5
	3	33,1	54,1	67,2	76,3
	4	25,0	41,4	57,0	68,2
	5	32,4	48,6	64,6	74,7

Table 4: Results of the simulations - 5 networks generated with $P_{Split} =$ 40% & *Initial parents number=10*

		Number of Firms			
	Network	4	8	16	32
Best Practice	1	0,389 (0,054)	0,342 (0,055)	0,286 (0,055)	0,224 (0,034)
	2	0,389 (0,058)	0,346 (0,049)	0,284 (0,051)	0,224 (0,029)
	3	0,391 (0,053)	0,336 (0,050)	0,273 (0,053)	0,222 (0,037)
	4	0,388 (0,055)	0,336 (0,053)	0,284 (0,050)	0,223 (0,035)
	5	0,386 (0,052)	0,347 (0,053)	0,272 (0,051)	0,220 (0,026)
Average Productivity	1	0,347 (0,048)	0,306 (0,049)	0,254 (0,045)	0,205 (0,027)
	2	0,352 (0,052)	0,318 (0,048)	0,259 (0,043)	0,210 (0,024)
	3	0,351 (0,049)	0,293 (0,044)	0,242 (0,043)	0,202 (0,027)
	4	0,349 (0,051)	0,301 (0,048)	0,252 (0,043)	0,201 (0,025)
	5	0,346 (0,048)	0,306 (0,045)	0,242 (0,043)	0,199 0,019)
Ratio Average Productivity: Imitators/Innovators	1	0,618 (0,189)	0,722 (0,196)	0,815 (0,157)	0,978 (0,080)
	2	0,699 (0,233)	0,796 (0,196)	0,901 (0,134)	1,008 (0,071)
	3	0,579 (0,180)	0,708 (0,161)	0,845 (0,142)	0,961 (0,107)
	4	0,619 (0,211)	0,721 (0,191)	0,853 (0,145)	0,960 (0,90)
	5	0,629 (0,179)	0,703 (0,171)	0,867 (0,132)	0,977 (0,077)
Industry Total Innovations N#	1	31,78 (6,04)	28,46 (7,41)	21,73 (7,20)	14,15 (5,02)
	2	30,39 (6,32)	27,95 (6,63)	19,82 (6,05)	12,54 (4,23)
	3	31,73 (5,92)	27,23 (6,60)	21,18 (6,88)	13,92 (4,97)
	4	31,95 (6,68)	28,26 (6,51)	21,19 (6,75)	13,49 (4,02)
	5	31,10 (6,14)	29,07 (6,46)	19,83 (6,90)	13,17 (4,47)

Table 4: Continued

Max N# Innovation by a single firm	1	19,77 (3,78)	14,32 (3,80)	8,87 (3,97)	4,14 (2,79)
	2	18,77 (3,66)	13,70 3,42)	8,07 (3,72)	3,27 (1,57)
	3	19,90 (3,52)	14,35 (3,84)	8,64 (4,12)	4,30 (2,58)
	4	19,59 (3,65)	14,02 (3,68)	9,25 (3,94)	4,23 (3,02)
	5	19,10 (3,27)	14,70 (3,78)	8,42 (3,72)	3,90 (2,21)
Imitators' Capital Share	1	0,167 (0,156)	0,230 (0,177)	0,398 (0,223)	0,760 (0,174)
	2	0,230 (0,179)	0,284 (0,172)	0,498 (0,209)	0,823 (0,145)
	3	0,144 (0,146)	0,227 (0,156)	0,428 (0,204)	0,742 (0,191)
	4	0,162 (0,164)	0,224 (0,159)	0,434 (0,194)	0,745 (0,170)
	5	0,174 (0,158)	0,223 (0,159)	0,469 (0,194)	0,783 (0,153)
Imitators' Market Share	1	0,134 (0,153)	0,201 (0,186)	0,366 (0,239)	0,753 (0,189)
	2	0,204 (0,184)	0,261 (0,183)	0,479 (0,224)	0,822 (0,154)
	3	0,111 (0,142)	0,191 (0,157)	0,398 (0,220)	0,732 (0,210)
	4	0,135 (0,170)	0,194 (0,166)	0,406 (0,209)	0,732 (0,188)
	5	0,141 (0,154)	0,188 (0,162)	0,442 (0,209)	0,775 (0,169)
Herfindahl Number Equivalents	1	2,225 (0,353)	2,883 (0,686)	4,281 (1,516)	7,069 (2,365)
	2	2,356 (0,421)	3,127 (0,734)	4,831 (1,546)	7,556 (1,797)
	3	2,153 (0,291)	2,770 (0,563)	4,550 (1,594)	7,013 (2,498)
	4	2,205 (0,315)	2,911 (0,672)	4,230 (1,519)	7,316 (2,551)
	5	2,263 (0,342)	2,843 (0,629)	4,632 (1,696)	7,486 (2,263)
Herfindahl % Decrease	1	44,4	64,0	73,2	77,9
	2	41,1	60,9	69,8	76,4
	3	46,2	65,4	71,6	78,1
	4	44,9	63,6	73,6	77,1
	5	43,4	64,5	71,0	76,6