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Abstract

We study monetary policy in a New Keynesian (NK) model with endogenous growth and knowledge spillovers external to each firm. We find the following results: (i) technology and government spending shocks have different effects on growth; (ii) disinflationary monetary policies entail positive effects on growth; (iii) the optimal long-run inflation rate is zero; (iv) the Ramsey dynamics implies deviation from full inflation targeting in response to technology and government spending shocks; (v) the optimal operational rule is backward looking and responds to inflation and output deviations from their long-run levels.
Keywords: Monetary Policy, Endogenous Growth, Disinflation, Ramsey Problem, Optimal Simple Rules.
JEL codes: E32, E52, O42.

1 Introduction

The traditional NK literature is characterized by exogenous growth or no growth at all. However, since the seminal paper by Ramey and Ramey (1995) many theoretical and empirical contributions have pointed out to the importance of the relationship between short-run dynamics and long-run growth.[1] In an endogenous growth model, uncertainty is likely to affect growth-enhancing activities (i.e. savings, learning process, R&D activities) and modifies the growth trend of the entire economy.[2] Despite these results, very few papers analyze the interaction between growth and business cycle in the context of monetary models (see e.g. Dotsey and Sarte 2000 and Varvarigos 2008). An even smaller subset introduce nominal rigidities (among others Blackburn and Pelloni 2004, 2005 and Annicchiarico et al. 2011a), but in the form of one-period nominal wage contracts. An exception are Annicchiarico et al. (2011b), who consider a NK model with staggered prices

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[1]See also Jones et al. (2000, 2005), Pelloni (1997) and Blackburn (1999), among others.
[2]On the welfare consequences of business cycle fluctuations see Barlvey (2004a) and Krebs (2003). For an overview see Barlevy (2004b).

together with staggered wages to study the interplay between the two rigidities, nominal short-run volatilities and growth under different Taylor-type rules.[3] These papers, however, neither study the implications for the Ramsey-optimal monetary policy, nor derives optimal operational interest-rate rules.[4] On the other hand, in recent years, a growing body of research has studied the steady state and the dynamic properties of the NK model and the related monetary policy issues. However, to the best of our knowledge the literature concentrates on models abstracting from growth, thus, neglecting any role for the interaction between short-run dynamics and growth which instead seems to be of particular interest for the policy analysis.

In this paper we fill this gap and investigate the interaction between growth and short-run dynamics along different aspects. First, we show how the endogenous growth mechanism affects the dynamics of the competitive economy, both in response to technology and government spending shocks. Second, we analyze the transitional dynamics of the economy when the monetary policy authority implements a disinflationary policy and show the non-trivial implication for long-run growth. Third, we look at the Ramsey optimal steady state and dynamics. Finally, we characterize the monetary policy rules that are optimal within a family of implementable and simple rules in a calibrated model of the business cycle under a positive steady state inflation rate. In this respect we depart from the standard NK literature which studies optimal monetary policy in economies where long-run inflation is nil or there is some form of wide-spread indexation.[5] From an empirical point of view, neither of these two assumptions is realistic for economies like the United States or the Euro Area. Thus, it is of interest to investigate the characteristics of optimal policy in their absence and their relationship with growth.

To this scope, we consider an NK model with endogenous growth *à la* Romer (1986), and nominal rigidities due to staggered prices *à la* Calvo (1983). This economy features three sources of inefficiency which provide a rationale for the conduct of monetary policy. The first two distortions are the ones which characterize the basic NK model, namely: (i) monopolistic competition, which generates an average markup, which lowers output with respect to the efficient economy; (ii) nominal rigidities due to staggered prices, which generate price dispersion. The third source of inefficiency is the one that differentiates the present model from standard NK model, i.e. the presence of *knowledge spillovers* which are external to each firm. In other words, a sort of serendipitous learning mechanism characterizes the production activity. In this context, the decentralized equilibrium is Pareto suboptimal and the economy grows at a lower rate than under the allocation that would maximize the representative household's lifetime utility. The following main results characterize our model economy.

First, in the competitive equilibrium, with a standard Taylor rule, productivity shocks are

[3]The authors find a non-negligible negative relationship between nominal volatility and growth, in accordance with the negative relationship found in the data in response to monetary volatility. As shown by the authors, this relationship strongly depend on type of nominal rigidity characterizing the economy and on the type of the Taylor rule adopted by the central bank.

[4]Blackburn and Pelloni (2005) is the only analysis on stabilization policies and derives an optimal feedback monetary rule. From a welfare perspective their findings suggest that policy makers should focus on a simple growth objective.

[5]An exception are Schmitt-Grohe and Uribe (2007a). The author show that by assuming zero steady state inflation or full price indexation, nominal rigidities have no real consequences for economic activity and thus welfare in the long-run. Thus, the assumptions of zero long-run inflation or indexation should not be expected to be inconsequential for the form that optimal monetary policy takes.

deflationary and increase output similarly to what happens in a standard NK model. Yet, in the present framework, households find it optimal to build up the capital stock during the early phases of the adjustment process, by increasing labor supply. In other words, agents are aware that the positive effects on productivity will vanish over time and thus choose to allocate more resources to growth-enhancing activities. In an endogenous growth model this mechanism, which characterizes a standard business cycle model without nominal rigidities, is particularly powerful since a productivity shock will result in a higher consumption stream along the balanced growth path the faster the capital will grow when productivity is higher. Government spending shocks, instead, crowd out consumption and reduces growth. However, the dynamic response of the growth rate is lower and less persistent following a government spending shock than following a productivity shock.

Second, under the disinflation experiments, while the adjustment dynamics of consumption and output resembles those found in models with Calvo pricing and no growth,[6] the dynamics in work hours is reversed. Indeed, hours increase sluggishly instead of jumping on impact and then decreasing to the new lower steady state value. This is due to the fact that in a model with capital and endogenous growth disinflation is followed by an increase in capital accumulation which, in turn, gives an additional boost to output so increasing labor demand. As a result, work hours will increase instead of decreasing.

Third, even in the presence of the additional distortion due to knowledge spillovers, we find that the Ramsey steady state inflation rate is zero.[7] The reason is the following. In the long run, higher inflation rate would imply a lower return on capital and thus lower savings and lower growth and, consequently, lower consumption and output. The increase in consumption and in growth rate more than compensate the increase in hours worked and thus households' welfare increases as trend inflation decreases.

Fourth, despite the long run value of inflation is zero, the Ramsey dynamics requires deviation from full inflation targeting in response to technology and government spending shocks. However, the intensity of the reaction crucially depends on the nature of the shock. Following a positive technology shock the central bank tolerates moderate deviations of the inflation rate from its optimal steady state in order to push the short-run economy growth rate toward the efficient one. In this case optimality calls for an increase in the real interest rate so as to moderate consumption, foster capital accumulation and so growth. Also in response to a government spending shock, the optimal monetary requires an increase in the real rate so as to generate a fall in consumption and mitigate the expansionary effects of the demand shock.

Finally, the optimal operational monetary rule is backward-looking, features a strong positive reaction to output movements and a mild response to inflation, with no inertia, contrary to the previous findings in the literature.[8] As will be clear in the paper, all these results strongly depend on the role played by the endogenous growth mechanism and the implied inefficiency due to the presence of external knowledge spillovers.

Summing up, while the NK literature assumes that growth is an exogenous and independent

[6] See for example Ascari and Rossi (2012).

[7] It is well known in the literature that in a model with Calvo pricing the first two distortions require a zero steady state inflation (see King and Wolman 1999).

[8] See for example Schmitt-Grohé and Uribe (2004a), (2007a,b) who, in different models, find that the optimal interest-rate rule features a mute response to output.

process with respect to the business cycle, the literature that studies the interplay between growth and business cycle concentrates on the relationship between volatility and growth and disregards the implied optimal monetary policy prescriptions. Thus, to the best of our knowledge we are the first to study the monetary policy implication of this setup. Overall, we find that the NK literature cannot disregard the additional transmission channel introduced by the endogenous growth mechanism.

The paper proceeds as follows. Section 2 describes the model. Section 3 provides a positive description of the dynamics of the competitive economy and explores the effects of disinflationary monetary policies. Section 4 analyzes the Ramsey optimal policy. Section 5 shows results from the search of an optimal operational interest rate rule. Section 6 concludes.

2 A Sticky Price Endogenous Growth Model

The economy is described by a standard New Keynesian model with nominal prices rigidities *à la* Calvo (1983), including an endogenous growth mechanism with serendipitous learning *à la* Romer (1986). There are two sources of uncertainty: the level of total factor productivity and government spending, which is assumed to be fully financed by lump-sum taxes.

2.1 Final Good-Sector

In each period, the final good Y_t is produced by perfectly competitive firms, using the intermediate inputs produced by the intermediate sector, with the standard CES technology: $Y_t = \left[\int_0^1 Y_{j,t}^{(\theta_p-1)/\theta_p} dj\right]^{\theta_p/(\theta_p-1)}$, with $\theta_p > 1$. Taking prices as given, the typical final good producer assembles intermediate good quantities $Y_{j,t}$ to maximize profits, resulting in the usual demand schedule: $Y_{j,t} = (P_{j,t}/P_t)^{-\theta_p} Y_t$. The zero-profit condition of final good producers leads the aggregate price index $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj\right)^{1/(1-\theta_p)}$.

2.2 Intermediate Good-Sector and Externalities

The market is populated by a continuum of firms acting as monopolistic competitors. We assume that this continuum of intermediate good-producing firms $j \in [0,1]$ employ labor N_t and capital K_t from households to produce Y_t units of the intermediate good using the following technology:

$$Y_{j,t} = A_t K_{j,t}^{1-\alpha} (Z_t N_{j,t})^{\alpha}, \quad \alpha \in (0,1), \, A > 0. \tag{1}$$

where Z_t represents an index of knowledge, taken as given by each firm, which is freely available to all firms and which is acquired through learning-by-doing. In particular, we assume $Z_t = K_t$, where $K_t = \int_0^1 K_{j,t} dj$. Following convention, productivity Z_t is taken as given by each firm, so that learning takes the form of a pure externality. The term A_t is an aggregate productivity shock, which follows the following process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \varepsilon_{A,t}, \tag{2}$$

with $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$.

Prices are modeled *à la* Calvo. In each period there is a fixed probability $1-\xi_p$ that a firm in the intermediate sector can set its optimal price $P^*_{j,t}$ otherwise the price is unchanged.

Cost minimization, taking the nominal wage rate W_t and the rental cost of capital R^K_t as given, yields the standard optimality conditions, $W_t = \alpha MC^N_{j,t}\frac{Y_{j,t}}{N_{j,t}}$ and $R^K_t = (1-\alpha)MC^N_{j,t}\frac{Y_{j,t}}{K_{j,t}}$ which, in turn, imply that real marginal cost, MC_t, is common to all firms:

$$MC_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}}\frac{1}{A_t Z^\alpha_t}\left(\frac{R^K_t}{P_t}\right)^{1-\alpha}\left(\frac{W_t}{P_t}\right)^\alpha. \tag{3}$$

The typical firm, able to reset its price at time t, will choose the price P^*_t so as to maximize the expected present discounted value of profits given the demand schedule and the marginal cost MC_t. At the optimum

$$\frac{P^*_t}{P_t} = \frac{\theta_p}{\theta_p - 1}\frac{E_t\sum_{i=0}^\infty \xi^i_p Q_{t,t+i} MC_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta_p} Y_{t+i}}{E_t\sum_{i=0}^\infty \xi^i_p Q_{t,t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta_p - 1} Y_{t+i}}, \tag{4}$$

where $Q_{t,t+i}$ is the stochastic discount factor used at time t by shareholders to value date $t+i$ profits.

Define the two artificial variables $x_t = E_t\sum_{i=0}^\infty (\xi_p\beta)^i Q_{t,t+i} MC_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta_p} Y_{t+i}$ and $z_t = E_t\sum_{i=0}^\infty (\xi_p\beta)^i Q_{t,t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta_p-1} Y_{t+i}$ and denote $p^*_t = \frac{P^*_t}{P_t}$. The optimal price equation (4) now reads as follows

$$p^*_t = \frac{\theta_p}{\theta_p - 1}\frac{x_t}{z_t}, \tag{5}$$

where x_t can be written recursively as:

$$x_t = C^{-1}_t Y_t MC_t + \xi_p \beta E_t \pi^{\theta_p}_{t+1} x_{t+1}, \tag{6}$$

while z_t can be written as:

$$z_t = C^{-1}_t Y_t + \xi_p \beta E_t \pi^{\theta_p - 1}_{t+1} z_{t+1}, \tag{7}$$

where $\pi_t = P_t/P_{t-1}$. Finally, the aggregate price level $P_t = \left(\int_0^1 P^{1-\theta_p}_{j,t} dj\right)^{1/(1-\theta_p)}$ evolves according to $P_t = \left[\xi_p P^{1-\theta_p}_{t-1} + (1-\xi_p) P^{*1-\theta_p}_t\right]^{1/(1-\theta_p)}$, that is to say that the price level is just a weighted average of the last period's price level and the price set by firms adjusting in the current period. This equation can be rewritten as follows:

$$1 = \xi_p \pi^{\theta_p - 1}_t + (1-\xi_p)(p^*_t)^{1-\theta_p}. \tag{8}$$

2.3 Households

The representative household maximizes the following lifetime utility subject to a sequence of flow budget constraints:

$$E_0 \sum_{t=0}^\infty \beta^t \left(\log C_t - \mu_n \frac{N^{1+\phi}_t}{1+\phi}\right),\ \phi,\ \mu_n > 0 \text{ and } \beta < 1, \tag{9}$$

$$P_tC_t + R_t^{-1}B_{t+1} = B_t + W_tN_t + D_t + R_t^K K_t - P_tI_t - T_t, \tag{10}$$

where C_t is consumption and N_t labor hours at time t, K_t is physical capital, I_t denotes investments and B_{t+1} represents purchases of riskless one-period bonds, and paying one unit of the numéraire in the following period $t+1$, while B_t is the quantity of bonds carried over from period $t-1$. R_t is the gross nominal return on riskless bonds purchased in period t, T_t denotes lump-sum taxation and D_t are dividends from ownership of firms. Physical capital accumulates according to:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{11}$$

where δ is the depreciation rate of capital. The first-order conditions for the consumer's problem can be written as:

$$\frac{1}{R_t} = E_tQ_{t,t+1}, \tag{12}$$

$$\frac{W_t}{P_t} = \mu_n C_t N_t^{\phi}, \tag{13}$$

$$C_t^{-1} = \beta E_t C_{t+1}^{-1}\left(\tilde{R}_{t+1}^k + 1 - \delta\right), \tag{14}$$

where $Q_{t,t+1} = \beta\frac{C_tP_t}{C_{t+1}P_{t+1}}$ is the stochastic discount factor for nominal payoffs and $\tilde{R}_{t+1}^k = R_{t+1}^k/P_{t+1}$.

2.4 Market Clearing

In equilibrium factor and good markets clear, hence the following conditions are satisfied for all t: $N_t = \int_0^1 N_{j,t}dj$, $K_t = \int_0^1 K_{j,t}dj$ and $Y_tD_{p,t} = \int_0^1 Y_{j,t}$ where $D_{p,t} = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} dj$ is a measure of price dispersion. Using (1) aggregate production is found to be:

$$Y_t = AK_tN_t^{\alpha}\left(D_{p,t}\right)^{-1}, \tag{15}$$

where it is easy to see that $D_{p,t}$ evolves according to a non-linear first-order difference equation:

$$D_{p,t} = \left(1-\xi_p\right)p_t^{*-\theta_p} + \xi_p\pi_t^{\theta_p}D_{p,t-1}. \tag{16}$$

Finally, the following aggregate resource constraint must hold:

$$Y_t = C_t + I_t + G_t, \tag{17}$$

where $G_t = T_t$.

2.5 Stationary Competitive Equilibrium

In this economy a number of variables, such as output, consumption etc. will not be stationary along the balanced-growth path. We therefore perform a change of variables, so as to obtain a set of equilibrium conditions that involve only stationary variables. We note that non stationary variables at time t are cointegrated with K_t, while the same variables at time $t+1$ are cointegrated with K_{t+1}. We divide variables by the appropriate cointegrating factor and denote the corresponding stationary variables with lowercase letters. Equations (5), (8), (16), (2), (28) are already expressed in terms of stationary variables.

Capital and labor demands are now expressed as

$$\tilde{R}_t^K = (1-\alpha)\, MC_t y_t, \tag{18}$$

$$w_t = \alpha MC_t \frac{y_t}{N_t}, \tag{19}$$

where $y = Y/K$ and $w = W/PK$. In terms of stationary variables the price related equations (6) and (7) are

$$x_t = c_t^{-1} y_t MC_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p} x_{t+1}, \tag{20}$$

$$z_t = c_t^{-1} y_t + \xi_p \beta E_t \pi_{t+1}^{\theta_p - 1} z_{t+1}. \tag{21}$$

The Euler equation (12) can be expressed as

$$c_t^{-1} = E_t \beta R_t \left(c_{t+1} g_{k,t+1}\right)^{-1} \frac{1}{\pi_{t+1}}, \tag{22}$$

where $c = C/K$ and $g_{k,t+1} = K_{t+1}/K_t$.

The labor supply (13) can be written as

$$w_t = \mu_n c_t N_t^{\phi}. \tag{23}$$

The capital Euler equation (14) becomes:

$$c_t^{-1} = E_t \beta \left(c_{t+1} g_{k,t+1}\right)^{-1} \left(\tilde{R}_{t+1}^K + 1 - \delta\right). \tag{24}$$

The capital accumulation equation (11) becomes

$$g_{k,t+1} = 1 - \delta + i_t, \tag{25}$$

where $i = I/K$.

The production function (15) is simply

$$y_t = A_t N_t^{\alpha} \left(D_{p,t}\right)^{-1}. \tag{26}$$

Finally, the resource constraint of the economy (17) in stationary terms is

$$y_t = c_t + i_t + g_t, \tag{27}$$

where $g_t = G_t/K_t$ and evolves as

$$\log g_t = (1-\rho_G) \log g + \rho_G \log g_{t-1} + \varepsilon_{G,t}, \tag{28}$$

where $0 < \rho_G < 1$ and $\varepsilon_t^G \sim N(0, \sigma_G^2)$.

The competitive equilibrium of the economy under study can now be formally defined.

Definition 1: *A stationary competitive equilibrium is a sequence of allocations and prices* $\{c_t, i_t, g_{k,t+1}, N_t, y_t, \tilde{R}_t^K, MC_t, w_t, \pi_t, D_{p,t}, p_t^*, x_t, z_t\}_{t=0}^{\infty}$ *that remain bounded in some neighborhood around the deterministic steady state and satisfy equations (5), (8), (16), (18)-(27), given a sequence of nominal interest rate* $\{R_t\}_{t=0}^{\infty}$*, initial value for* $D_{p,t-1}$ *and a set of exogenous stochastic processes* $\{A_t, g_t\}_{t=0}^{\infty}$.

The competitive economy considered so far is distorted. In particular, it features three sources of inefficiency providing a rationale for the conduct of monetary policy: (i) monopolistic competition in the intermediate goods sector; (ii) nominal rigidities due to staggered prices introduced *à la* Calvo (1983); (iii) the presence of knowledge spillovers which are external to each firm.

The first two distortions are the ones which characterize the basic NK model and act as follows. Monopolistic competition in the intermediate goods sector generates an *average markup*, which lowers output with respect to the competitive economy. Nominal rigidities due to staggered prices generates *price dispersion* which, in turn, results in an inefficiency loss in aggregate production (the higher is price dispersion the more inputs are needed to produce a given level of output).[9]

The third additional source of inefficiency differentiates the present model from a standard New Keynesian model, i.e. the presence of *knowledge spillovers* which are external to each firm. In this context the decentralized equilibrium is Pareto suboptimal and the economy grows at a slower rate than under the allocation that would maximize the representative household's lifetime utility.

3 The Dynamics of the Competitive Economy

Before analyzing the optimal policy problem we will consider the dynamic properties of the model in response to a one-percent shock in technology and government spending.[10] The monetary policy is described by a Taylor-type interest rate rule, i.e.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_r} \left[\left(\frac{\pi_{t+i}}{\pi}\right)^{\phi_\pi} \left(\frac{y_{t+i}}{y}\right)^{\phi_y}\right]^{1-\phi_r} \text{with } i = 0, +1, -1, \tag{29}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$, π is the deterministic balanced growth path (BGP) value of π_t, y is the deterministic BGP value of $y_t = Y_t/K_t$. R is the deterministic BGP value of R_t and ϕ_r, ϕ_π, ϕ_y are policy parameters.

We set the benchmark parameters in line with the existing literature. Time is measured in quarters. We consider three different interest rate rules reacting to current variables (i.e. $i = 0$): (i) a standard Taylor rule with coefficients of 1.5 on inflation and 0.125 on output; (i) an augmented

[9] Schmitt-Grohé and Uribe (2007a) show that price dispersion is bounded below at one, so that it is always costly in terms of aggregate output. The intuition is that price dispersion causes firms, despite being symmetric, charge different prices and thus produce different levels of output. This, in turn, decreases the level of aggregate output by Jensen inequality because the elasticity of substitution among goods is larger than one (see Ascari 2004, King and Wolman 1998, Graham and Snower 2004).

[10] All the simulation results for the competitive economy, as well as for the optimal monetary policy, have been obtained by following the 'pure' perturbation method devised by Schmitt-Grohé and Uribe (2004c). In particular, the results on the optimal simple rules have been obtained adapting the codes developed by Faia (2008) and Faia and Rossi (2012).

Taylor rule with a degree of smoothing equal to 0.8; (iii) an inflation targeting rule, whereby the short-term interest rate is set as an increasing function of inflation deviations from its target level with a coefficient equal to 5, while the response to output is set to zero.

The discount factor β is set to 0.99, so that the annual interest rate is equal to 4 percent. The steady state inflation rate is set equal to 4% at annual level. The inverse of the Frisch elasticity ϕ, is set equal to 2. The parameter μ_n on labor disutility is calibrated to get a steady state value of labor hours equal to 1/3.[11] The price elasticity θ_p is set equal to 6 and the probability that firms do not revise prices ξ_p is set equal to 0.75. Labor return to scale α is set equal to 2/3. Finally, capital depreciation rate δ is 0.025. We calibrate the remaining parameters to have $C/Y = 0.65$ in steady state and a quarterly growth rate of output of 0.5% along the BGP.

Similarly to Schmitt-Grohé and Uribe (2007a) the persistence of the technology shock is $\rho_a = 0.8556$, while that of the government spending shock is $\rho_g = 0.87$. The standard deviations of productivity and of the government purchases processes are set equal to $\sigma_a = 0.0064$ and $\sigma_g = 0.016$, respectively.

We show the impulse response functions for a set of selected variables (consumption, inflation, hours, output, rate of growth and nominal interest rates) under the three different monetary policy rules.

Figure 1 shows the impulse response functions to a one percent increase in productivity under the inflation targeting rule (continuous lines), the Taylor rule (dashed lines) and the Taylor rule with smoothing (dotted-dashed lines). As expected, an increase in productivity raises output, consumption and the capital growth rate.

When the monetary policy takes the form of a Taylor rule, with or without smoothing, an increase in productivity is deflationary. In this case, in fact, the higher productivity brings about a fall in firms' marginal costs so pushing down the inflation rate. The fall in inflation is associated with a less than proportional fall in the nominal interest rates since the monetary policy also reacts to output. We also observe a contractionary effect of productivity shocks on aggregate employment. Intuitively, since prices adjust slowly in the short run, preventing an increase in aggregate demand, firms take advantage of the productivity increase by reducing labor demand.

Conversely, when the monetary authority adopts an inflation targeting rule, an increase in productivity becomes slightly inflationary and the monetary authority reacts by raising the nominal interest rate, so that real interest rate increases. In this case the inflationary effects of a productivity shock results from higher marginal costs.[12] This apparently counterintuitive result can be explained as follows. Agents are aware of the fact that by targeting inflation, and not responding to output, the monetary authority fully accommodates the productivity shock. As a consequence, firms will increase their investment expenditure even more than under a standard Taylor rule. The larger investments expenditure will raise aggregate demand, so that firms able to re-set their prices will find it optimal to increase prices. For the same reason, under an inflation targeting rule the demand for hours will increase instead of decreasing. At the same time, in this framework households find it optimal to build up the capital stock during the early phases of the adjustment process, by increasing labor supply. In other words, agents are aware that the positive effects on productivity

[11] Sensitivity analysis have been done on alternative preference parameters and results are qualitatively unchanged.

[12] This result is consistent with the analysis of Schmitt-Grohé and Uribe (2004a) who argue that when the monetary policy is conducted according to a simple Taylor rule responding only to inflation deviations from its target, in the absence of adjustment costs on investments, a positive shock to productivity is inflationary.

will dissipate over time, that is why they are willing to allocate more resources to growth-enhancing activities. In an endogenous growth model, this mechanism which characterizes a standard business cycle model, is particulary powerful since a productivity shock will result in a higher consumption stream along the balanced growth path the faster capital will grow during the adjustment phase characterized by higher productivity.

- Figure 1 about here -

Figure 2 shows the responses of the six selected variables to a one-percent increase in government spending under the three rules. Overall, higher government spending crowds out consumption and investments, while labor hours increase. The crowding out of investments reduces capital accumulation and consequently its growth rate falls down. The effect on consumption is modest, reflecting the fact that households almost succeed in smoothing out consumption expenditure. The resulting changes in the real interest rates are small, so implying a weak reaction to inflation. However the direction of this change in inflation depends on the rule adopted. Under an inflation targeting rule, inflation increases, consistently with the observed higher marginal costs, and implies an increase in the nominal interest rate. Conversely, when short-run interest rate also reacts to output, a positive government spending shock will be associated with a fall in inflation. The reason is the following. The standard Taylor rule combined with the Fisher relation, up to a first order, implies that inflation follows a process of the form, $\widehat{\pi}_{t+1} = 1.5\widehat{\pi}_t + 0.125\widehat{y}_t - \widehat{rr}_t$ where all variables are expressed as deviations from their steady state level and $\widehat{rr}_t$ is the real interest rate. It can be shown that following an increase in public spending the term $0.125\widehat{y}_t - \widehat{rr}_t$ becomes positive. Since in equilibrium the inflation rate cannot explode, it must be the case that inflation is below its steady value along the adjustment process. Putting it differently, the stronger response of the monetary policy to the aggregate demand increase will drive down the expected marginal costs, so that firms revising their prices will find it optimal to decrease prices.

- Figure 2 about here -

3.1 Permanent Disinflation

In this section we look at an unanticipated and permanent reduction in the inflation target of the Central Bank. The Central Bank follows the standard Taylor rule (i.e. $\phi_\pi = 1.5$, $\phi_y = 0.125$, $\phi_r = 0$). Figure 3 shows the path for output, inflation, nominal interest rate, consumption, hours and the growth rate in response to such a change in the Central Bank policy regime. We consider three cases: a disinflation from 4%, 6% and 8% trend inflation to zero steady state inflation, i.e. $\pi_{new} = 1$.[13]

[13] The simulation results of this section have been obtained by numerically solving the non-linear deterministic model in DYNARE which uses a Newton-type algorithm. For details, http://www.cepremap.cnrs.fr/dynare/ and Adjemian et al. (2010).

- Figure 3 about here -

First, we notice that the dynamic adjustment of the model after a disinflation is inertial and depends on the initial value of trend inflation, consistently with Ascari and Ropele (2012) and Ascari and Rossi (2012). Indeed, under Calvo pricing the model displays price dispersion, i.e., $D_{p,t}$, that is a backward-looking variable that adjusts sluggishly after a disinflationary policy. Thus, the standard NK model features an endogenous state variable, and the model dynamics is inertial. However, our model has an additional source of inertia given by capital accumulation, which turns out to be of particular interest for the dynamics of an endogenous growth model. As a result of the two inertial components, output, consumption, hours and the rate of growth increase sluggishly in response to a permanent disinflationary policy, while the nominal interest rate decreases.

It should be emphasized that the response of hours is the result of the interaction of two opposing forces. On the one hand, in a model with Calvo pricing, inflation creates a wedge between production inputs and aggregate output, through price dispersion. Clearly, the lower price dispersion, the less the inputs are needed for a given level of output. In the absence of capital, the lower price dispersion under the new trend inflation will imply a lower level of hours worked in the long run.[14] On the other hand, in a model with capital accumulation and endogenous growth, disinflation is followed by an increase in capital accumulation which gives an additional boost to output, so increasing labor demand. In this context the latter effect prevails so that, as a result of the disinflationary policy, work hours will increase instead of decreasing. Given the above results, it comes as no surprise that disinflationary policies are unambiguously beneficial for growth.

4 Ramsey Monetary Policy

The Ramsey optimal policy is determined by the central bank which maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The Ramsey approach allows to study the optimal policy around a distorted steady state, as it is in our model.[15] We assume that *ex-ante commitment* is feasible. In most New Keynesian models it is not possible to combine all constraints in a single implementability constraint, thus, as common in the literature, we follow an hybrid approach in which the competitive equilibrium conditions are summarized via a minimal set of equations, as reported in the Appendix.

In this context, the central bank chooses the policy instrument, namely the inflation rate, to implement the optimal allocation obtained as solution to the Ramsey problem.

4.1 The Ramsey Optimal Steady State

In what follows we analyze the optimal monetary policy in the long-run by looking at the Ramsey optimal steady state inflation rate. This amounts to computing the *modified golden rule* steady

[14] As shown by Ascari and Rossi (2012) along the adjustment, output is increasing, while price dispersion is decreasing. From period 2 onwards, the latter effect dominates, making aggregate production more efficient and thus saving hours worked, despite the rise in output.

[15] See Khan et al. (2003), Schmitt-Grohé and Uribe (2007a), Faia (2009) for a discussion on welfare analysis with a distorted steady state.

state inflation, i.e. the steady state inflation rate obtained imposing steady state conditions ex post on the first order conditions of the Ramsey plan.[16]

We find that the steady state inflation rate associated with the Ramsey optimal policy turns out to be zero in an NK economy characterized by endogenous growth and knowledge spillovers. This happens, because the central bank chooses the inflation rate that allows to push the steady state growth rate toward the efficient one. Specifically, given the parametrization used in the previous Section, under the Ramsey plan, the balanced-growth path growth rate is slightly higher than under the competitive equilibrium and equal to 0.53% at quarterly level, so confirming that zero steady state inflation is beneficial for growth.[17]

The reason is the following. First, it is worth to recall that the distortions due to monopolistic competition and the one due to staggered prices, i.e. the *average markup* and the *price dispersion respectively,* go in the direction of decreasing output as the steady inflation rate increases. More in detail, as shown by King and Wolman (1996), the average markup, i.e. $\frac{P}{MC^N}$ can be split in two terms: $\frac{P}{MC^N} = \frac{P}{P_i^*}\frac{P_i^*}{MC^N}$, with MC^N denoting the nominal marginal cost. The first term, i.e., $\frac{P}{P_i^*}$, is the "*price adjustment gap*", i.e. the ratio between the CPI index P and the newly adjusted price P_i^*. King and Wolman (1996) show that this ratio decreases with steady state inflation because trend inflation erodes the markups and the relative prices that were set by non-adjusting firms in past periods. The second term, i.e., $\frac{P_i^*}{MC^N}$, instead, is called the "*marginal markup*" by King and Wolman (1996), and it is an increasing function of the steady state inflation rate. Indeed, price resetting forward-looking firms will set higher prices relative to their current marginal costs, exactly to offset the erosion of markups and relative prices that trend inflation creates, if they will not be allowed to change their price in the future. The "*marginal markup*" effect generally dominates the "*price adjustment gap*" effect.[18] This means that higher trend inflation yields a larger average markup, hence a larger distortion and a lower steady-state output. Analogously, price dispersion increases rapidly with trend inflation, so that the negative effect of trend inflation on output through this channel is quite powerful (see Ascari 2004 and Yun 2005). Overall, both the average markup and price dispersion are therefore increasing with trend inflation, generating a negative steady state relationship between inflation and output. Therefore, if these two distortions were the only ones affecting the economy, the central bank would not face any trade off and would reach the first best allocation by setting the steady state inflation rate equal to zero (see Galí 2008, and Blanchard and Galí 2007 for a nice and detailed explanation on the so called *divine coincidence*).

In a model with knowledge spillovers, capital accumulation is below its social optimum value since agents do not price the role of capital stock in increasing productivity. Hence, although, the aggregate production function has constant returns to scale in capital, due to knowledge spillovers, firms will accumulate capital as if they are facing a production function with decreasing returns to scale and therefore investments will be sub-optimal and the economy will grow at a lower level than the optimal one. With respect to capital accumulation and growth, zero steady state inflation turns

[16] The Ramsey steady-state equilibrium was calculated numerically by using an OLS approach, as described by Schmitt-Grohé and Uribe (2012).

[17] The optimality of the zero steady inflation turns out to be confirmed also under alternative parametrization. Results are available from the authors upon request.

[18] Actually the price adjustment gap is stronger for extremely low values of trend inflation, such that average markup first slightly decreases and then increases with trend inflation (see King and Wolman 1996).

out to be the optimal one. Intuitively and consistently with the above findings on the implications of disinflationary policies, a positive inflation rate would imply a lower return on capital and thus lower savings and lower growth and, consequently, lower steady state consumption and output levels and less worked hours. Under a zero steady state inflation, instead, the higher growth rate and the higher level of consumption will more than compensate the utility loss deriving from more hours worked. That is why households' welfare increases as trend inflation decreases.

4.2 The Ramsey Optimal Dynamics

Let's now analyze the dynamic properties of the Ramsey plan in a calibrated version of the model. The dynamic responses of the Ramsey plan are computed by taking second order approximations of the set of first order conditions around the steady state.[19] The calibration of the model follows the one presented in Section 3.1.

Figure 4 shows the Ramsey optimal impulse response functions to a one percent positive productivity shock for consumption, inflation, employment, output, rate of growth and nominal interest rates. As in the competitive economy, output and consumption increase. Inflation rate decreases, but the nominal interest rate is above its long run level implying a higher real interest rate. Intuitively, since the beneficial effects of a positive innovation on productivity are temporary, the Ramsey planner, will find it optimal to create the conditions that induce households to build up the capital stock during the early phases of the adjustment process and to increase labor supply. As a result of this the rate of growth will be higher.

- Figure 4 about here -

Figure 5 shows impulse response functions to a one percent positive government spending shock. The government spending shock crowds out consumption and investments. The inflation and the nominal interest rate responses are such that the real rate is always positive along the adjustment path. In this context, the optimal monetary policy calls for an increase in the real rate so as to moderate the temporary expansionary effects of aggregate demand on output and the corresponding increase in hours. Intuitively, following an increase in public consumption (a pure waste in this economy), the welfare-maximizing planner will find it preferable to suffer a higher short-run decrease of the growth rate (than for instance that observed under an inflation targeting rule), than a sharp increase in the disutility deriving from non-leisure activity.

- Figure 5 about here -

Summing up, the optimal Ramsey dynamics requires a deviation from price stability in an economy characterized by endogenous growth and knowledge spillovers. The central bank tolerates a moderately negative inflation rate in order to push the short-run economy growth rate toward the

[19]See Schmitt-Grohé and Uribe (2004c).

efficient one in response to a positive technology, while increases the real interest rate to mitigate the effects of a positive government spending shock on output and labor hours.

Overall, a model with knowledge spillovers, and thus endogenous growth, is characterized by having the capital growth rate into the welfare function. As a consequence, it turns out that a full inflation targeting policy is far from being optimal.

5 Optimal Operational Interest Rate Rules

Despite the Ramsey policy delivers the optimal policy functions in response to shocks, in practice most of the central banks nowadays implement simple feedback interest rate rules. For this reason, we now study the optimal operational interest rate rules. Such a rule is obtained by searching, within the class of Taylor-type rules, for the parameters that maximize households conditional welfare subject to the competitive equilibrium conditions that characterize the model economy. As well explained by Schmitt-Grohé and Uribe (2004a, 2007a,b) and Faia (2008) among others, this class of rules must satisfy the following four criteria: a) they must be simple, i.e. they must involve only observable variables; b) they have to guarantee the uniqueness of the rational expectation equilibrium; c) they must be optimal, i.e. they have to maximize the expected life-time utility of the representative agent; d) they must respect the zero bound on nominal interest rates.

Following Schmitt-Grohé and Uribe (2004a, 2007a,b) we focus on the *conditional* expected discounted utility of the representative agent, that is:[20]

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + \xi \log \frac{M_t}{P_t} - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} \right), \tag{30}$$

however, in a model with endogenous growth the value function (30) needs to be stationarized. Thus, after some algebra we find the following expression:

$$\upsilon_t = \log c_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{k,t+1} + \beta E_t \upsilon_{t+1}. \tag{31}$$

where $\upsilon_t = V_t - \frac{1}{1-\beta} \ln (K_t)$.

The analysis of the optimal rules and the welfare comparison with ad-hoc rules is done based on the Taylor-type class of rules as in equation (29), where R, π and y are set equal to their respective competitive steady-state levels.

The central bank searches for the optimal rule by maximizing the welfare (31), subject to the constraints represented by the competitive economy relations. Numerically, the search is conducted over the parameter space given by $\left\{ \phi_\pi, \phi_y, \phi_r \right\}$. The parameter i in the Taylor rule is alternatively set to $i = 0$ for contemporary rules, $i = -1$ for backward-looking rules and finally to $i = 1$ for forward-looking rules. Further,

[20] Specifically, the conditional measure of welfare assumes a initial state of the economy and allows to take into account the transitional effects from that initial condition to the stochastic steady state implied by the policy rule adopted by the monetary authorities. As commonly assumed, the initial state of the economy coincides with the deterministic steady state.

We resort to constrained optimal rules, that is we restrict the grid-search to consider only empirically relevant values for the policy parameters ϕ_π and ϕ_y : we search for ϕ_π restricted to lie in the interval $[1, 3]$ and for ϕ_y in $[0, 2]$ with a step of size 0.0625 and for ϕ_r restricted in $[0, 0.9]$ with a step of size 0.1. As in Schmitt-Grohé and Uribe (2007a) we approximate the non-negativity constraint on the nominal interest rate by requiring that a rule must induce a low volatility of the nominal interest rate around its target level, that is we impose the condition: $2\sigma_R < R$, where σ_R denotes the standard deviation of the nominal interest rate. Table 1 summarizes the results.

The optimal operational rule takes the form:

$$\ln\left(\frac{R_t}{R}\right) = 1.3125 \ln\left(\frac{\pi_{t-1}}{\pi}\right) + 0.9375 \ln\left(\frac{y_{t-1}}{y}\right). \tag{32}$$

Thus, optimized interest-rate rule: i) is a backward-looking rule; ii) requires a vigorous response to output and a quite aggressive reaction to inflation; iii) does not feature inertia.

First, it is worth to notice the following. Differently, from the existing literature,[21] we find that the coefficient attached to past output is positive and strongly different from zero. The coefficient relative to the response to past inflation is instead higher than 1, but not too strong. From this point of view this result differs from what is generally found in the literature.[22] Intuitively, a rule such as (32) entails a strong coutercyclical component which is in line with the behaviour of the Ramsey planner who tends to increase the real interest rate in response to positive demand and supply shocks.

Second, as shown in Table 1, the optimal operational rule exhibits a strong level of inertia in interest rates when the feedback part of the rule is current-looking or forward-looking. Thus, overall, the optimal operational interests rules require either to target past inflation and past output, or a high response to past interest rate when the policy is current-looking or forward-looking. Overall, the strong interest-rate inertia or, alternatively, the strong response to past output together with the one-to-one response to past inflation, implies that the monetary authority must be inertial so to stabilize inflation expectations.[23] An economy with capital and knowledge externalities is particularly sensitive to changes in inflation expectations since they will alter expectations on the real return on capital accumulation.

Finally, as argued by many authors, both the current and the forward-looking rule are not truly operational. If on the one hand, the forward-looking rules require information on future inflation expectation, which is not directly observable, on the other hand the current rule requires information on the current value of inflation and output which are not in the information set of the central bank. Thus, according to these critics, a rule is truly operational only when is based on past information. Thus, we can claim that the backward-looking rule implied by our model with endogenous growth is not only the optimal one, but it is indeed the truly operational rule.

[21] See for example Schmitt-Grohé and Uribe (2004a, 2007a, 2007b) among others.

[22] See for example Schmitt-Grohé and Uribe (2007b) who, in a medium scale model, find that the optimal interest-rate rule responds to current price and wage inflation, while it is mostly mute in output, and implies only moderate inertia. In Schmitt-Grohé and Uribe (2007a, 2004a) optimized policy rules feature mute response to output and no inertia.

[23] By doing so, the monetary authority stabilizes inflation expectations toward its non-zero inflation target, i.e. the one ensuring higher long-run growth.

6 Conclusions

We consider an NK model characterized by endogenous growth with serendipitous learning *à la* Romer, and nominal rigidities due to staggered price *à la* Calvo. An additional source of inefficiency differentiates our model from the standard NK model, i.e. knowledge spillovers which are external to each firm. The decentralized equilibrium is Pareto suboptimal and the economy grows at a lower rate than under the allocation that would maximize the representative household's lifetime utility. In this context we find the following results. First, in the competitive equilibrium, under a standard Taylor rule productivity shocks are likely to be deflationary and tend to increase output as usual, while, at the same time, labor hours and the growth rate increase. Intuitively, since the beneficial effects of the positive productivity shock will dissipate over time, agents find it optimal to allocate more resources to growth-enhancing activities during the expansionary phase of the business cycle. Positive government spending shocks, instead, will reduce growth.

Second, under the disinflation experiments consumption, output and hours inertially increase along the transitional dynamics. The increase in work hours is due to the increase in capital accumulation which, in turn, pushes up output so boosting labor demand and work hours.

Third, despite the optimal long-run value of inflation is zero, the Ramsey dynamics requires deviation from full inflation targeting in response to technology and government spending shocks. However, the intensity of the reaction crucially depends on the source of fluctuations. Following a positive technology shock the central bank tolerates moderate deviations of the inflation rate below its optimal steady state coupled with a higher nominal rate in order to foster savings and push up the short-run economy growth rate. In response to a positive government shock, optimality calls for an increase in the real interest rate so as to moderate the effects of the expansionary policy.

Finally, the optimal operational monetary rule is found to be backward-looking, featuring a strong response to output deviations and a mild reaction to inflation movements.

Overall, we find that macroeconomic stabilization policy must explicitly consider the additional transmission channel introduced by an endogenous growth mechanism. In this sense, our analysis provides a further step towards the understanding of the non-trivial interconnections between macroeconomic fluctuations and growth. The analysis of the present paper has been deliberately restricted to the analysis of monetary policy in the context of a very simple endogenous growth model. We argue that future research should be oriented to explore in more depth these issues considering different and more realistic growth models as well as the implications for both monetary and fiscal policy.

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Appendix

Welfare Measure

The welfare of the typical individual (30) can be written in recursive form as:

$$V_t = \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \beta E_t V_{t+1}. \tag{33}$$

By adding and subtracting $\frac{1}{1-\beta} \log K_t$ and $\frac{\beta}{1-\beta} \log K_{t+1}$ we get

$$\begin{aligned} V_t \;\; = \;\; & \log C_t - \mu_n \frac{{N_t}^{1+\phi}}{1+\phi} + \\ & - \log K_t + \frac{1}{1-\beta} \log K_t - \frac{\beta}{1-\beta} \log K_t + \\ & + \frac{\beta}{1-\beta} \log K_{t+1} - \frac{\beta}{1-\beta} \log K_{t+1} + \beta E_t V_{t+1}, \end{aligned} \tag{34}$$

where we have used the fact that $\frac{1}{1-\beta} \log K_t = \log K_t + \frac{\beta}{1-\beta} \log K_t$. Collecting terms and defining $\upsilon_t = V_t - \frac{1}{1-\beta} \ln K_t$ yield (31) which can be also expressed as:

$$\upsilon_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\log c_{t+j} - \mu_n \frac{{N_{t+j}}^{1+\phi}}{1+\phi} + \frac{\beta}{1-\beta} \log g_{k,t+1+j} \right). \tag{35}$$

Ramsey Problem

The Ramsey optimal policy is determined by the central bank which maximizes the discounted sum of utilities of all agents, (35), given the constraints of the competitive economy, namely (5), (8), (16), (18)-(27).

The number of constraints to the Ramsey planner's optimal problem can be reduced by substitution so to have:

$$\beta E_t \frac{1}{c_{t+1}} \left[(1-\alpha) \frac{\mu_n N_{t+1}^{\phi+1} c_{t+1}}{\alpha} + 1 - \delta \right] - \frac{g_{k,t+1}}{c_t} = 0, \tag{36}$$

$$c_t + g_{k,t+1} + g_t - A_t N_t^{\alpha} \left(D_{p,t}\right)^{-1} - (1-\delta) = 0, \tag{37}$$

$$D_{p,t} - \left(1-\xi_p\right) \left(\frac{\theta_p}{\theta_p - 1} \frac{x_t}{z_t} \right)^{-\theta_p} - \xi_p \pi_t^{\theta_p} D_{p,t-1} = 0, \tag{38}$$

$$\xi_p \pi_t^{\theta_p - 1} - 1 + \left(1-\xi_p\right) \left(\frac{\theta_p}{\theta_p - 1} \frac{x_t}{z_t} \right)^{1-\theta_p} = 0, \tag{39}$$

$$x_t - \frac{\mu_n N_t^{\phi+1}}{\alpha} - \xi_p \beta E_t \pi_{t+1}^{\theta_p} x_{t+1} = 0, \tag{40}$$

$$\xi_p \beta E_t \pi_{t+1}^{\theta_p - 1} z_{t+1} - z_t + c_t^{-1} A_t N_t^{\alpha} \left(D_{p,t}\right)^{-1} = 0, \tag{41}$$

$$E_t\beta\left(c_{t+1}g_{k,t+1}\right)^{-1}\frac{1}{\pi_{t+1}}-\frac{c_t^{-1}}{R_t}=0. \tag{42}$$

Let $\{\lambda_{1,t},\ \lambda_{2,t},\ \lambda_{3,t},\ \lambda_{4,t},\ \lambda_{5,t},\ \lambda_{6,t},\lambda_{7,t}\}$ represent sequences of the Lagrange multipliers on the constraints (36), (37), (38), (39), (40), (41), (42), respectively. Given an initial value for the price dispersion, $D_{p,t-1}$, and a set of exogenous stochastic processes for productivity and public consumption, $\{A_t, g_t\}_{t=0}^{\infty}$, the allocations plans for the control variables $\mathbf{d}_t \equiv \{c_t,\ g_{k,t+1},\ N_t,\ \pi_t, D_{p,t},\ x_t,\ z_t, R_t\}_{t=0}^{\infty}$ and for the co-state variables $\mathbf{\Lambda}_t \equiv \{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t},\lambda_{5,t},\lambda_{6,t}\}_{t=0}^{\infty}$ represent an optimal allocation if they solve the following maximization problem:

$$Min_{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}Max_{\{\mathbf{d}_t\}_{t=0}^{\infty}}E_0\sum_{t=0}^{\infty}\beta^t\left(\log c_t-\mu_n\frac{N_t^{1+\phi}}{1+\phi}+\frac{\beta}{1-\beta}\log g_{k,t+1}\right), \tag{43}$$

subject to constraints (36)-(42).

Since the above maximization problem exhibits forward-looking constraints is intrinsically non-recursive (see Kydland and Precott 1980). As common practice in the literature (i.e. Khan et al. 2003), we circumvent this problem by augmenting the policy problem with a full set of lagged multipliers, corresponding to the constraints exhibiting future expectations of control variables.

The augmented Lagrangian for the optimal policy problem then reads as follows:

$$\begin{aligned}
&Min_{\{\mathbf{\Lambda}_t\}_{t=0}^{\infty}}Max_{\{\mathbf{d}_t\}_{t=0}^{\infty}}E_0\left\{\sum_{t=0}^{\infty}\beta^t E_t\left[\left(\log c_t-\mu_n\frac{N_t^{1+\phi}}{1+\phi}+\frac{\beta}{1-\beta}\log g_{k,t+1}\right)+\right.\right.\\
&+\lambda_{1,t-1}\frac{1}{c_t}\left((1-\alpha)\frac{\mu_n N_t^{\phi+1}c_t}{\alpha}+1-\delta\right)-\lambda_{1,t}\frac{g_{k,t+1}}{c_t}+\\
&+\lambda_{2,t}\left(c_t+g_{k,t+1}+g_t-A_tN_t^{\alpha}\left(D_{p,t}\right)^{-1}-(1-\delta)\right)+\\
&+\lambda_{3,t}\left(\frac{D_{p,t}}{\pi_t^{\theta_p}}-\left(1-\xi_p\right)\frac{1}{\pi_t^{\theta_p}}\left(\frac{\theta_p}{\theta_p-1}\frac{x_t}{z_t}\right)^{-\theta_p}-\xi_pD_{p,t-1}\right)+\\
&+\lambda_{4,t}\left(\xi_p\pi_t^{\theta_p-1}-1+\left(1-\xi_p\right)\left(\frac{\theta_p}{\theta_p-1}\frac{x_t}{z_t}\right)^{1-\theta_p}\right)+\\
&+\lambda_{5,t}\left(x_t-\frac{\mu_nN_t^{\phi+1}}{\alpha}\right)-\lambda_{5,t-1}\xi_p\beta\pi_t^{\theta_p}x_t+\\
&+\lambda_{6,t-1}\xi_p\pi_t^{\theta_p-1}z_t-\lambda_{6,t}\left(z_t-c_t^{-1}A_tN_t^{\alpha}\left(D_{p,t}\right)^{-1}\right)+\\
&\left.\left.\lambda_{7,t-1}\left(c_tg_{k,t}\right)^{-1}\frac{1}{\pi_t}-\lambda_{7,t}\frac{c_t^{-1}}{R_t}\right]\right\}.
\end{aligned}$$

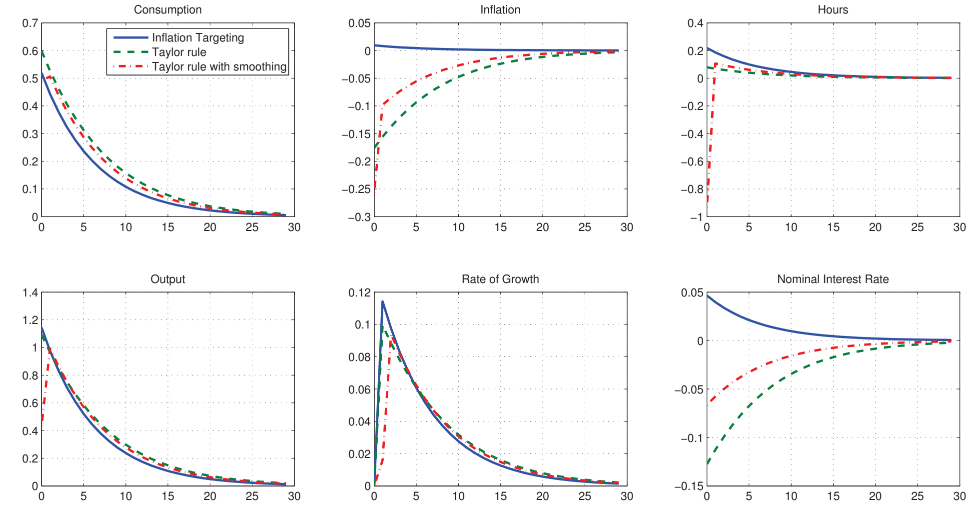

Figure 1: Dynamic responses to a 1% increase in productivity under different interest rules.

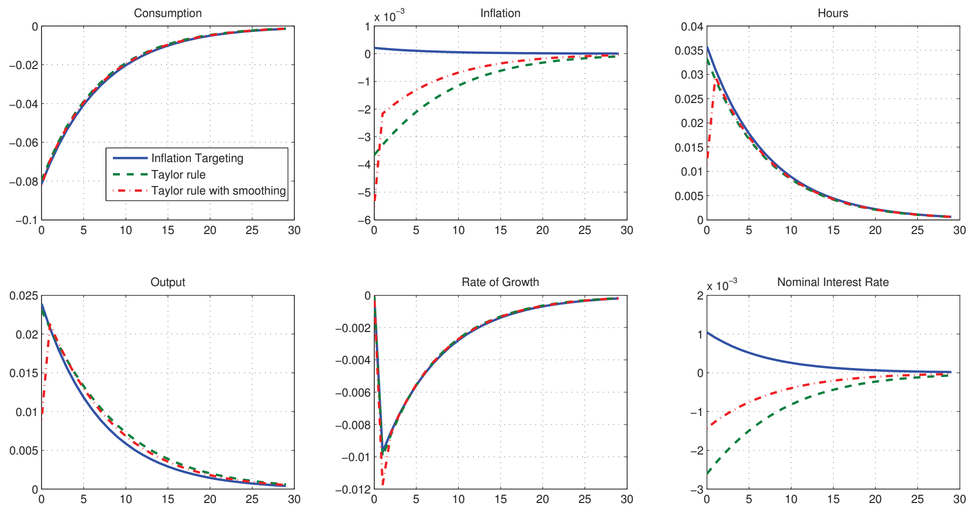

Figure 2: Dynamic responses to a 1% increase in government spending under different interest rules.

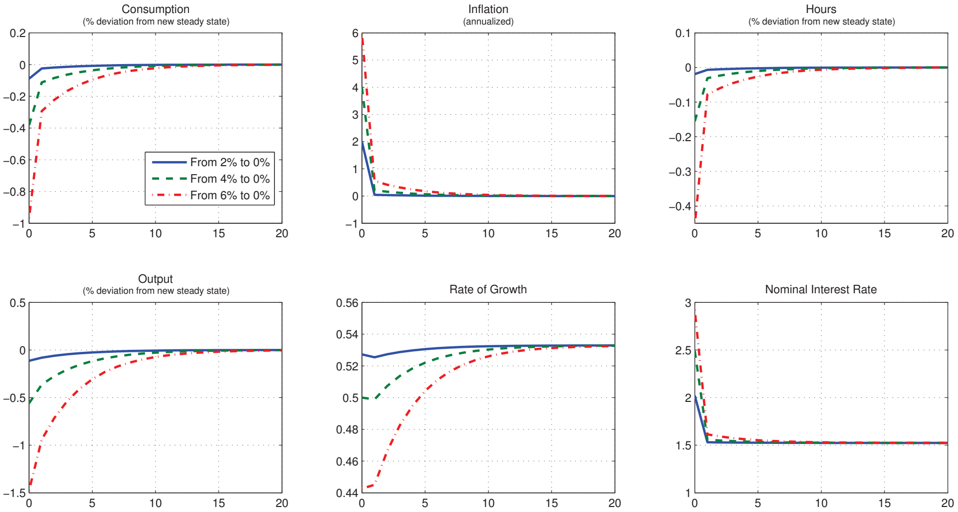

Figure 3: Permanent disinflation

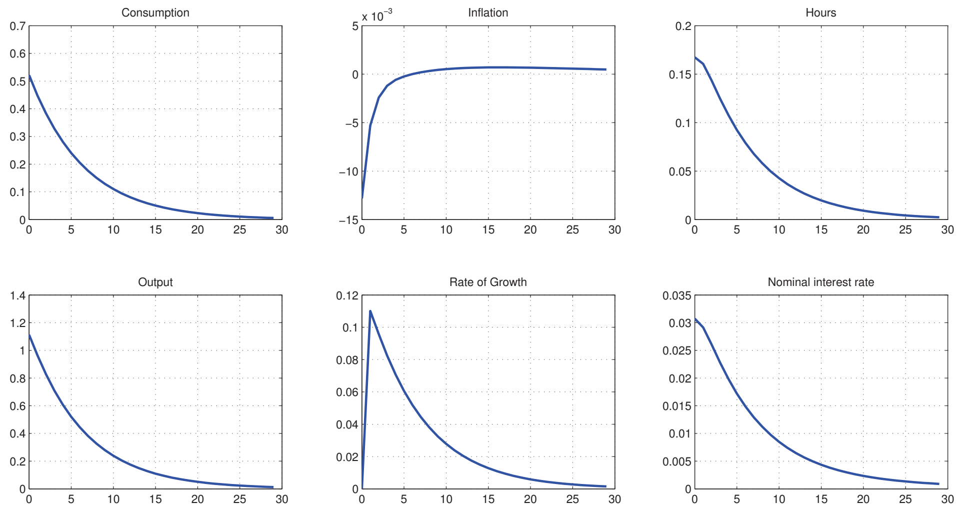

Figure 4: Dynamic responses to a 1% increase in productivity under Ramsey monetary policy

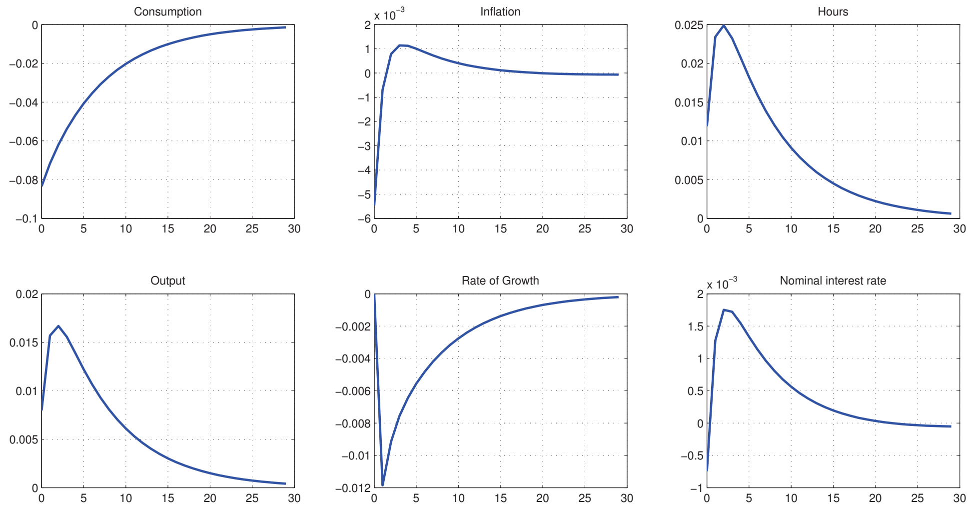

Figure 5: Ramsey dynamics to a 1% increase in government spending under Ramsey monetary policy

Table 1: Optimal Monetary Policy

	ϕ_π	ϕ_y	ϕ_r	**Conditional Welfare**
Ramsey Policy	–	–	–	-217.6216
Optimal Rules				
Current looking	1.4375	0.0625	0.9000	-221.3196
Backward looking	1.3125	0.9375	0	-217.6375
Forward looking	1.3125	0.0625	0.70000	-220.0160
Non-Optimized Rules				
Inflation targeting	5	0	0	-221.2846
Standard Taylor	1.50	0.125	0	-221.6647
Taylor with smoothing	1.50	0.125	0.8	-221.6066