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**Transmission Lags and Optimal Monetary Policy**

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# Transmission Lags and Optimal Monetary Policy

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## Abstract

Real world monetary policy is complicated by long and variable lags in the transmission of the policy to the economy. Most of the policy models, however, abstracts from policy lags. This paper presents a model where transmission lags depend on the behaviour of a two-sector supply side of the economy and focuses on how lag *length* and *variability* affect optimal monetary policy. The paper shows that optimal monetary policy should respond more to the sector with the shortest transmission lag and that the presence of production links among sectors amplifies this response. Furthermore, the shorter or more variable the *aggregate* transmission lag, the more active the overall policy and the larger the response to the sector with the shortest transmission lag. Finally, the relative strength of the response to inflation and output gap depends on the intensity of the sectoral production links, and on the length of the transmission lags. Only with reasonable production links should the optimal policy respond more to inflation than to the output gap in line with the empirical evidence.

JEL Classification: E52, E58, F41.

Key Words: Inflation targeting; monetary policy transmission mechanism; policy transmission lags; multiplicative uncertainty; Markov jump linear quadratic systems; optimal monetary policy.

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# 1 Introduction

*"(...) Monetary action takes a longer time to affect the price level than to affect the monetary totals and both the time lag and the magnitude of effect vary with circumstances. As a result, we cannot predict at all accurately just what effect a particular monetary action will have on the price level and, equally important, just when it will have that effect. Attempting to control directly the price level is therefore likely to make monetary policy itself a source of economic disturbance because of false stops and starts. Perhaps, as our understanding of monetary phenomena advances, the situation will change."* Friedman (1968), The Role of Monetary Policy, AER.

Since Friedman's seminal paper, the understanding of monetary phenomena has remarkably improved, yet long and variable lags in the transmission of the monetary policy to the economy keep on posing a major challenge to policymakers. Indeed, according to central banks experience, it takes 2-6 quarters for monetary action to influence the output gap with a peak effect in around one year, and 4-10 quarters to affect inflation with peak effect in around two years.

Motivated by this policy problem, the question this paper focuses on is how *long* and *variable* transmission lags affect the design of the optimal monetary policy. To address the question the paper presents a model with transmission lags in the demand and supply side of the economy. While the former is constant, the latter can vary resulting in an overhaul variable lag from policy action to inflation. The *aggregate* transmission lag in the supply side is determined by two sectors that produce different *composite* goods and differ in their sectoral transmission lag and, possibly, in the number of firms. Attempting to gain a general picture of the working of the economy, the two composite goods are assumed to be a services and a manufacturing good. In each sector there are many firms and it is the firms' pricing behaviour that determines the time length of the response of sectoral inflation to sectoral marginal costs. This length, based on the possibility that prices can be chosen either when they take effect or before, generates the *sectoral* transmission lag of policy action to inflation. Sectoral transmission lags, in turn, generate the aggregate transmission lag whose variability depends on the time-varying number of firms in each sector.

A distinctive characteristic of the model is the possibility that production links among sectors exist. The interest in this further dimension of analysis is that it allows studying the impact of transmission lags on optimal policy when the goods produced in one sector can be only consumed or used as an input by the two sectors, or both.

It is worth noting that the use of a two-sector model is also consistent with the important role that sectoral analysis plays in the policy process at central banks. Indeed, as noted by Sims (2002) in his description of the policy process at the Swedish Riksbank, the ECB, the Bank of England, and the FED, sectoral experts play a major role in the policy decisions by providing sectoral analysis and forecasts.

The current model draws on Rotemberg and Woodford (1998), Benigno (2004), Boivin and Giannoni (2006), and Flamini (2007). The innovation with respect to the previous New-Keynesian literature is twofold. On the one hand the paper embeds the possibility that in a two-sector economy firms in one sector use the output produced by the other sector as an input along with labour. On the other hand, the model is set up in a framework in which optimal monetary policy is determined considering model uncertainty, that is relaxing the Certainty Equivalence principle. The former is important in that it captures the production link among sectors, which is a key feature of real world economy playing a significant role in the transmission mechanism of monetary policy and exogenous shocks. The latter matters as the impact of lag uncertainty on policymakers' optimal decision rule can be investigated if and only if lag uncertainty is formalised as model uncertainty.

The contribution of the paper lies in showing that accounting for policy transmission lag plays an important role in the design of the optimal policy rule. Structural features in the behaviour of heterogeneous firms result in different sectoral lags. When the design of the optimal policy takes these differences into account, the sector with the shortest transmission lags stands out as the sector to which the optimal rule should respond the most.

The rest of the paper is structured as follows. Section 2 presents a two-sector New-Keynesian model where the aggregate transmission policy lag to CPI inflation depends on the sectoral-specific pricing behaviour of the firms. Section 3 presents and discusses the results assuming that the central bank knows the aggregate transmission lag. An important novelty of the current model with respect to the canonical two-sector New-Keynesian model is that it allows considering production links among sectors. Section 4 relaxes the assumption of perfect information on the aggregate transmission lag and extends the analysis to the more realistic scenario that policymakers have normally to face. Section 5 concludes.

## 2 The model

In this economy the representative household seeks to maximize

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \tilde{u}(C_T; \bar{C}_T) - \int_0^1 \tilde{v}[h_t(j)] dj \right\},$$

where  $\beta$  is the intertemporal discount factor,  $C_t$  represents all interest-rate-sensitive expenditure including investments and is defined, following Woodford (2003, Chapter 3), as a CES aggregate

$$C_t \equiv \left[ (n^s \varphi_t^s)^{1/\rho} (C_t^s)^{(\rho-1)/\rho} + (n^m \varphi_t^m)^{1/\rho} (C_t^m)^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)}, \quad (1)$$

of the goods  $C_t^s$  and  $C_t^m$  which are produced, respectively, by the s and m-sector, with  $\rho$  defining their elasticity of substitution,  $n^s$  and  $n^m$  ( $n^s \equiv 1 - n^m$ ) the number of goods of each sector in  $C_t$ , and  $\varphi_t^s$  and  $\varphi_t^m$  positive random coefficients satisfying  $n^s \varphi_t^s + n^m \varphi_t^m = 1$ . The presence of  $\varphi_t^s$  and  $\varphi_t^m$  is motivated by the interest to study exogenous disturbances to the relative demand for the two sectors' goods and their effect on the aggregate transmission lag. Assuming a unit interval continuum of differentiated goods indexed by  $i$ , let each sectoral good be a Dixit-Stiglitz aggregate of the continuum of differentiated goods produced in the sector:

$$C_t^s \equiv \left[ n^{s-\frac{1}{\theta}} \int_0^{n^s} (C_t^s(i))^{1-\frac{1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad C_t^m \equiv \left[ n^{m-\frac{1}{\theta}} \int_{n^s}^1 (C_t^m(i))^{1-\frac{1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (2)$$

where  $\theta > 1$  is the elasticity of substitution between any two differentiated goods. Finally,  $\bar{C}_t$  is an exogenous preference shock and  $h_t(j)$  is the quantity supplied of labour of type  $j$ . Period preferences on consumption and labour are modeled as CRRA functions

$$\tilde{u}(C_t; \bar{C}_t) \equiv \bar{C}_t^{\frac{1}{\tilde{\sigma}}} \frac{C_t^{1-\frac{1}{\tilde{\sigma}}} - 1}{1 - \frac{1}{\tilde{\sigma}}}, \quad (3)$$

$$\tilde{v}(h_t) \equiv \frac{h_t^{1+\nu}}{1+\nu}, \quad (4)$$

where  $\tilde{\sigma} > 0$  captures the intertemporal elasticity of substitution in consumption and  $\nu > 0$  is the inverse of the elasticity of goods production.

The price index for the minimum cost of a unit of  $C_t$  is given by

$$P_t \equiv \left[ n^s \varphi_t^s (P_t^s)^{1-\rho} + n^m \varphi_t^m (P_t^m)^{1-\rho} \right]^{1/(1-\rho)}, \quad (5)$$

with  $P^s$ ,  $P^m$  denoting, respectively, the Dixit-Stiglitz price index for goods produced in the  $s$  and  $m$  sector

$$P_t^s \equiv \left[ n^{s-1} \int_0^n p^s(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad P_t^m \equiv \left[ n^{m-1} \int_n^1 p^m(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

Preferences captured by equation (1) imply that the optimal sectoral consumptions level are given by

$$C_t^s = n^s \varphi_t^s C_t \left( \frac{P_t^s}{P_t} \right)^{-\rho}, \quad (6)$$

$$C_t^m = n^m \varphi_t^m C_t \left( \frac{P_t^m}{P_t} \right)^{-\rho}. \quad (7)$$

Financial markets are assumed to be complete so that at any date all households face the same budget constraint and consume the same amount. Thus, utility maximization subject to the budget constraint and the no-Ponzi scheme requirement yields the condition for optimal consumption

$$Q_{t,t+1} = \beta \frac{U_c(C_{t+1}; \bar{C}_{t+1})}{U_c(C_t; \bar{C}_t)} \frac{P_t}{P_{t+1}}, \quad (8)$$

where  $Q_{t,t+1}$  is the stochastic discount factor by which financial markets discount random nominal income in period  $t+1$ . Recalling that with no arbitrage opportunities the riskless short term nominal interest rate  $i_t$  and the stochastic discount factor  $Q_{t,t+1}$  are related by

$$\frac{1}{1+i_t} = E_t[Q_{t,t+1}],$$

equation (8) implies

$$\frac{1}{1+i_t} = \beta E_t \left[ \frac{U_c(C_{t+1}; \bar{C}_{t+1})}{U_c(C_t; \bar{C}_t)} \frac{P_t}{P_{t+1}} \right]. \quad (9)$$

Finally, utility maximization requires that the optimal supply of labour of type  $j$  is given by

$$\Omega_t(j) = \frac{\tilde{v}_h[h_t(j)]}{\tilde{u}_c(C_t; \bar{C}_t)}, \quad (10)$$

where  $\Omega_t(j)$  is the real wage demanded for labour of type  $j$ .

Turning to production, each household  $i$  is assumed to supply all type of labour and is a monopolistically competitive producer of one differentiated good, either  $y^m(i)$  or  $y^s(i)$ . In this economy any producer  $i$  belongs to an industry  $j$  which, in turn, belongs either to sector  $s$  or  $m$ . Furthermore, there is unit interval continuum of industries indexed by  $j$  and in each industry there is a unit interval continuum

of good indexed by  $i$  so that the total number of goods is one. Since in equilibrium all the firms belonging to an industry will supply the same amount, they will also demand the same amount of labor. As a result, the total demand of labour in an industry is equal the demand of labor of any differentiated firm in the industry.

I now model the connection between the manufacturing and services sectors by assuming that in each sector the differentiated good  $i$  is produced with two inputs: labor of type  $j$ , which is industry-specific, and the composite good produced by the other sector, that is either  $Y_t^m$  or  $Y_t^s$ , which are given by the appropriate Dixit-Stiglitz aggregators

$$Y_t^s \equiv \left[ \frac{1}{n^s} \int_0^{n^s} [y_t^s(i)]^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad Y_t^m \equiv \left[ \frac{1}{n^m} \int_{n^s}^1 [y_t^m(i)]^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \quad (11)$$

Describing the production function, to capture the feature that the combination of labour and the other sector composite output tends to be rigid at business cycle frequency, I assume that each firm uses a concave-Leontief technology of the form

$$y_t^m(i) = A_t \left[ \min \left( \frac{h_t^m(i)}{1 - \mu^{s,m}}, \frac{Y_t^{s,m}(i)}{\mu^{s,m}} \right) \right]^{\frac{1}{\phi}},$$

$$y_t^s(i) = A_t \left[ \min \left( \frac{h_t^s(i)}{1 - \mu^{m,s}}, \frac{Y_t^{m,s}(i)}{\mu^{m,s}} \right) \right]^{\frac{1}{\phi}},$$

where  $\phi > 1$ ,  $h_t^m$  and  $Y_t^{s,m}$  denote, respectively, the quantity of labour and of the (composite) s-sector good used as inputs in the m-sector and  $(1 - \mu^{s,m})$  and  $\mu^{s,m}$  denote, respectively, the shares of labour and s-sector good in the composite input required to produce the differentiated m-sector good  $y_t^m(i)$ ; similarly,  $h_t^s$  and  $Y_t^{m,s}$  are, respectively, the quantities of labour and the composite good  $Y_t^m$  used to produce the service good  $y_t^s(i)$  and  $(1 - \mu^{m,s})$  and  $\mu^{m,s}$  denote, respectively, the shares of labour and m-sector good in the composite input required to produce the differentiated s-sector good  $i$ . Thus the input requirement functions are

$$h_t^m(i) = (1 - \mu^{s,m}) \left[ \frac{y_t^m(i)}{A_t} \right]^{\phi}, \quad (12)$$

$$Y_t^{s,m}(i) = \mu^{s,m} \left[ \frac{y_t^m(i)}{A_t} \right]^{\phi}, \quad (13)$$

for firm  $i$  in the  $m$  sector and

$$h_t^s(i) = (1 - \mu^{m,s}) \left[ \frac{y_t^s(i)}{A_t} \right]^{\phi}, \quad (14)$$

$$Y_t^{m,s}(i) = \mu^{m,s} \left[ \frac{y_t^s(i)}{A_t} \right]^{\phi}, \quad (15)$$

for firm  $i$  in the  $s$  sector.

We now turn to the determination of the demand curves for the differentiated goods. It is worth noting that a monopolistically competitive supplier in one sector will demand a quantity of any *differentiated good* in the other sector equal to the quantity of the required *composite good* produced by the other sector. Indeed, in equilibrium, any firm  $i$  in sector  $k \in \{s, m\}$  will produce the same quantity,  $y_t^k(i) = y_t^k$ . Then, given (11), it follows that  $y_t^k = Y_t^k$ . Hence, accounting for (13), the demand for the differentiated good  $i$  belonging to sector  $s$  for production in sector  $m$  is given by  $n^m \mu^{s,m} \left[ \frac{Y_t^m}{A_t} \right]^\phi$ . Similarly, the demand for the differentiated good  $i$  belonging to sector  $m$  for production in sector  $s$  is given by  $n^s \mu^{m,s} \left[ \frac{Y_t^s}{A_t} \right]^\phi$ . Furthermore, accounting for the preferences (1-2), the total quantity demanded for each differentiated good in the manufacturing and services sector are, respectively,

$$\begin{aligned} y_t^m(i) &= C_t^m(i) + n^s Y_t^{m,s}(i) \\ &= C_t \left( \frac{p_t^m(i)}{P_t^m} \right)^{-\theta} \left( \frac{P_t^m}{P_t} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_t^s}{A_t} \right]^\phi \end{aligned} \quad (16)$$

$$\begin{aligned} y_t^s(i) &= C_t^s(i) + n^m Y_t^{s,m}(i) \\ &= C_t \left( \frac{p_t^s(i)}{P_t^s} \right)^{-\theta} \left( \frac{P_t^s}{P_t} \right)^{-\rho} + n^m \mu^{s,m} \left[ \frac{Y_t^m}{A_t} \right]^\phi \end{aligned} \quad (17)$$

In equilibrium, market clearing in the goods market requires

$$Y_t^m = C_t^m + n^s \mu^{m,s} \left( \frac{Y_t^s}{A_t} \right)^\phi, \quad (18)$$

$$Y_t^s = C_t^s + n^m \mu^{s,m} \left( \frac{Y_t^m}{A_t} \right)^\phi, \quad (19)$$

Then, combining (3), (9), (6-7) and (18-19) we obtain the nonlinear version of the aggregate demand in the  $m$  and  $s$ -sector respectively

$$\frac{1}{1+i_t} = \beta E_t \left[ \frac{\bar{C}_{t+1}^{\frac{1}{\sigma}} \left[ \left( Y_{t+1}^m - n^s \mu^{m,s} \left( \frac{Y_{t+1}^s}{A_{t+1}} \right)^\phi \right) \left( \frac{P_{t+1}^m}{P_{t+1}} \right)^\rho \right]^{-\frac{1}{\sigma}}}{\bar{C}_t^{\frac{1}{\sigma}} \left[ \left( Y_t^m - n^s \mu^{m,s} \left( \frac{Y_t^s}{A_t} \right)^\phi \right) \left( \frac{P_t^m}{P_t} \right)^\rho \right]^{-\frac{1}{\sigma}}} \frac{P_t}{P_{t+1}} \right], \quad (20)$$

$$\frac{1}{1+i_t} = \beta E_t \left[ \frac{\bar{C}_{t+1}^{\frac{1}{\sigma}} \left[ \left( Y_{t+1}^s - n^m \mu^{s,m} \left( \frac{Y_{t+1}^m}{A_{t+1}} \right)^\phi \right) \left( \frac{P_{t+1}^s}{P_{t+1}} \right)^\rho \right]^{-\frac{1}{\sigma}}}{\bar{C}_t^{\frac{1}{\sigma}} \left[ \left( Y_t^s - n^m \mu^{s,m} \left( \frac{Y_t^m}{A_t} \right)^\phi \right) \left( \frac{P_t^s}{P_t} \right)^\rho \right]^{-\frac{1}{\sigma}}} \frac{P_t}{P_{t+1}} \right]. \quad (21)$$



## 2.1 Firm problem with sticky prices and monopolistic competition

It is important to note that in this economy the marginal costs of the firm  $i$  in a given sector, say  $m$ , depend also on the *quantity* of the composite output produced by the other sector, i.e.  $s$ . Indeed, due to sectoral interdependencies (working through the technological parameters  $\mu^{m,s}$  and  $\mu^{s,m}$ ), when the production in the  $s$  sector increases, more of the input  $Y_t^m$  is required by the  $s$  sector. This, in turns, increases the demand for firm  $i$  in the  $m$  sector and therefore increases its marginal costs.

Moving to the producers' pricing behaviour, firms in both sectors fix their prices at random intervals following the Calvo (1983) staggered price scheme and have the opportunity to change their prices with probability  $(1 - \alpha)$ . Starting with firms in the manufacturing sector, a producer that is allowed to set its price in period  $t$  chooses its new price for the random period starting in  $t$ ,  $\tilde{p}_t^m$ , to maximize the flow of expected profits:

$$\max_{\tilde{p}_t^m} E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ \tilde{p}_t^m y_T^m(i) - \left[ \frac{y_T^m(i)}{A_T} \right]^\phi \left[ (1 - \mu^{s,m}) \frac{\tilde{v}_h[h_T(j)]}{\tilde{u}_c(C_T; \bar{C}_T)} P_T + \mu^{s,m} P_T^s \right] \right\}.$$

Substituting the demand function (16) and accounting for the CRRA period preferences specification (3-4) yields

$$\begin{aligned} & \max_{\tilde{p}_t^m} E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ \tilde{p}_t^m C_T \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + \tilde{p}_t^m n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^\phi \right. \\ & \quad - \left[ \frac{C_T \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^\phi}{A_T} \right]^\phi \\ & \quad \times \left. \left[ \frac{(1 - \mu^{s,m})^\nu \left[ \frac{C_T \left( \frac{p_t^m(j)}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^\phi}{A_T} \right]^\nu}{\bar{C}_T^\sigma C_T^{-\sigma}} P_T + \mu^{s,m} P_T^s \right] \right\} \end{aligned}$$

where to ease the notation I set  $\sigma \equiv \tilde{\sigma}^{-1}$ . Note that the total demand of labour in an industry is equal the demand of labor of any differentiated firm in the industry<sup>1</sup>. Note

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<sup>1</sup>In this economy there is unit interval continuum of industries indexed by  $j$  and in each industry there is a unit interval continuum of good indexed by  $i$ . Thus the total number of goods is one and since in equilibrium all the firms belonging to an industry will supply the same amount, they will also demand the same amount of labor. As a result, the total demand of labour in an industry is equal the demand of labor of any differentiated firm in the industry. For this reason we have  $h_T(j) = (1 - \mu^{s,m}) \left[ \frac{y_t^m(j)}{A_t} \right]^\phi$ .

also that  $p_t^m(j)$  is at time  $t$  because all the firms in the same industry are assumed to change their price at the same time. Turning to profit maximization, the f.o.c. is

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ C_T \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} - \theta C_T \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi} \right. \\ \left. + \phi \left[ \frac{C_T \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\phi-1} \theta C_T^m \left( \frac{\tilde{p}_t^m}{P_T^m} \right)^{-\theta-1} \left( \frac{P_T^m}{P_T} \right)^{-\rho} \frac{1}{P_T^m} \right. \\ \left. \left[ \frac{(1 - \mu^{s,m})^{\nu} \left[ \frac{C_T \left( \frac{p_t^m(j)}{P_T^m} \right)^{-\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\nu\phi}}{\overline{C}_T^{\sigma} C_T^{-\sigma}} P_T + \mu^{s,m} P_T^s \right] \right\} = 0$$

Then we replace  $\tilde{p}_t^m$  and  $p_t^m(j)$  with  $p_t^{m*}$  and in order to substitute  $\frac{p_t^{m*}}{P_T^m}$  with  $\frac{p_t^{m*}}{P_t^m}$  we multiply the powers of  $\frac{p_t^{m*}}{P_T^m}$  by terms of the type  $\left( \frac{P_T^m}{P_t^m} \right)^{-a} \left( \frac{P_T^m}{P_t^m} \right)^a$  where  $a$  is an appropriate exponent. After simplification, this leads to

$$\left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} (1 - \theta) C_T \left( \frac{P_T^m}{P_t^m} \right)^{\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} \\ + \phi \theta E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \frac{C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} \left( \frac{P_T^m}{P_t^m} \right)^{\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\phi-1} C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta-1} \left( \frac{P_T^m}{P_t^m} \right)^{\theta+1} \\ \left( \frac{P_T^m}{P_T} \right)^{-\rho} \left\{ \frac{(1 - \mu^{s,m})^{\nu+1} \left[ \frac{C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} \left( \frac{P_T^m}{P_t^m} \right)^{\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\nu\phi}}{\overline{C}_T^{\sigma} C_T^{-\sigma}} \frac{P_T}{P_T^m} + \mu^{s,m} \frac{P_T^s}{P_T^m} \right\} \\ = -E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}$$

Now let

$$F_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} f_T \left( \frac{P_T^m}{P_t^m} \right)^{\theta}, \quad (22)$$

$$R_t \equiv -E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} r_T, \quad (23)$$

$$G_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} L_{t,T} M_{t,T} N_{t,T}, \quad (24)$$

and

$$f_t \equiv (1 - \theta) C_t \left( \frac{P_t^m}{P_t} \right)^{-\rho}, \quad (25)$$

$$r_t \equiv n^s \mu^{m,s} \left[ \frac{Y_t^s}{A_t} \right]^{\phi} \quad (26)$$

$$L_{t,T} \equiv \left[ \frac{C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} \left( \frac{P_T^m}{P_t^m} \right)^{\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\phi-1} \quad (27)$$

$$M_{t,T} \equiv C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta-1} \left( \frac{P_T^m}{P_t^m} \right)^{\theta+1} \left( \frac{P_T^m}{P_T} \right)^{-\rho} \quad (28)$$

$$N_{t,T} \equiv \left\{ \frac{(1 - \mu^{s,m})^{\nu+1} \left[ \frac{C_T \left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} \left( \frac{P_T^m}{P_t^m} \right)^{\theta} \left( \frac{P_T^m}{P_T} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_T^s}{A_T} \right]^{\phi}}{A_T} \right]^{\nu\phi}}{\bar{C}_T^{\sigma} C_T^{-\sigma}} \frac{P_T}{P_T^m} + \mu^{s,m} \frac{P_T^s}{P_T^m} \right\} \quad (29)$$

then the f.o.c. can be rewritten as

$$\left( \frac{p_t^{m*}}{P_t^m} \right)^{-\theta} = \frac{R_t}{F_t} - \phi\theta \frac{G_t}{F_t}. \quad (30)$$

Finally, note that the price index in the m-sector evolves following the law of motion

$$P_t^m = \left[ (1 - \alpha) (p_t^{m*})^{1-\theta} + \alpha (P_{t-1}^m)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

which is convenient to rewrite as

$$\frac{p_t^{m*}}{P_t^m} = \left[ \frac{1 - \alpha (\Pi_t^m)^{\theta-1}}{(1 - \alpha)} \right]^{\frac{1}{1-\theta}}. \quad (31)$$

Hence, the first order condition (30) together with the law of motion for  $P_t^m$ , (31), and the definitions (22-29) represent a non linear version of the New-Keynesian Phillips curve in presence of two sectors with production links.

## 2.2 Existence and uniqueness of the steady state equilibrium

Recall that

$$y_t^m(i) = C_t \left( \frac{p_t^m(i)}{P_t^m} \right)^{-\theta} \left( \frac{P_t^m}{P_t} \right)^{-\rho} + n^s \mu^{m,s} \left[ \frac{Y_t^s}{A_t} \right]^\phi \quad (32)$$

In presence of flexible prices, the monopolistic competitive representative firm  $i$  in sector  $m$  sets the optimal price  $\tilde{p}_t^m$  in any period to maximize the period profit

$$\max_{\tilde{p}_t^m} \left\{ \tilde{p}_t^m y_t^m(i) - \left[ \frac{y_t^m(i)}{A_t} \right]^\phi \left[ (1 - \mu^{s,m}) \frac{\tilde{v}_h[h_t(j)]}{\tilde{u}_c(C_t; \bar{C}_t)} P_t + \mu^{s,m} P_t^s \right] \right\}$$

and the f.o.c. consists of setting the price as a mark-up on marginal costs

$$\tilde{p}_t^m = \frac{\theta}{(\theta - 1)} \phi \left[ \frac{y_t^m(i)}{A_t} \right]^{\phi-1} \left[ (1 - \mu^{s,m}) \frac{\tilde{v}_h[h_t(j)]}{\tilde{u}_c(C_t; \bar{C}_t)} P_t + \mu^{s,m} P_t^s \right].$$

Let  $s^m$  be the real marginal cost in the  $m$ -sector

$$s^m \left( y_t^m(i), C_t, \frac{P_t^s}{P_t^m}; \xi_t \right) \equiv \phi \left[ \frac{y_t^m(i)}{A_t} \right]^{\phi-1} \left[ (1 - \mu^{s,m}) \frac{\tilde{v}_h[y_t^m(i)]}{\tilde{u}_c(C_t; \bar{C}_t)} \frac{P_t}{P_t^m} + \mu^{s,m} \frac{P_t^s}{P_t^m} \right]$$

where "real" is with respect to the price of the composite good in the  $m$  sector.

Notice that accounting for (5) we obtain

$$\left( \frac{P_t}{P_t^m} \right)^{1-\rho} = n^s \varphi_t^s \left[ \left( \frac{P_t^s}{P_t^m} \right)^{1-\rho} - 1 \right] + 1, \quad (33)$$

so that  $s^m$  turns out to be a function only of  $(y_t^m(i), C_t, \frac{P_t^s}{P_t^m}; \xi_t)$ . Finally,  $\xi_t \equiv (A_t, \bar{C}_t, \varphi_t^s)'$  is a vector of shocks. Then the f.o.c. can be rewritten as

$$\frac{\tilde{p}_t^m}{P_t^m} = \frac{\theta}{(\theta - 1)} s^m(y_t^m(i), C_t; \xi_t).$$

Now, considering the demand for good  $i$  in sector  $m$ , we have that

$$\frac{p_t^m(i)}{P_t^m} = \frac{\left[ y_t^m(i) - n^s \mu^{m,s} \left( \frac{Y_t^s}{A_t} \right)^\phi \right]^{-\frac{1}{\theta}}}{C_t^{-\frac{1}{\theta}}} \left( \frac{P_t^m}{P_t} \right)^{-\frac{\rho}{\theta}}$$

Then, the supply of good  $i$  must satisfy

$$\frac{\left[ y_t^m(i) - n^s \mu^{m,s} \left( \frac{Y_t^s}{A_t} \right)^\phi \right]^{-\frac{1}{\theta}}}{C_t^{-\frac{1}{\theta}}} \left( \frac{P_t^m}{P_t} \right)^{-\frac{\rho}{\theta}} = \frac{\theta}{(\theta - 1)} s^m \left( y_t^m(i), C_t, \frac{P_t^s}{P_t^m}; \xi_t \right).$$

Now notice that the LHS and the RHS are, respectively, decreasing and increasing in  $y_t^m(i)$ . Thus there is only one value of  $y_t^m(i)$  that satisfies the previous equation

given  $\left(C_t, Y_t^s, \frac{P_t^s}{P_t^m}\right)$ . In equilibrium all the firms in the  $m$ -sector produce the same quantity so that it must be that  $y_t^m(i) = Y_t^m$ . Hence

$$\frac{\left[Y_t^m - n^s \mu^{m,s} \left(\frac{Y_t^s}{A_t}\right)^\phi\right]^{-\frac{1}{\theta}}}{C_t^{-\frac{1}{\theta}}} \left(\frac{P_t^m}{P_t}\right)^{-\frac{\rho}{\theta}} = \frac{\theta}{(\theta-1)} s^m \left(Y_t^m, C_t, \frac{P_t^s}{P_t^m}; \xi_t\right), \quad (34)$$

which means that equilibrium output in the  $m$  sector is given by

$$Y_t^m = Y^m \left(C_t, Y_t^s, \frac{P_t^s}{P_t^m}; \xi_t\right).$$

Similarly, repeating the derivation for the  $s$  sector we obtain

$$\frac{\left[Y_t^s - n^m \mu^{s,m} \left(\frac{Y_t^m}{A_t}\right)^\phi\right]^{-\frac{1}{\theta}}}{C_t^{-\frac{1}{\theta}}} \left(\frac{P_t^s}{P_t}\right)^{-\frac{\rho}{\theta}} = \frac{\theta}{(\theta-1)} s^s \left(Y_t^s, C_t, \frac{P_t^s}{P_t^m}; \xi_t\right), \quad (35)$$

and

$$Y_t^s = Y^s \left(C_t, Y_t^m, \frac{P_t^s}{P_t^m}; \xi_t\right).$$

Now accounting for (6-7) and the sectoral market clearing conditions (18-19) we obtain

$$Y_t^m = n^m \varphi_t^m C_t \left(\frac{P_t^m}{P_t}\right)^{-\rho} + n^s \mu^{m,s} \left(\frac{Y_t^s}{A_t}\right)^\phi \quad (36)$$

$$Y_t^s = n^s \varphi_t^s C_t \left(\frac{P_t^s}{P_t}\right)^{-\rho} + n^m \mu^{s,m} \left(\frac{Y_t^m}{A_t}\right)^\phi \quad (37)$$

thus accounting for (33), we have to solve a system of four equations (34-37) in four unknowns, which defining  $Q_t \equiv \frac{P_t^s}{P_t^m}$ , are  $(Y_t^m, Y_t^s, C_t, Q_t)$ . Summing up and focusing on the deterministic case, the nonlinear system characterizing the steady state boils down to

$$\frac{(\theta-1)}{\theta\phi} - \mu^{s,m} Q [Y^m]^{\phi-1} = \frac{(1 - \mu^{s,m})^{\nu+1} [Y^m]^{\nu\phi+\phi-1}}{C^{-\sigma}} \left[ n^s (Q)^{1-\rho} + n^m \right]^{\frac{1}{1-\rho}}$$

$$\frac{(\theta-1)}{\theta\phi} - \mu^{m,s} Q^{-1} [Y^s]^{\phi-1} = \frac{(1 - \mu^{m,s})^{\nu+1} [Y^s]^{\nu\phi+\phi-1}}{C^{-\sigma}} \left[ n^s + n^m (Q)^{\rho-1} \right]^{\frac{1}{1-\rho}}$$

$$Y^m = n^m C \left[ n^s (Q)^{1-\rho} + n^m \right]^{\frac{\rho}{1-\rho}} + n^s \mu^{m,s} (Y^s)^\phi$$

$$Y^s = n^s C \left[ n^s + n^m (Q)^{\rho-1} \right]^{\frac{\rho}{1-\rho}} + n^m \mu^{s,m} (Y^m)^\phi$$

### 2.3 Loglinearized AS

The log linearization of the New-Keynesian Phillips curve around the deterministic steady state (see appendix for the derivation) results in

$$\pi_t^m = \delta_q q_t + \delta_{y^s} y_t^s + \delta_{y^m} y_t^m + \beta \pi_{t+1|t}^m + \delta_\xi^m \xi_t,$$

where  $\xi_t \equiv (\bar{c}_t, a_t)'$  and  $\delta_\xi \equiv (\delta_{\bar{c}}, \delta_a)$ . Repeating the derivation assuming two-period ahead predetermined decisions in the manufacturing sector, i.e.  $\tilde{p}_{t+2}^m = E_t \tilde{p}_{t+2}^m$ , yields

$$\pi_{t+2}^m = \beta_{y^m} y_{t+2|t}^m + \beta_{y^s} y_{t+2|t}^s + \beta_{m,q} q_{t+2|t} + \beta_\pi \pi_{t+3|t}^m + \delta_\xi^m \xi_{t+2}. \quad (38)$$

The same procedure applied to the s-sector but assuming one-period ahead predetermined pricing decisions yields

$$\pi_{t+1}^s = \beta_{y^s} y_{t+1|t}^s + \beta_{y^m} y_{t+1|t}^m + \beta_{s,q} q_{t+1|t} + \beta_\pi \pi_{t+2|t}^s + \delta_\xi^s \xi_{t+1}. \quad (39)$$

Moving to demand side of the economy, loglinearizing the nonlinear sectoral aggregate demands (21-20) and embedding the assumption of one-period ahead predetermined consumption decisions, i.e.  $C_{t+1} = E_t C_{t+1}$ , we obtain

$$y_{t+1}^m = \alpha_{y^s} y_{t+1|t}^s + \alpha_{m,q} q_{t+1|t} - \alpha_{m,r} \tilde{i}_{t+1|t} + \eta_{t+1}^m, \quad (40)$$

$$y_{t+1}^s = \alpha_{y^m} y_{t+1|t}^m + \alpha_{s,q} q_{t+1|t} - \alpha_{s,r} \tilde{i}_{t+1|t} + \eta_{t+1}^s, \quad (41)$$

where

$$\tilde{i}_t \equiv \sum_{\tau=0}^{\infty} (i_{t+\tau|t} - \pi_{t+1+\tau|t})$$

can be interpreted as the long real interest rate,  $y_t^m$  denotes the output gap in the m-sector defined as  $\log(Y_t^m/Y^m) - \log(\tilde{Y}_t^m/Y^m)$  where  $\tilde{Y}_t^m$  is the level of output with flexible prices (natural level of output) and  $Y^m$  is the steady state level of output (natural level of output assuming absence of shocks),  $\pi_t^m \equiv \log(P_t^m/P_{t-1}^m)$ ,  $q_t \equiv \log(P_t^s/P_t^m)$ ,  $\pi_t \equiv \log(P_t/P_{t-1})$ ,  $\eta_t^m$  is a composite disturbance depending on shocks to preferences and technology, and corresponding definitions hold for the output gap and inflation in the s-sector.

It is important to note that in this work prices in the m-sector are predetermined two-period in advance while in the s-sector one-period in advance. This difference allows studying how optimal monetary policy should respond to sectoral information according to the extent to which sectoral prices are predetermined. It is also worth noting that for each sector the model allows to choose to what extent, if any, prices are predetermined. Thus the model is suitable to study optimal monetary policy in a continuum of economies each characterised by a specific aggregate transmission lag generated in the supply side.

## 2.4 Central bank

The central bank optimization problem consists of finding the interest rate path that maximizes its preferences subject to the AD and AS in the m and s sectors. Turning to central bank preferences, they are described by the following operational loss function

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau \left[ \varphi_t^m n^m \pi_{t+\tau}^{m2} + (1 - n^m) \varphi_t^s \pi_{t+\tau}^{s2} + \lambda [\varphi_t^m n^m y_{t+\tau}^{m2} + (1 - n^m) \varphi_t^s y_{t+\tau}^{s2}] + \nu (i_{t+\tau} - i_{t+\tau-1})^2 \right], \quad (42)$$

where the weights given to the stabilization of the two sectors depend on their relative sizes, and the weights for the stabilization of real activity relative to inflation is captured by  $\lambda$ . Finally,  $\nu$  expresses the preferences of the central bank for interest rate smoothing<sup>2</sup>. The optimal problem for the central bank is therefore to minimize (42) subject to (38-41).

## 2.5 Calibration

The choice of the structural parameters follows Rotemberg and Woodford (1997). The choice of the new parameters,  $n^m$  and  $\mu^{m,s}$  and  $\mu^{s,m}$  depends on the cases explored below.

## 3 Optimal monetary policy knowing the aggregate transmission lag

We now start optimal monetary analysis assuming that policymakers have full information on the size of the sectors. Importantly, this hypothesis implies that the length of the *aggregate* transmission lag is known. Indeed, assuming one-period ahead predetermined prices for the s-sector and two-period for the m-sector, the lag for the aggregate inflation belongs to the  $[1, 2]$  interval and depends on the number of firms in the sectors,  $(n^m, n^s)$ <sup>3</sup>. Although the assumption that policymakers know the aggregate transmission lag is unrealistic, it is useful to study

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<sup>2</sup>Regarding the presence of interest rate smoothing in the preferences of the central bank see, for example, Svensson (2010), p. 2, Holmsen et al. (2008), and Woodford (2003).

<sup>3</sup>Considering that  $n^s + n^m = 1$ , the aggregate lag  $L$  is given by

$$L = n^s + 2(1 - n^s).$$

1. how the length of the aggregate lag affects the degree of policy activism and
2. how sectoral lags, sizes and production links affect optimal monetary policy.

In section 4 this assumption will be relaxed to focus on how lag variability affects the optimal policy.

Thus the current section studies first the relation between aggregate lag and policy activism and then explores various cases that differ for the degree of asymmetries in sectoral consumption (which occurs when  $n^s \neq n^m$ ), and for the degree of intensity in symmetric production links (which occurs when  $\mu^{m,s} = \mu^{s,m} \in [0, 0.5]$ ). All the following cases consider the long run monetary policy.

### 3.1 Policy activism and length of the aggregate transmission lag

The first step to study how the length of the aggregate lag affects the degree of monetary policy activism is to introduce a measure of policy activism, henceforth  $PA$ . A straightforward candidate is provided by the average of the optimal policy coefficients, i.e.  $(i_{\pi^m}, i_{\pi^s}, i_{y^m}, i_{y^s})$ . Figure 1 reports the relation between the aggregate transmission lag,  $L$ , and the measure of policy activism,  $PA$ , for various degree of symmetric production links, i.e.  $\mu^{s,m} = \mu^{m,s} \in \{0, 0.25, 0.5\}$ .

[Figure 1 here]

Figure 1 shows an inverse relation between the aggregate transmission lag and the degree of policy activism which is amplified by the degree of production links.

### 3.2 The role played by sectoral production links and sectors' size

Figure 2 shows the optimal policy responses to sectoral output gaps,  $i_{y^m}$ ,  $i_{y^s}$ , and inflations,  $i_{\pi^m}$ ,  $i_{\pi^s}$ , for various degree of symmetric production links,  $\mu^{s,m} = \mu^{m,s} \in [0, 0.5]$ . Panel a focuses on the responses to sectoral output gaps and reveals that they behave in opposite ways when production links increase, specifically  $i_{y^s}$  increases and  $y^m$  falls. Moving to the response to sectoral inflations, panel b reveals that both responses tend to increase with production links but the one related to the services sector is much more sensitive to production links. Thus both panels suggest that the importance given by optimal monetary policy to the services sector (i.e. the sector with the shortest lag) relative to the manufacturing sector is increasing with the degree of production links.



[Figure 2 here]

We then considered to what extent optimal monetary policy is sensitive to the sizes of the sectors. Figure 3 (Figure 4) reports the optimal policy responses when the services (manufacturing) sector is one and half time larger than the manufacturing (services) sector. Both figures show that the optimal response to sectoral statistics is increasing in the size of the sector as we would expect. Yet the impact of the sectoral size is more pronounced for the s sector. Furthermore, Figure 4 shows that when the m-sector is larger than the s-sector, for a sufficiently high level of production links optimal policy is still responding more to the s-sector. Put it differently, *production links* seems to play a more important role than *sectoral sizes* in determining to which sector policy should respond more.

[Figure 3 here]

[Figure 4 here]

Considering that production links among sectors are a standard characteristic of an economy, the main result of this analysis is that when optimal monetary policy is allowed to respond to sectoral statistics, it responds more to the supply sector with the shortest transmission lag. The mechanics at work is based on two factors. The first is that production links tend to offset the costs associated with the central bank policy. Indeed, while with no production links responding to a sectoral shock implies that the other sector is perturbed by the policy action, in presence of production links responding to a sectoral shock limits the spreading of the shock to the sector that has not been initially affected.

The second factor is that the length of the transmission lag alters the costs associated to the policy, specifically the longer the policy lag in the sector not initially hit by the shock, the lower the perturbing impact of the policy, that is the lower the cost. Thus the benefits associated with the policy consist of buffering sectoral shocks and limiting their propagation due to production links while the cost lies in perturbing the sector not initially hit by the shock. Given the tension between benefits and costs, at the margin their equalization leads to a policy that is more aggressive the longer the lag in the sector not initially hit by the shock.

## 4 Optimal monetary policy with a variable aggregate transmission lag

A natural question to ask is how optimal monetary policy changes if we relax the full information assumption on the sectors' size. First and foremost, it is worth noting that now the aggregate inflation lag depends also on the realization of the shocks to the number of firms in the m and s sectors, i.e.  $\varphi_t^m$ ,  $\varphi_t^s$ . Thus, the length of the aggregate inflation lag is a random variable with support on the real line depending on the sectoral pricing behaviours and with distribution depending on both  $n^m$  and  $n^s$  and the stochastic properties of  $\varphi_t^m$ ,  $\varphi_t^s$ . Hence, assuming imperfect information on the size of the sectors allows considering a *long* and *variable* lag from policy action to inflation, which is an important problem facing central banks.

In order to study the impact of the uncertainty related to the sectors' sizes and therefore to the aggregate transmission lag on the optimal monetary policy, the current model follows the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007)<sup>4</sup>. This route is followed by assuming a discrete support for  $\varphi_t^m$  and  $\varphi_t^s$  and that in any period these shocks can take  $n_j$  different values corresponding to  $n_j$  exogenous modes drawn by nature and indexed by  $j_t \in \{1, 2, \dots, n_j\}$ . Thus,  $\varphi_t^m$ ,  $\varphi_t^s$  corresponds to  $\varphi_{j_t}^m$ ,  $\varphi_{j_t}^s$ , respectively. Then, the mode  $j_t$  is let following a Markov Process with constant transition probabilities and I assume that each transition probability has the same value

$$P_{jk} \equiv \Pr \{j_{t+1} = k | j_t = j\} = \frac{1}{n_j}, \quad \forall j, k \in \{1, 2, \dots, n_j\}. \quad (43)$$

Next, let  $P \equiv [P_{jk}]$  be the Markov transition matrix and  $p \equiv (p_{1t}, \dots, p_{n_jt})'$  (with  $p_{jt} \equiv \Pr \{j_t = j\}$ ) the central bank's subjective probability distribution over the modes in period  $t$ . I now assume that the central bank does not observe the modes and that does not update its subjective distribution of modes through the observation of the economy<sup>5</sup>. Thus its subjective distribution  $p_t \equiv (p_{1t}, \dots, p_{n_jt})'$  evolves according to the exogenous

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<sup>4</sup>This approach relaxes the certainty equivalence assumption which is a common feature of the linear-quadratic setup and allows to analyze the impact of uncertainty on the optimal monetary policy.

<sup>5</sup>Svensson and Williams (2007a) base the no-learning perspective on the *forgetting the past* assumption according to which, when the central bank and the private sector make decisions in period  $t$ , they forget past observations of the economy so that they cannot use current observations to update their beliefs.

transition probabilities, that is

$$p_{t+\tau} = (P')^\tau p_t, \quad \tau \geq 0. \quad (44)$$

As to the central bank knowledge before choosing the instrument  $i_t$  at the beginning of period  $t$ , the information set consists of the transition matrix  $P$ , the central bank's subjective probability distribution over the modes in period  $t$  and subsequent periods via (44), the  $n_j$  different values that each of the shocks can take in any mode and, finally, the realizations of the predetermined variables. It is worth noting that given (43), the unique stationary distribution of the modes associated with the Markov transition matrix  $P$  is a uniform distribution. Thus the transition probabilities described by (43) capture the case of generalized modes uncertainty in which modes are serially i.i.d. The motivation to consider this case lies in the interest of studying optimal monetary policy when the central bank has a minimal knowledge on the multiplicative shocks, specifically it only knows their bands and considers any realization as equally likely.

This implies that the central bank only knows the band of the effective size of the sectors which is now captured by  $n^s \varphi_{j_t}^s$  and  $n^m \varphi_{j_t}^m$ , and considers any effective size on this band as equally likely.

Turning to the results, Figure 5 reports the optimal policy responses to sectoral statistics for various degrees of symmetric production links under the assumption that the effective number of firms in the s-sector  $n^s \varphi_{j_t}^s$  can take any value on the interval  $[0.3, 0.6]$  with the same probability. It follows that the aggregate lag is a random variable uniformly distributed on the interval  $[1.4, 1.7]$ . Comparing Figure 5 with Figure 2 shows that accounting for the uncertainty on the sectors' size leads to respond more to the s-sector than the m-sector even with no production links. Furthermore, in presence of production links, accounting for uncertainty magnifies the role played by production links. Indeed, moving from certainty to uncertainty, the optimal response to the output gap in the manufacturing sector shifts down approximately 30% and the optimal response to inflation in the services sector shifts up markedly. This shows that uncertainty plays down the role of the manufacturing sector which features longer transmission lags.

[Figure 5 here]

Summing up, results in sections 3 and 4 show that

1. there is an inverse relation between the aggregate transmission lag and the degree of policy activism which is amplified by the degree of production links.
2. optimal monetary policy reacts to the services sector relatively more than to the manufacturing sector the higher the production links and the larger the uncertainty on the length of the aggregate transmission lag.
3. even if the m-sector is larger than the s-sector, optimal monetary policy tends to react more to the s-sector.

## 5 Conclusions

This paper presents a model where transmission lags are substantially affected by a two-sector supply side of the economy and focuses on how lag length and variability affect optimal monetary policy. In the economy at issue the two sectors are manufacturing and services, where the former features a longer policy transmission lag than the latter. The paper shows that with symmetry in consumption, optimal policy responds more to the service-sector the more the s- and m-sectors are interlinked in terms of production. With asymmetries in consumption, sectoral policy response is increasing in the size of the sector but tends to respond more to the sector with the shortest transmission lag even if this sector is smaller than the other. Furthermore, the presence of sectoral production links is necessary for optimal policy reacting more to inflation than the output gap in line with the conventional wisdom.

The paper also shows that the shorter or more variable the aggregate transmission lag, the more active the overall policy and the larger the response to the sector with the shortest transmission lag.

These results suggest two policy implications. First, optimal monetary policy should respond more to the supply sector with the shortest transmission lag and the larger the uncertainty on the aggregate lag, the larger the response. Second, in a multi-sector economy, monetary policy should be cast in terms of sectoral information; policy based only on aggregate information is no longer optimal. Future analysis will compare welfare associated with policies in terms of aggregate and sectoral statistics and will consider the uncertainty about the size of the impact of the policy action.

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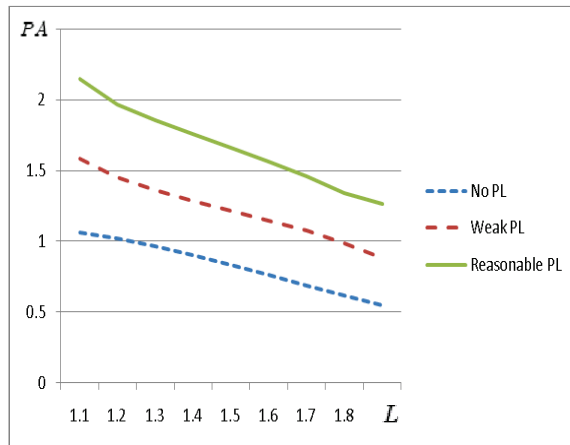
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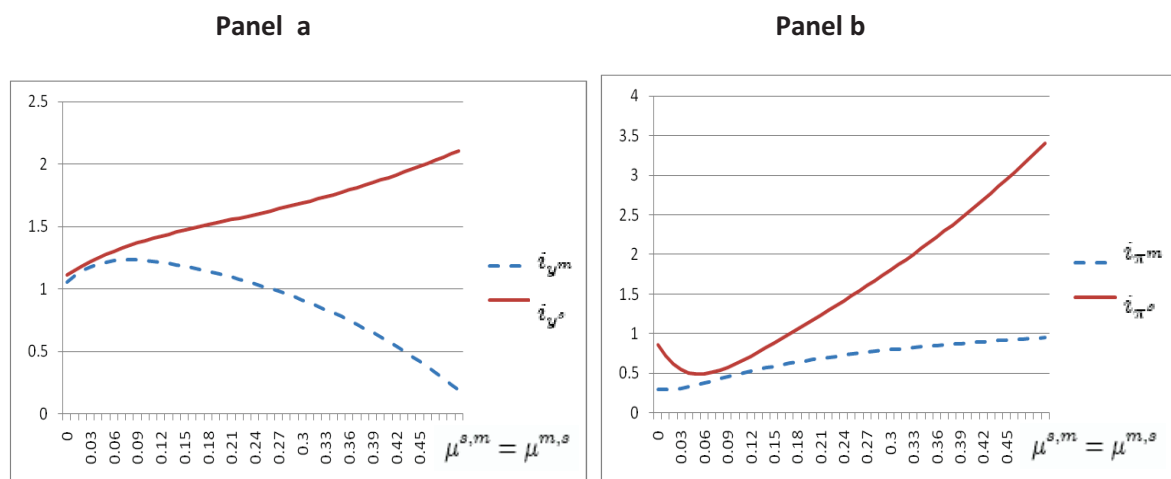
## Relation between the degree of policy activism, PA, and the aggregate transmission lag, L, for various degrees of production links, PL.

Fig 1

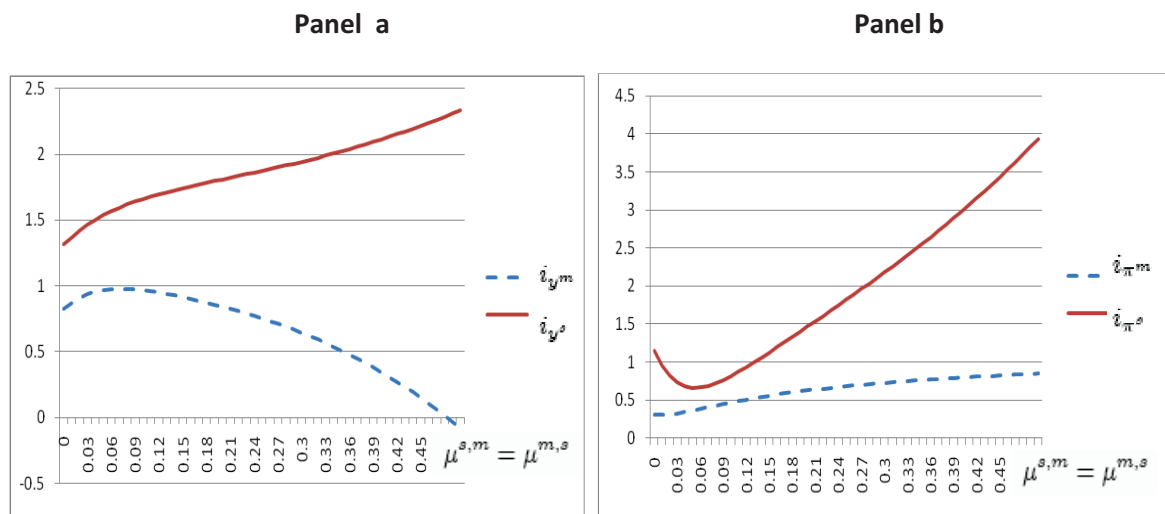


## Optimal policy responses to sectoral inflations and output gaps with no uncertainty and for an increasing degree of symmetric production links.

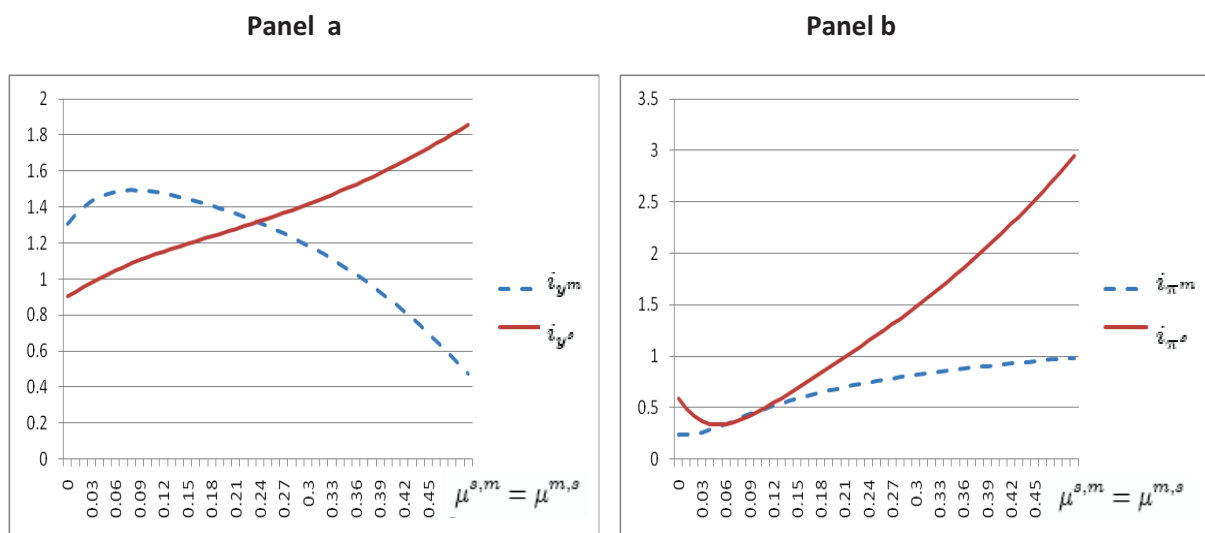
Fig 2: Optimal policy responses assuming equal-sectoral sizes.



**Fig 3: Optimal policy responses assuming  $n^s=0.6$ .**



**Fig 4: Optimal policy responses assuming  $n^s=0.4$ .**



# Optimal policy responses assuming uncertainty on the aggregate transmission lag for an increasing degree of symmetric production links.

Fig 5: Optimal policy responses assuming policymakers take  $n^s$  distributed uniformly on  $[0.3, 0.6]$ .

