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Implementing Disinflations in a Medium-Scale Dynamic General Equilibrium Model: Money Supply vis-à-vis Interest Rate Rules

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Abstract

Successful disinflation episodes have been shown to involve a sustained period of output contraction. We revisit the largely debated issue on the costs of different speed and timing of disinflations when monetary policy is implemented either via a money supply rule (MSR) or an interest rate rule (IRR). In terms of transitional costs, cold-turkey IRR disinflations are less expensive than those under MSR, with theoretical sacrifice ratios averaging 1.0 and 2.8 respectively, and are accomplished more rapidly. Gradual and anticipated disinflations deliver further lower sacrifice ratios. From a welfare perspective, despite the temporary economic contraction, disinflations are welfare improving. More interestingly, the overall welfare gain from disinflation is not affected by the actual policy implementation: what really matters is the achievement of a permanent lower inflation rather than how this is practically accomplished.

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1 Introduction

Since at least 1970s the analysis of disinflation and how to practically implement a permanent reduction in inflation have been topical in economics. The relevance of these issues has received further attention as the monetary policy literature has largely emphasized the benefits of achieving price stability and many central banks worldwide have committed to low inflation targets.

The empirical literature on disinflations tells us quite robustly that successful disinflationary programmes are accompanied by temporary economic downturns (e.g., Gordon and King, 1982, Ball, 1994b, or Cecchetti and Rich, 2001). Estimates of the so-called sacrifice ratio, which measures the cumulative output loss for each percentage reduction in inflation, exhibit considerable variation across countries, historical episodes and use of different econometric techniques. Furthermore, institutional factors such as the monetary policy regime have also been shown to affect the cost of disinflation. For the famous Volcker disinflation, often referred to as a monetarist experiment, Mankiw (1999) estimates a sacrifice ratio of 2.8. Corbo et al. (2001) study instead a broad group of countries that adopted inflation targeting and find a lower average sacrifice ratio, 0.6. In general, from the available empirical evidence on the real costs of disinflations a plausible range for the sacrifice ratio is $0.5 - 3$.

In the empirical literature, most of the disinflations took place at times when the monetarist dictum was prominent. Indeed, the most analysed disinflation episode in history is the Volcker disinflation. This is often referred to as a monetarist experiment, following the celebrated monetary policy reform in October 1979 that abandoned the federal funds targeting in favor of nonborrowed reserves targeting to control the money supply.[1] Since then the theory and practice of monetary policy has radically changed. Nowadays, it is standard in theoretical models to assume an inflation targeting framework, where monetary policy is conducted through a simple Taylor-type nominal interest rate rule. Often these theoretical models are even cashless. In light of these considera-

[1] The extent to which the Volcker disinflation can actually be considered as a monetarist experiment is discussed in length in Lindsey et al. (2005) (see also the other papers in the same FED of St. Louis Review issue) and Goodfriend and King (2005).

tions, it would be important to assess the implications of these two different monetary policy strategies for disinflation dynamics.

In discussing conditions for a successful disinflation monetary policy without too much output loss, several authors have emphasized the role played by the speed and timing of disinflation. Taylor (1983) argues that a gradual approach to disinflation entails less output cost since inertial prices and wages take time to adjust after the monetary tightening. In opposition, Sargent (1983) and Ball (1994b) favour a quick (also known as "cold-turkey") disinflation on the ground that a rapid disinflation enhances credibility and hence a shift in expectations.

In this paper, we address the issues raised above using a medium-scale dynamic general equilibrium model with nominal and real frictions à la Christiano et al. (2005). As in Ascari and Ropele (2010), where it is shown that such a theoretical model successfully accounts for the main stylized facts of disinflations without resorting to imperfect credibility or irrational expectations, we develop our analysis focusing on fully credible disinflation monetary policy.[2]

Our main contributions are twofold. First, we examine to what extent the costs of disinflation depend on the monetary policy strategy, i.e., money supply vis-à-vis interest rate rule, and on the operational procedure, i.e., cold-turkey, gradualism and anticipation. On this regard, our results show that the monetary policy strategy for disinflation substantially matters for the sacrifice ratio and for the dynamics of the model. Disinflating by controlling the nominal money supply is more costly than under an interest rate rule. On the transitional costs of disinflation we find that: (*i*) disinflations implemented through a money supply rule or an interest rate rule involve a long-lasting decline in output; (*ii*) the theoretical sacrifice ratios are in line with empirical estimates, with those under an interest rate rule being in general lower; (*iii*) gradual and anticipated cold-turkey disinflations yield even lower sacrifice ratios; (*iv*) the theoretical sacrifice ratios tend to decrease with average inflation.

[2] Credibility is certainly an important aspect for a policy change as a disinflation. In recent contributions, several authors addressed this issue by assuming a learning behaviour on the part of private agents, see e.g. Erceg and Levine (2003), Goodfriend and King (2005) and Cogley et al. (2010).

Second, we integrate the study on the short-run output costs of disinflation with a rigorous welfare analysis. Despite the prolonged output downturn, disinflationary monetary policies are overall welfare improving. The long-run welfare gains of permanently lower inflation prevail on the short-run welfare costs. Yet, given our benchmark parameters calibration, the magnitude of these welfare effects is rather small. In terms of consumption equivalent units, each percentage point of diminished inflation increases the representative household's initial steady-state consumption by about 0.07% each period. Interestingly, this finding is quite robust with regards to the practical implementation of disinflation. Although, alternative disinflation strategies or procedures involve different effects on the short-run dynamics of output and on the sacrifice ratio, from a welfare perspective there are no sizable differences. Thus, at least from a welfare perspective what really matters is the achievement of a permanent lower inflation rate, while it less important how this goal is achieved in practice.

Last but not least, our analysis also contributes to the literature along a methodological dimension. As in Ascari and Ropele (2010), we have chosen not to linearize the structural equations of the model but instead decided to work with the non-linear first order conditions. We have done this for at least two reasons. First, Ascari and Merkl (2009) shows that the use of log-linear approximations to study disinflations may lead to misleading results due to the fact that these monetary experiments entail a transition from one steady-state to another. Second, the standard approach of taking linear or log-linear approximations may rule out some important transmission mechanisms. Yun (2005), for instance, emphasizes the role of relative price dispersion, often neglected in linear models, in driving his results for optimal monetary policy.

2 Empirical evidence on disinflations

Here we discuss the basic facts that characterize disinflationary monetary policies. First, we examine the real costs of disinflations and survey how empirical studies have in practice tackled this issue. Second, we review the transmission mechanism of a disinflationary monetary policy by focusing on the adjustment pattern of output and inflation.

Short-run costs of disinflations: the sacrifice ratio. Most of the empirical studies on disinflation focused on the sacrifice ratio, defined as the ratio of the cumulative percentage output loss to the disinflation size. Hence, the sacrifice ratio measures the cumulative output costs per unit of permanent decrease in steady-state inflation.

In broad terms, three alternative approaches have been used to estimate the sacrifice ratio. The first approach builds on the estimation of autoregressive Phillips curve regressions. Using this methodology, Gordon and King (1982) estimates on quarterly U.S. data from 1947 to 1981 sacrifice ratios ranging from 0 to 8. More recently, Andersen and Wascher (1999) provide a comprehensive analysis for 19 industrialized countries and illustrates that sacrifice ratio estimates are sensitive to the specification of the Phillips curve and to historical periods. They also report that the average sacrifice ratio has risen to 2.5 from 1.5 in tandem with average inflation decreasing throughout 1980s and 1990s and thus with the flattening of aggregate supply curve. Filardo (1998) highlights instead the non-linearities in the costs of disinflation as the estimated slope of the Phillips curve differs in periods of sustained economic growth vis-à-vis periods of weak economic activity. Finally, using quarterly data for EMU countries in the period 1960-2001, Cuñado and Gracia (2003) estimate sacrifice ratios between 0.6 and 2.0 and, as in Andersen and Wascher (1999), find a negative relation between average inflation and the sacrifice ratio.

A second approach was popularized by Ball (1994a). In essence, Ball's approach relies on the analysis of single disinflation episodes, identified by locating peaks and troughs in trend inflation.[3] Ball (1994a) examines 19 moderate-inflation OECD countries from 1960 to 1991 and reports estimates of the sacrifice ratio between 1.8 and 3.3. Analyzing the Volcker disinflation in 1982-1985, Ball (1994a) obtains a sacrifice ratio of 1.8, which is close the recent estimate of 1.7 by Erceg and Levin (2003) but somewhat smaller than Mankiw's (1999) estimate of 2.8. More recently, Zhang (2005) generalizes Ball's approach showing that the estimates of the sacrifice ratio are larger when long-lived effects on output are taken into account. Also Zhang (2005) finds evidence supporting a negative relation between the sacrifice ratio and the level of inflation at the beginning

[3] Ball (1994a) defines trend inflation as the centered nine-quarter moving average of actual inflation.

of the disinflation.

Andersen and Wascher (1999) and Cecchetti and Rich (2001) criticize Ball's (1994a) approach and advocate the use of structural models in order to disentangle structural supply and demand shocks. Cecchetti and Rich (2001) uses a structural VAR model on quarterly US data for the period 1959-1997, using different identification schemes, and calculates the cumulative decline in output. Estimates of the sacrifice ratio vary from 1 to nearly 10. More recently, Durand et al. (2008) performs a study along the same lines for twelve EMU countries, finding values of the sacrifice ratio ranging from 0.2 to 0.8. Interestingly, also Durand et al. (2008) finds evidence in favour of a negative relationship between the average inflation and the sacrifice ratio. Finally, Collard, Fève, and Matheron (2007) and Fève et al. (2010) use a structural VAR model for euro area countries and find an average sacrifice ratio of 4.3.

Dynamic adjustment after a disinflation. Structural VAR models have also illustrated the dynamic adjustment after a disinflation. As one might expect, impulse response functions show that after a disinflation output declines temporarily while inflation decreases permanently. Yet, the use of different model specifications, identification schemes and data set have been shown to impinge on the estimated impact effect of output and inflation and the overall adjustment pattern. Two different transmission mechanisms are worthy to mention. Using U.S. data, Cecchetti and Rich (2001) (in one of their model's specifications) and Collard et al. (2007) find that inflation exhibits a markedly sluggish adjustment pattern. Inflation increases on impact and then slowly declines towards the new lower level. Meanwhile, output falls leading to a severe and protracted recession. Instead, Cecchetti and Rich (2001) in their benchmark model specification for U.S. data and Fève et al. (2010) for euro area data find that after a disinflation inflation abruptly falls on impact, then surges mildly and eventually converges to its lower level, through an oscillatory path. The adjustment of output is similar to the one described above although the economic contraction is smaller in size and less prolonged. In both cases, the dynamic paths of consumption and hours worked qualitatively follow the adjustment of output.

3 A medium-scale dynamic general equilibrium model

Our investigation on the effects of disinflation relies on a medium-scale dynamic general equilibrium model, for many aspects similar to Christiano et al. (2005), Schmitt-Grohé and Uribe (2004) and Smets and Wouters (2003). In particular, the model extends the basic three-equation New Keynesian model à la Clarida et al. (1999) by adding a broader set of real and nominal frictions and bringing in endogenous capital accumulation. Real frictions include: monopolistic competition in goods and labor markets, internal habit in consumption, variable capital utilization and adjustment costs in investment decisions. As for nominal frictions: prices and wages are sticky as in the Calvo staggered adjustment mechanism, but with a clause of indexation to past inflation. Thus, those prices and wages that cannot be reoptimized are automatically adjusted to keep up with the inflation rate occurred in previous period. Finally, money balances enter the model in two ways: households derive direct utility from holding real money balances (i.e., assumption of money-in-the-utility function) and firms hold nominal money balances to pay wages before production (i.e., assumption of working capital).[4]

Instead of detailing any further the nuts and bolts of the model, we highlight the major differences from our reference model.[5] First, as regards the characterization of monetary policy, we assume that the central bank operates either under a nominal money supply rule (MSR) or under a simple interest rate rule (IRR). Under MSR, the central bank uses as policy instrument the growth rate of nominal money supply π_t^*. Hence, resembling the Friedman's k-percent rule, the stock of nominal money evolves as

$$M_t = (1 + \pi_t^*)\, M_{t-1}. \tag{1}$$

In this case, the steady-state inflation rate is clearly pinned down by π_t^*. Under IRR, the central bank sets the nominal interest rate according to the following targeting rule

$$\frac{1 + i_t}{1 + i_t^*} = \left(\frac{1 + \pi_t}{1 + \pi_t^*} \right)^{\phi} \tag{2}$$

[4] Christiano et al. (2005) for the U.S. economy and Smets and Wouters (2003) for the euro area document the importance of these rigidities to match the business cycle empirical regularities.

[5] Appendix A reports the analytical description of the model and the calibration of parameters. For further details, the interested reader is referred to Schmitt-Grohé and Uribe (2004).

with $\phi > 1$ and where π_t, π_t^* and i_t^* represent the inflation rate, the inflation target and the nominal interest rate target, respectively. In particular, the nominal interest rate target is given by $1 + i_t^* = (1 + \pi_t^*)/\beta$, where β is the representative household's subjective discount factor. Unlike more conventional Taylor-type rules, our postulated policy neither responds to some measure of excess demand (e.g., the output gap) nor to lagged interest rate. The rationale for this choice is that we would otherwise find hard to justify the hypothesis of perfectly credible disinflations with countercyclical or history dependent monetary policy. This is particularly true when the disinflation is actually implemented and the nominal interest rate ought to be adjusted in accordance to the new target without any restriction.

Second, although the degree of price and wage indexation is calibrated to one, money is non-superneutral. This is due to the assumption of working capital, according to which firms pay the wage bill before production and the real marginal costs depend positively on the nominal interest rate. Albeit this feature contributes to increase the empirical fit of the model (see Christiano et al., 2005), it also affects the steady-state relationship between output and inflation. Indeed, the higher the level of steady-state inflation, the larger the labor costs for the firms; hence, *ceteris paribus*, the lower the wage paid to workers. In response, households reduce their labor supply and employment falls. Firms in turn decrease their capital stock, because labor and capital are complements in the production function. Eventually, the level of output decreases. The long-run Phillips Curve is not vertical. Using the calibration reported in Schmitt-Grohé and Uribe (2004): a permanent 1 per cent reduction in inflation implies roughly a 0.1 per cent increase in steady-state output.[6]

Third, from a methodological perspective we propose a non-linear solution of the model. As discussed above, the non-superneutrality of money implies that changes in

[6] The assumption of 100% price and wage indexation to past inflation rules out any potential real effect originating from the Calvo nominal friction. With partial indexation, a positive level of steady-state inflation would raise prices and wages dispersion, yielding an inefficiency output loss (e.g., Ascari, 2004, Schmitt-Grohé and Uribe, 2004). In this case, the real effects of steady-state inflation would be significantly larger.

steady-state inflation have a level effect on output in the long run. Then, whenever a policy action entails long-run effects one should restrain from using solution methods based on linear approximation. In these events, we think it is preferable and more accurate to use non-linear solution method. And this is what we do: we numerically simulate the transition adjustment path by solving the non linear model in DYNARE.[7] Throughout our analysis, we, thus, work with a perfect foresight model and examine the transition from one steady-state to another, abstracting from stochastic shocks.[8]

Last, as our main focus here is on the effects of disinflations we deliberately restrain from addressing issues related to fiscal policy, e.g., the use of government spending or distortionary taxation. Under the MSR we only make the technical assumption that seigniorage revenues are returned to households via a lump-sum transfer.

4 Designing disinflationary monetary policy

Throughout, a disinflationary monetary policy corresponds to a permanent reduction of the policy target π_t^*. In particular, we assume for $t = -\infty, \cdots, -2, -1$ the economy was in a steady-state characterized by positive inflation denoted by π_{old}^*. At $t = 0$, the central bank decides to disinflate the economy by lowering the policy target from π_{old}^* to π_{new}^*. Furthermore, we also assume that the shift of the target is permanent and agents do not expect any other policy surprise.

As largely debated in the literature, disinflations may be designed in several ways. Under the so-called cold-turkey disinflation the reduction of policy target is immediate,

$$\pi_t^* = \begin{cases} \pi_{\text{old}}^* & t = -\infty, ..., -2, -1 \\ \pi_{\text{new}}^* & t = 0, 1, ..., \infty \end{cases} .$$

Under gradualism, central bank steadily reduces the policy target in k periods, by setting

[7] For further details on DYNARE see the webpage: http://www.cepremap.cnrs.fr/dynare/.

[8] Broadly speaking, one may think of disinflation policies as triggered by the realization of a permanent shock to the policy target π^*.

a *time-varying* target such that,

$$\pi^* = \begin{cases} \pi^*_{\text{old}} & t = -\infty, ..., -2, -1 \\ \pi^*_{t-1} - k^{-1}\left(\pi^*_{\text{old}} - \pi^*_{\text{new}}\right), \text{ with } k \geqslant 1 & t = 0, 1, ..., k-1 \\ \pi^*_{\text{new}} & t = k, k+1, ..., \infty \end{cases}$$

Effectively, the parameter k controls the speed of disinflation: the lower k the faster the reduction of the policy target. The cold-turkey disinflation attains for $k = 1$.

Another interesting case to look at is the anticipated disinflation, whereby at time $t = 0$ the central banker credibly announces the intention to implement a cold-turkey disinflation after k periods.

To gauge the real output costs of disinflations we conform to the empirical literature and calculate a model consistent sacrifice ratio (SR), using the following formula

$$\text{SR} = -\frac{1}{\pi^*_{\text{old}} - \pi^*_{\text{new}}} \sum_{t=0}^{T} \left(\frac{Y_t - Y_{\text{new}}}{Y_{\text{new}}} \right), \tag{3}$$

where Y_{new} represents the steady-state level of output at π^*_{new}. Thus, our measure indicates the cumulative percentage output loss the economy has to sacrifice to achieve a 1% permanent reduction of steady-state inflation. Two features of (3) are noteworthy. First, we compute the sacrifice ratio by calculating the output loss in deviation from the new steady-state. Second, we sum up the percentage output losses over the first T periods, where T indicates the number of periods inflation takes to settle down to the new steady-state.

In the next two following sections, we examine how well different disinflation programmes replicate the stylized facts reviewed in section 2 and compare the outcomes between MSR and IRR. More specifically, we address the following questions: (*i*) do cold-turkey, gradual and announced disinflations involve recessionary effects? (*ii*) how large are the real costs in terms of sacrifice ratio? (*iii*) does the disinflation size, i.e., $\pi^*_{\text{old}} - \pi^*_{\text{new}}$, impinge on the disinflation costs? (*iv*) do initial and final values of steady-state inflation count for the disinflation costs?

5 Cold-turkey disinflations

In this section we compare the effects of cold-turkey disinflationary monetary policies under MSR and IRR. In both cases, we consider disinflations from moderately inflated steady-states, i.e., $\pi^*_{\text{old}} = \{2\%, 4\%, 6\%, 8\%\}$, to $\pi^*_{\text{new}} = 0\%$, that is full price stability.

Cold-turkey disinflations under money supply rule. As shown in figure 1, under MSR cold-turkey disinflations come with a notable recession. Output decreases following a hump-shaped pattern and eventually converges to the new steady-state through dying oscillations. Inflation immediately falls yielding a long-lasting deflation. Real money balances gradually build up while the nominal interest rate decreases. The ex-ante real interest rate rises and then reverts to steady-state.

To better understand the mechanism underlying these adjustment paths consider the disinflation from $\pi^*_{\text{old}} = 2\%$.

At time $t = 0$ when the central bank halts printing money (recall that $\pi^*_{\text{new}} = 0\%$), only a random fraction of firms optimize prices: aware of the new inflation target and the ensuing output contraction (necessary to curb inflation), these firms lower prices. The remaining fraction of firms instead mechanically adjust their prices one-to-one with previous period's inflation rate, hence raise prices by $1 + \pi^*_{\text{old}}$. As shown in figure 1, the former pricing decision prevails on the latter, with the result that the aggregate price index decreases. The ensuing deflation boosts real money balances and drives down the nominal interest rate. The ex-ante real interest rate rises significantly, mainly reflecting the long-lasting future deflation, leading to gradual reduction in consumption and investment spending. Output falls.

At time t=1, optimizing as well as non-optimizing firms lower prices. The former do so as they anticipate a hump-shape decline in aggregate demand driven by habit in consumption and investment adjustment costs. The latter firms instead lower prices because of indexation to a negative inflation rate. As a result, deflation exacerbates: the ex-ante real interest rate peaks up and output reaches the trough. From then on the ex-ante real interest rate slowly reverts and as it stays below steady-state the economy experiences a temporary and mild output expansion. At last, the cold-turkey disinflation is completed in about 28 quarters.

Cold-turkey disinflations from higher inflation rates, i.e., $\pi^*_{\text{old}} = \{4\%, 6\%, 8\%\}$, exhibits qualitatively similar dynamics. Neither the transmission mechanism nor the timing of turning points in output and inflation are affected. Yet, higher initial levels of steady-state inflation have remarkable effects on the amplitude of output and inflation declines. In percentage deviations from the new steady-state, the fall in output at the trough, which occurs after two quarters following the disinflation, is nearly 2% for $\pi^*_{\text{old}} = 2\%$, then nearly doubles for $\pi^*_{\text{old}} = 4\%$, and becomes 6% for $\pi^*_{\text{old}} = 8\%$. Likewise, the decline in inflation at the trough, which instead occurs after four quarters, intensifies as π^*_{old} increases. Intuitively, as the initial level of steady-state inflation rises optimizing firms lower prices more intensely, leading to a more profound deflation and a greater rise of ex-ante real interest rate.

So, MSR cold-turkey disinflations are accompanied by a decline in output, but how costly is the disinflation? The top panel of Table 2 reports the theoretical sacrifice ratios, calculated with $T = 28$. First, the values of the sacrifice ratios are approximately equal to 2.8, in line with empirical evidence: to achieve a permanent reduction of steady-state inflation from 2% to zero the economy has to incur a cumulative output loss of 5.6%. Second, varying the size of disinflation has minor affects on the sacrifice ratio.

As documented in Section 2, a robust empirical finding indicates that the sacrifice ratio tend to decrease as the initial steady-state level of inflation rises. On this account, we study fixed size cold-turkey disinflations from $\pi^*_{\text{old}} = \{4\%, 6\%, 8\%\}$ to $\pi^*_{\text{new}} \equiv \pi^*_{\text{old}} - 2\%$. As shown by the right-hand side entries in Table 2, sacrifice ratios tend to decrease as π^*_{old} rises. Disinflating from 4 to 2% entails a sacrifice ratio of 2.2, i.e., 0.8 percentage points lower than from 2 to 0%. Sacrifice ratios decrease even more for $\pi^*_{\text{old}} = 6\%$ and $\pi^*_{\text{old}} = 8\%$, standing at 1.8 and 1.6, respectively. Hence, fixed size cold-turkey disinflations have notable and non-linear effects on the sacrifice ratio. Figure 2 shows disinflation paths in these cases.

Cold-turkey disinflation under interest rate rule. Next, we replicate the same disinflation experiments above assuming the central bank operates under an interest rate rule. As in Taylor (1993) we set $\phi = 1.5$. As shown in Figure 3 also in this case cold-turkey disinflations come with a notable recession. Although the transmission mechanism is broadly similar to that under MSR, a number of qualitative and quantitative differences stand out. First, under IRR cold-turkey disinflations involve an immediate rise of nominal interest rate. So, as one would expect, the central bank starts off the disinflation with a contractionary monetary policy.[9] The prolonged phase of ex-ante real interest rate staying above steady-state lowers aggregate demand and yields a decline of output. As inflation steadily starts decreasing the central bank cuts the policy rate. Second, the rate of inflation converges to the new steady-state through a gradual adjustment path. This is in stark contrast with the deflation that instead characterizes cold-turkey disinflations under MSR (see figure 1). Third, for a given disinflation size, cold-turkey disinflations under IRR yield less macro volatility than under MSR. At the trough the fall of output under MSR is approximately three times larger than under IRR. Fourth, under IRR cold-turkey disinflations are accomplished in 15 quarters, roughly half the length of time it takes under MSR.

In figure 3 we also report the time-varying growth rate of money supply implied by the interest rate rule.[10] At time $t = 0$, the growth rate of nominal money supply suddenly declines, then increases overshooting the initial growth rate, and thereafter gradually converges to the new equilibrium. The immediate response of M_t is due to the initial rise of the nominal interest rate, which temporarily depresses the demand of real money balances.

The bottom panel of Table 2 reports the theoretical sacrifice ratios, calculated for $T = 15$. Not surprisingly, the sacrifice ratios are substantially lower than under MSR

[9] The reduction of the policy target π^* has two opposing effects on the nominal interest rate under IRR. On the one hand, the permanent decline in π^* opens up a temporary inflation gap (given that the inflation does not adjust immediately) and this calls for an increase in the policy rate. On the other hand, the permanent decline in π^* leads to a permanent decline in the nominal interest rate target, i.e., i^*. This latter effect calls instead for a cut of the policy rate. Ceteris paribus, which of the two forces prevails crucially depends on the policy parameter ϕ. For low values of ϕ the initial increase in the nominal interest rate would be absent. Nonetheless, we think that in the context of disinflation it would be more realistic to assume instead a value of ϕ larger than 1.5.

[10] Under IRR, the growth rate of money supply is recovered by using $\pi_t^* = (m_t/m_{t-1})\,\pi_t$, where m_t denontes holding of real money balances and π_t is the inflation rate.

(approximately equal to 1) and still in line with estimated values. Moreover, it turns out that sacrifice ratios are quite insensitive to different disinflation sizes or fixed size disinflations.

Discussion: the role of policy and of indexation. Inflation dynamics is rather different under the two disinflationary policies. Under MSR, the monetary authority freezes the nominal money supply; however, real money balances have to increase to reach the new steady-state level. The only possible way these patterns can square is for inflation to decrease more than the growth rate of nominal money supply, which is zero in Figure 1 and positive in Figure 2. Thus, the needed aggregate price dynamics is brought about by a long-lasting output contraction, which induces firms to lower prices yielding a decrease of inflation. This, in turn, rationalizes the lack of inflation persistence under MSR, despite the assumption of full indexation of prices to past inflation. Backward-looking price indexation just makes the inflation adjustment relatively more sluggish and the economic downturn more profound, but it is not the fundamental driver for the recession. Even without price indexation, the same qualitative dynamic adjustment illustrated in Figures 1 and 2 would carry on. In other words, it is the particular disinflationary monetary policy rule that actually eradicates the persistence of inflation.

Under IRR, instead, the backward-looking indexation assumption in prices and wages is indeed the main cause of the recession. There will be no recession, in this case, with zero indexation. This is because under a Taylor rule the money supply is endogenous, and can adjust in order to satisfy the increase in the demand for real money balances. Indeed, in Figure 3 the rate of growth of money supply jumps upwards, after an initial sharp fall, and diminishes only gradually. Monetary policy has to fight the inertia in indexation in the initial period, and it cuts abruptly the rate of growth of the money supply, leading to an increase in the nominal interest rate. In the following period, however, it accommodates money demand, since a relatively high rate of growth of money would be absorbed by money demand without inflationary pressure (and it is actually needed to avoid a deflationary period). With no indexation, instead, monetary policy would just increase the money supply initially to satisfy the new level of money demand, since a pure forward-looking inflation does not need a recession to adjust.

The important message here is that the way monetary policy implements a disinflation matters. Compared to MSR, an IRR disinflation greatly decreases the output cost of disinflation, reflecting the different paths of the money supply implied by the two monetary policy strategies.

What about indexation? Looking at short-run dynamics, indexation[11] makes the adjustment in inflation more sluggish and the recession deeper, regardless of the policy strategy. However, it causes the recession only if the central bank is following an inflation targeting rule. If the Volcker disinflation can in truth be thought as a "monetarist experiment", then backward-looking indexation is not really needed to explain the large cost of a disinflation.[12]

6 Designing the timing of disinflation programmes

In this section we examine gradual and announced disinflationary monetary policies. To save space we report disinflations from $\pi^*_{\text{old}} = \{2\%, 4\%, 8\%\}$ to $\pi^*_{\text{new}} = 0\%$, and show adjustment paths of output and inflation.

Gradualism versus cold-turkey. Figures 4 and 5 illustrate the effects of gradual disinflations for $k = \{4, 8, 12\}$ under MSR and IRR respectively. In general, gradual disinflations are accompanied by a hump-shaped decline in output, though for a given π^*_{old} a slower reduction of the policy target implies less output volatility. Indeed, as k increases optimizing firms lower prices less aggressively and the ex-ante real interest rate rises less.

Entries of Table 2 confirm that gradualism unambiguously reduces real output costs. This is particularly true under MSR, in which for a given π^*_{old} more gradual disinflations reduce monotonically the sacrifice ratio. For example, a three-year disinflation programme, i.e., $k = 12$, involves a sacrifice ratio that is roughly half that obtainable under a cold-turkey policy. Also, for given disinflation speed, i.e., a given value of k,

[11] Or lack of credibility, since backward-looking indexation can also be thought as a reduced form short-cut for sluggish adjustment in expectations.

[12] This obviously does not rule out the possibility that the lack of credibility could have added significant costs to the Volcker disinflation.

it is less costly to disinflate starting from higher levels of steady-state inflation. With regards to IRR, a gradual approach to disinflation still delivers sacrifice ratios that are lower than those under a cold-turkey, though the effects are somewhat less pronounced. Furthermore, the reduction of the sacrifice ratio due to more gradual disinflation is not monotonic in k; for a given π^*_{old} the sacrifice ratio steadily decreases until $k = 8$ and then starts rising somewhat. Finally, with regards to fixed-size disinflations, for a given speed of disinflation the sacrifice ratios decrease with higher levels of initial steady-state inflation. Notably, this is particularly true under MSR.

Announcement versus cold-turkey. Figures 6 and 7 illustrate the effects of announced cold-turkey disinflation policies under MSR and IRR, respectively. In each case we consider disinflations announced with 1, 2 and 4 quarters in advance. The main result here is that anticipated cold-turkey disinflations entail long-lasting output downturns. Under MSR, the effects of announcing a future cold-turkey disinflation reflect upon output and inflation dynamics. Regardless of the disinflation size, output contraction gets smaller (see for instance the percentage fall of output at the trough) and inflation may even converge smoothly to steady-state without any deflation (see for instance the case with $\pi^*_{\text{old}} = 8\%$ and a 1-year anticipated cold-turkey disinflation). In general, top entries of Table 2 confirm that the anticipation of future cold-turkey disinflation brings about monotonic declines of sacrifice ratios, regardless of any disinflation size and fixed-size disinflation.[13]

Under IRR, the announcement of future cold-turkey disinflations has stabilizing effects on output, while, and this is in contrast with MSR, has barely any effect on inflation (see table 2). As shown in Figure 8, these policies tend to de-stabilize the nominal interest rate as well as the ex-ante real interest rate. This is actually an artifact of the peculiar experiment we are considering since the central banker keeps targeting the old inflation target π^*_{old} until the disinflation is truly implemented. Right after the central

[13] In other disinflation experiments not reported here, we found that for announced cold-turkey disinflation longer than two years the sacrifice ratio starts to increase. This naturally calls for the question into the optimal design of fully credible anticipated disinflation policies. The answer to this question is beyond the scope of this paper and is the subject of ongoing research.

bank announces the intention to disinflate, optimizing firms lower prices. As inflation moderately falls relatively to the old target the monetary authority reduces the nominal interest rate. The ex-ante real interest rate slightly increases leading to a more muted output contraction. When the central bank actually executes the reduction of the inflation target, the nominal as well as the ex-ante real interest rates peak and thereafter monotonically decrease towards their respective steady-state.[14]

To summarize our results, both MSR and IRR gradual or announced disinflationary monetary policies deliver lower sacrifice ratios than under cold-turkey. However, the relation between sacrifice ratio and disinflation speed under gradualism is not necessarily monotonic. Finally, for a given disinflation speed, sacrifice ratios are negatively related to the disinflation sizes and to initial inflation rate in fixed-size experiments.

7 A welfare based measure of the cost of disinflation

As already noted in Gordon and King (1982), the mere existence of output losses following a disinflation does not by itself contain policy implications. A thorough balance should be made of the welfare cost of foregone output and the welfare benefits of lower inflation. On this later point, the recent new Keynesian monetary policy literature has largely emphasized under which conditions and why the achievement of full price stability is socially desirable (see Woodford, 2003 and the references therein).

In this section we tackle this issue and follow Ascari and Ropele (2010) to calculate a synthetic indicator that summarize the whole welfare implications that occur throughout a disinflation. In Appendix B we report the derivation of the welfare-based indicator and furthermore show how to disentangle the short-run (or transitional) from the long-run effects of disinflation. Three remarks are in order. First, the welfare-based measure we propose directly builds on the comparison of representative household's value func-

[14] The sudden reversal and zigzagged behavior of nominal interest rate may lead to peculiar adjustment dynamics, especially for announcement experiments longer than one year. In a 2-year announcement case, on impact the real interest rate may indeed decrease thus yielding an expansion in output.

tion in two distinct situations: (1) the situation in which the disinflation policy is not implemented and the economy remains in a steady-state characterized by π^*_{old} and (2) the situation in which the central bank does disinflate the economy from π^*_{old} to π^*_{new}. Second, for sake of interpretation, welfare results are expressed in terms of consumption equivalent units. In practice, the consumption equivalent measure is defined as the constant fraction of the initial consumption level that the representative household should give away each period in order to obtain the same level of value function the representative household would obtain if the disinflationary policy were implemented. Note that this is an accurate measure of the costs of disinflation in terms of consumption: indeed it measures how much the representative household suffers in terms of foregone consumption in exchange of a permanent reduction in inflation. Third, our welfare-based indicator is constructed echoing the standard sacrifice ratio. Thus, a positive (negative) value of the welfare-based sacrifice ratio has to be interpreted as a welfare loss (gain).

Table 3 reports the results for cold-turkey disinflations under MSR and IRR. For all disinflation experiments, the welfare-based indicator is negative meaning that disinflations are welfare improving.[15] We think this is an interesting result: empirical studies on disinflation focus only on the short-run costs in terms of foregone output but neglect, often by construction, potential long-run benefits. We have demonstrated, however, that in a medium-scale DSGE model of the business cycle cold-turkey disinflationary policy is welfare improving. Yet, the order of magnitude of these gains are rather small and amount to an extra 0.06 per cent of consumption each period. Overall, and this extends the results in Ascari and Ropele (2010) to MSR as well, it would be preferable to emphasize the welfare gain rather than the sacrifice ratio as in the empirical literature.

The above results are even more striking when the total welfare gain is decomposed between short-run (or transitional) and long-run effects. Two key findings discussed in previous sections are that cold-turkey disinflations under MSR or IRR entail a large and

[15] Qualitatively, this result does not depend on the inclusion of real money balances in the utility function. We have also calculated welfare-based measures without accounting for the utility gain coming from the long-run increase in real money balances. In this case, the welfare results would be smaller than the ones reported in the table by 30%. .

lasting output decline and that sacrifice ratios are in line with empirical estimates. From a welfare perspective, however, table 3 indicates that these short-run costs are quantitatively negligible and equal to 0.01% fall of initial consumption for each following period. The analysis of short-run welfare losses also reveals that disinflations under IRR are relatively less costly than under MSR. Though, the differences are very small. Disinflation size seems to matter only for long-run welfare gains, which tend to almost linearly increase with π^*_{old}. For fixed-size disinflation experiments, both short-run welfare costs decrease and long-run welfare gains increase as π^*_{old} rises. Table 4 reports welfare-based indicators for gradual and announced disinflations. More gradual or longer anticipated disinflations deliver larger welfare gains; whereas, for a given disinflation speed or disinflation announcement total welfare gain tend to decrease as π^*_{old} rises. In any case, these effects are quantitatively very small.

The key result here is that no matter how a disinflation is designed (through a MSR or IRR, or under cold-turkey, gradualism, announcement) the overall effects on the representative household's utility enhance welfare. Yet, the order magnitude of welfare gains is quite small and corresponds for each percentage point of diminished inflation to an increase of 0.06-0.07% of initial consumption in each period. As this result stands in stark contrast with the general view about the effects of disinflations, we next illustrate the intuition for this finding by considering cold-turkey disinflations under MSR. Figure 9 displays adjustment paths of consumption, employment and the utility function, with and without real money balances. The cold-turkey disinflation induces a prolonged recession fostered by a lasting decrease in consumption and employment. The levels of consumption and employment, however, has opposite effects on the representative household's utility function. Thus, the net effect of the representative agent's utility function is ambiguous. As a matter of fact, on impact the fall in consumption dominates dragging down utility. However, already in the second quarter the effect of falling employment takes over driving utility above the new steady-state level. Moreover, utility will remain above steady-state throughout the recession, mainly because the decline in employment is larger in percentage terms and relatively more sluggish. Hence, the positive effect of employment is quite effective in counterbalancing the negative effect of lower consumption. Overall the transition, thus, entails a short-run cost, as shown above, but of a negligible order of magnitude in terms of utility. Figure 9 also illustrates the adjustment path of the utility function net of real money balances to make clear that our results do not depend on the dynamics of real money balances.

The previous analysis shows that the result that disinflations are welfare improving hinges on the representative agent framework, which cannot account for the fact that some individuals may experience sharp drops in utility during recessions as they lose their jobs. Nonetheless, our results show two important aspects. On the one hand, they cast shadow on the use of DSGE models for welfare evaluation without "inspecting the mechanism". In particular, the ranking across different monetary policy rules or the optimal policy problems are bound to be based on mechanism similar to ours. On the other hand, if markets were complete (and agents are the same ex-ante), then all agents will have the same marginal utility from consumption. Hence, our results simply show once again that, if the economy could provide an efficient risk-sharing across agents (either through capital markets or some public welfare system), then disinflation, in particular, and recession, in general, could be less of a problem than we normally think.

8 Conclusions

A rich empirical evidence indicates that successful disinflation episodes in actual economies entail a sustained period of economic downturn. A classical policy issue regards the disinflation design to minimize the output loss associated with a period of disinflation. On the one hand, Taylor (1983) argued that a gradual disinflation is less expensive as allows wages and prices to have enough time to adjust to the new policy target. Likewise, disinflations announced farther in advance may deliver even lower costs. On the other hand, Sargent (1983) advocated that a fast disinflation, i.e., the so-called "cold-turkey approach", is more desirable because expectations adjust faster.

In this paper we revisited the largely debated issue in monetary economics of the effects of different speed and timing of disinflations by means of a medium-scale New Keynesian dynamic general equilibrium model. In particular, we explored which disinflation approach is less costly when the monetary policy is implemented either through a nominal money supply rule or an interest rate rule. Our comparative analysis on the costs of disinflation offered two perspectives. First, we evaluated the real costs of disinflation by constructing a theoretical sacrifice ratio that measures the cumulative output loss for each permanent percentage point reduction in inflation. Second, we used a novel metric based on the representative agent's welfare function. Such an indicator in practice balances the short-run welfare losses, related to the economic contractions, and the long-run welfare gains, brought about by the fact that a lower steady-state inflation increases the levels of real variables.

Our results can be summarized as follows. On the short-run costs of disinflation, we found that cold-turkey disinflations implemented through an interest rate rule are in general less costly, in terms of the sacrifice ratio, than those achieved by means of a money supply rule. Furthermore, in the former case, the permanent reduction in inflation is accomplished more rapidly. In either cases, gradual and anticipated disinflations deliver even lower sacrifice ratios, though in the case of an interest rate rule the relation between the sacrifice ratio and the speed of disinflation is not monotonically decreasing. On the welfare analysis our results showed that despite the substantial output contraction disinflations are overall welfare enhancing. The long-run welfare gains of permanently lower inflation prevail on the short-run welfare costs. Yet, given our benchmark parameters calibration of the model, welfare effects are quantitatively rather small. In terms of consumption equivalent units, each percentage point of diminished inflation increases the representative household's initial steady-state consumption by about 0.07% each period. Interestingly, this finding is quite robust with regards to the practical implementation of disinflation.

The two main results of the paper offers both a useful benchmark for future research.

First, the result that disinflating by controlling the money supply is more costly than disinflating by changing the inflation target, partly hinges on the way money demand is modelled. So to address further this issue one must carefully think about the money and the financial markets. This is surely a promising avenue for future research, especially given the fact that the financial crisis stimulated recent developments of DSGE macromodels with a banking sector and financial frictions.

Second, the result that a disinflation is welfare improving despite the short-run recession needs to be taken cautiously. The last Section of the paper clarifies that an heterogeneous agent framework that can account for different costs of the recession across agents can overturn the result, which is basically another side of the coin of the Lucas' low cost of business cycles in a representative agent framework. This direction of research is thus urgent, not only for evaluating the cost of disinflation, but also for the cost of business cycles and thus the optimal policy literature.

References

Andersen, P. S. and W. L. Wascher (1999). Sacrifice ratios and the conduct of monetary policy in conditions of low inflation. Bank of International Settlements wp No 82.

Ascari, G. (2004). Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics 7*, 642–667.

Ascari, G. and C. Merkl (2009). Real wage rigidities and the cost of disinflations. *Journal of Money, Credit and Banking 41*, 417–435.

Ascari, G. and T. Ropele (2010). Sacrifice ratio or welfare gain ratio? Disinflation in a DSGE monetary model. Temi di Discussione n.736, Banca d'Italia.

Ball, L. (1994a). Credible disinflation with staggered price-setting. *American Economic Review 84*, 282–289.

Ball, L. (1994b). What determines the sacrifcie ratio? In N. G. Mankiw (Ed.), *Monetary Policy*, pp. 155–182. University of Chicago Press, Chicago.

Cecchetti, S. G. and R. W. Rich (2001). Structural estimates of the U.S. sacrifice ratio. *Journal of Business and Economic Statistics 19*, 416–427.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature 37*, 1661–1707.

Cogley, T., C. Matthes, and A. M. Sbordone (2010). Optimal disinflation under learning. mimeo.

Collard, F., P. Fève, and J. Matheron (2007). The dynamic effects of disinflation policies. IDEI wp 426, Toulouse.

Corbo, V., O. Landerretche, and K. Schmidt-Hebbel (2001). Assessing inflation targeting after a decade of world experience. *International Journal of Finance and*

Economics 6, 343–368.

Cuñado, J. and F. P. D. Gracia (2003). Sacrifice ratios: Some lessons from EMU countries, 1960-2001. *International Review of Applied Economics 17*, 327–337.

Durand, J.-J., M. Huchet-Bourdon, and J. Licheron (2008). Sacrifice ratio dispersion within the Euro Zone: What can be learned about implementing a single monetary policy? *International Review of Applied Economics 22*, 601–621.

Erceg, C. J. and A. T. Levin (2003). Imperfect credibility and inflation persistence. *Journal of Monetary Economics 50*, 915–944.

Fève, P., J. Matheron, and J.-G. Sahuc (2010). Disinflation shocks in the eurozone: A DSGE perspective. *Journal of Money, Credit and Banking 42*, 289–323.

Filardo, A. J. (1998). New evidence on the output cost of fighting inflation. *Federal Reserve Bank of Kansas City Economic Review 3rd quarter*, 33–61.

Goodfriend, M. and R. G. King (2005). The incredible Volcker disinflation. *Journal of Monetary Economics 52*, 981–1015.

Gordon, R. J. and S. King (1982). The output cost of disinflation in traditional and vector autoregressive models. *Brookings Papers on Economic Activity 1*, 205–242.

Lindsey, D. E., A. Orphanides, and R. H. Rasche (2005). The reform of October 1979: How it happened and why. *Federal Reserve Bank of St. Louis Review March/April*(Part 2), 187–236.

Mankiw, G. N. (1999). *Macroeconomics*. New York: Worth.

Sargent, T. J. (1983). Stopping moderate inflations: The methods of Poincare and Thatcher. In R. Dornbusch and M. Simonsen (Eds.), *Inflation, Debt and Indexation*, pp. 54–96. MIT Press: Cambridge, MA.

Schmitt-Grohé, S. and M. Uribe (2004). Optimal operational monetary policy in the Christiano-Eichenbaum-Evans model of the U.S. business cycle. NBER wp No. 10724.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*, 1123–1175.

Taylor, J. B. (1983). Union wage settlement during a disinflation. *American Economic Review 73*, 981–993.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Yun, T. (2005). Optimal monetary policy with relative price distortions. *American Economic Review 95*, 89–108.

Zhang, L. H. (2005). Sacrifice ratios with long-lived effects. *International Finance 8*, 231–262.

A The Christiano, Eichenbaum and Evans (2005) Model

In this Appendix we describe the Christiano, Eichenbaum and Evans (2005) model, following closely the outline in Schmitt-Grhoe and Uribe (2004).

Households

There is a continuum of infinitely-lived households whose expected intertemporal utility function is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) \right\}. \tag{4}$$

where E_0 defines the mathematical expectation operator conditional on the information set available at time 0, β is the subjective discount factor, function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, while decreasing in hours worked h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods indexed by $i \in [0,1]$, that are aggregated in the usual CES consumption bundle c_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \tag{5}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

There is a continuum of labour services h_{jt}, $j \in [0,1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \tag{6}$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. The standard cost minimization problem for the firms yield the labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage paid to labor type j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. The total labor supply is found by integrating labour-specific demand functions, to obtain h_t^s

$$h_t^s \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} dj. \tag{7}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is

$$k_{t+1} = (1 - \delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right], \tag{8}$$

where the function S introduce the adjustment cost on investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$. The model features also variable capacity utilization of physical capital, denoted by u_t,. The cost of capital then depends on the degree of utilization and it is given by $a(u_t)$. Agents rent capital to firms at a real interest rate r_t^k and decide also over the utilization rate. There are complete markets for state contingent assets, such that all agents choose the same level of consumption.

Household first order conditions are hence given by

$$u_{c_t} \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) + u_{c_t} \left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h \right) = \lambda_t \tag{9}$$

$$u_{h_t} \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \tag{10}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} (1 - \delta) + r_{t+1}^k u_{t+1} - a(u_{t+1}) \right] \tag{11}$$

$$q_t \lambda_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) - \left[S_i \left(\frac{i_t}{i_{t-1}} \right) \right] i_t \right] - \beta q_{t+1} \lambda_{t+1} S_i \left(\frac{i_{t+1}}{i_t} \right) i_{t+1} = \lambda_t \tag{12}$$

$$a_{u_t} (u_t) = r_t^k \tag{13}$$

$$u_{m_t^h} \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \tag{14}$$

Wages are sticky à la Calvo, and $1 - \tilde{\alpha}$ is the probability of being able to reset wages next period. If wages can not be re-optimized then are automatically updated according to past inflation, such that: $w_{j,t+1} = w_{j,t} \pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal wage set every period t. The union chooses the optimal wage maximizing its the utility function given by equation (5), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h^d_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi^{\tilde{\chi}}_{t+k-1}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi^{\tilde{\chi}}_{t+k-1}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{15}$$

All the reset optimal wages are identical in all labour markets.

Firms

Each good is produced by a firm which monopolistically supply its own variety using a production technology of the form

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate technology factor common across firms, and ψ represents a fixed cost of production. The production function $F(k_{it}, h_{it})$ is well-behaved and it's the same across firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{16}$$

where

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{17}$$

Firms rent capital from the households on a competitive market, and must pay a fraction ν of wages at the beginning of the period by cash. Therefore their money demand function is

$$m^f_{it} = \nu w_t h_{it} \tag{18}$$

The firms' problem is then to maximize the expected value of future profits, under their demand function (16) and the cash-in-advance constraint (18). The first order conditions with respect to capital and labour services are

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r^k_t \tag{19}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{20}$$

Since F is homogeneous of degree one, equation (19) and equation (20) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky à la Calvo. Every period each firm can choose a new price of its own good with a probability $1-\alpha$. As for wages, also the prices that can not be resetted optimally, are automatically updated according to past inflation, such that: $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$, where χ is the degree of price indexation. The first order condition for the optimal price is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta-1}{\eta} \frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0. \tag{21}$$

Again, all the reset optimal prices are identical for all goods.

The Government

Government expenditure is financed through lump-sum taxes and seigniorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{22}$$

where m_t denotes real money balances, and $\pi_t \equiv P_t/P_{t-1}$ is the (gross) inflation rate at time t. Government minimizes the costs of acquiring the composite good, hence given public expenditure, government's absorption of a single type of good is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.To close the model we postulate the monetary policy uses the simple non-linear nominal interest rate rule as described in the paper.

Equilibrium

The model equilibrium conditions are

Money market:	$m_t = m_t^h + m_t^f$
Labor market:	$h_t^s = \int_0^1 h_{it}^d di$
Capital market:	$\int_0^1 k_{it} di = u_t k_t$
Good i market:	$z_t F(k_{it}, h_{it}) = (c_t + g_t + i + a(u_t) k_t) \left(\frac{P_{it}}{P_t}\right)^{-\eta}$
Aggregate Goods market :	$z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = (c_t + g_t + i + a(u_t) k_t) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ is the price dispersion generated by price staggering, causing a wedge between aggregate supply and aggregate absorption. Similarly wage staggering gives rise to wage dispersion, given by $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, see (7).

Functional forms and calibration

As in Schmitt-Grohé and Uribe (2004), we assume the following functional forms:

$$\begin{aligned} u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\ F\left(u_t k_t, h_t^d\right) &= \left(u_t k_t\right)^{\theta} \left(h_t^d\right)^{1-\theta} \\ S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\ a\left(u_t\right) &= \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2. \end{aligned}$$

Calibration is also as in Schmitt-Grohé and Uribe (2004) and Christiano et al. (2005). The parameters values are listed in the Table 1.

B A welfare-based measure of disinflation costs

In presenting our welfare-based measure, we closely follow the derivation in Ascari and Ropele (2010). One notable advantage of working with structural model is that they provide a natural metric to evaluate the welfare implication of disinflation: this is the representative household's value function. Mimicking the construction of the sacrifice ratio, a measure of the loss in welfare due to a disinflation is given by the difference between the level of the value function in period 1, and the level of the value function if the policy was not implemented, that is, the starting steady-state value. So a *microfounded sacrifice ratio* could be defined as

$$MSR = -\frac{1}{\Delta_{\mathrm{H,L}}}\left(V_1 - V_H\right) \tag{23}$$

where V_H = starting steady-state value function, and V_1 = value function the first period after the implementation of the disinflationary policy. Note that, as in the standard sacrifice ratio definition, $MSR > 0$, if $V_1 - V_H < 0$, that is, if a disinflation brings about a welfare loss, and vice versa. It is important to note that V_1 includes both the transition dynamics and the long-run effects.

The consumption equivalent measure

A policy maker is interested in the welfare cost of implementing a disinflationary policy, but given that the utility function is not cardinal, a measure based on V is not very revealing. The difference $(V_1 - V_H)$ can, as usual, be expressed in terms of consumption equivalent units. The consumption equivalent measure is then defined as that constant fraction of consumption that households should give away in each period in the starting steady-state, in order to obtain the same level of value function that households would get if the disinflationary policy is implemented. Note that this is a true measure of the costs of disinflation in terms of consumption: indeed it measures how much households have to suffer in terms of consumption loss, in order to reduce the inflation rate permanently of a certain amount.

Finally, this measure is very easy to get. The starting initial value function is

$$V_H = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \frac{m_t^{h^{1-\sigma_m}}}{1-\sigma_m} \right], \tag{24}$$

that in steady-state this reduces to

$$V_H = V_{ss0} = \frac{1}{1-\beta} \left[\ln\left((1-b)\bar{c}\right) - \frac{\phi_0}{2} \bar{h}^2 + \frac{\bar{m}^{h^{1-\sigma_m}}}{1-\sigma_m} \right]. \tag{25}$$

Given the value of V_1 from our simulations, then we just need to solve for what constant fraction of steady-state consumption households should give away in each period in the starting steady-state to obtain the same level of value function as V_1. This ends up to find the solution for λ in the following equation

$$V_1 = \frac{1}{1-\beta} \left[\ln\left((1-b)\bar{c}(1-\lambda)\right) - \frac{\phi_0}{2} \bar{h}^2 + \frac{\bar{m}^{h^{1-\sigma_m}}}{1-\sigma_m} \right], \tag{26}$$

where λ measure exactly that constant fraction. The consumption equivalent measure is thus simply given by $\lambda = 1 - \exp\left[(1-\beta)(V_1 - V_H)\right]$. Finally, the proposed *welfare based sacrifice ratio* measure is $\mathbb{W}_{SR} = \lambda/\Delta_{\mathrm{H,L}}$. Note that there is no minus in front of $\lambda/\Delta_{\mathrm{H,L}}$ to mantain a positive sign for a loss. Indeed, if $V_1 - V_{ss} < 0$, i.e. the disinflation brings about a welfare loss, then $\lambda > 0$, and vice versa.

Parameter	Value	Description
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady-state)
δ	0.025	Depreciation of capital
ν	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
χ	1	Price indexation
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock

Table 1: Calibration of parameters in the Christiano, Eichenbaum and Evans (2005).

Panel A: Money Supply Rule (T=28)								
$\pi^*_{\text{old}} - \pi^*_{\text{new}}$		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
Cold-turkey	k=0	2.94	2.85	2.78	2.73	2.22	1.89	1.69
	k=4	2.32	2.12	1.96	1.83	1.63	1.35	1.19
Gradualism	k=8	1.78	1.52	1.34	1.21	1.22	1.03	0.93
	k=12	1.55	1.32	1.16	1.06	1.16	1.06	1.00
	j=1	2.49	2.31	2.18	2.08	1.80	1.49	1.31
Announcement	j=2	2.09	1.86	1.70	1.82	1.45	1.16	1.00
	j=4	1.50	1.24	1.07	1.15	0.98	0.77	0.67

Panel B: Interest Rate Rule (T=15)								
$\pi^*_{\text{old}} - \pi^*_{\text{new}}$		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
Cold-turkey	k=0	1.06	1.05	1.04	1.03	1.03	1.01	0.99
	k=4	0.95	0.93	0.92	0.91	0.91	0.90	0.88
Gradualism	k=8	0.86	0.85	0.84	0.83	0.83	0.82	0.81
	k=12	1.05	1.04	1.03	1.02	1.02	1.01	1.00
	j=1	1.00	0.98	0.98	0.97	0.98	0.98	0.95
Announcement	j=2	0.90	0.89	0.88	0.87	0.88	0.87	0.86
	j=4	0.71	0.71	0.71	0.70	0.69	0.69	0.66

Table 2: Theoretical sacrifice ratio.

π^*_{old}	π^*_{new}	Money supply rule ($\times 10^{-2}$) Total	Long-Run	Short-Run	Interest rate rule ($\times 10^{-2}$) Total	Long-Run	Short-Run
2%	0	-6.40	-7.40	1.00	-6.59	-7.40	0.81
4%	0	-6.29	-7.30	1.00	-6.49	-7.30	0.81
6%	0	-6.20	-7.20	1.00	-6.39	-7.20	0.81
8%	0	-6.12	-7.11	1.00	-6.31	-7.11	0.80
4%	2%	-6.26	-7.18	0.91	-6.39	-7.18	0.79
6%	4%	-6.12	-7.00	0.87	-6.23	-7.00	0.76
8%	6%	-5.99	-6.82	0.83	-6.08	-6.82	0.74

Table 3: Welfare-based indicator of the effects of cold-turkey disinflations.

		Money supply rule (×0.01)						
Disinflation		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
	k=4	-6.42	-6.36	-6.28	-6.20	-6.32	-6.18	-6.04
Gradualism	k=8	-6.51	-6.42	-6.35	-6.27	-6.36	-6.21	-6.07
	k=12	-6.53	-6.45	-6.37	-6.30	-6.37	-6.21	-6.07
	j=1	-6.44	-6.34	-6.26	-6.18	-6.30	-6.16	-6.02
Anticipation	j=2	-6.48	-6.38	-6.30	-6.23	-6.33	-6.19	-6.05
	j=4	-6.53	-6.45	-6.37	-6.30	-6.38	-6.23	-6.09
		Interest rate rule (× 0.01)						
Disinflation		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
	k=4	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
Gradualism	k=8	-6.62	-6.51	-6.42	-6.34	-6.42	-6.25	-6.10
	k=12	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
	j=1	-6.60	-6.49	-6.40	-6.31	-6.40	-6.23	-6.08
Anticipation	j=2	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
	j=4	-6.63	-6.51	-6.43	-6.35	-6.43	-6.26	-6.11

Table 4: Welfare analysis: gradual and announced disinflationary monetary policies.

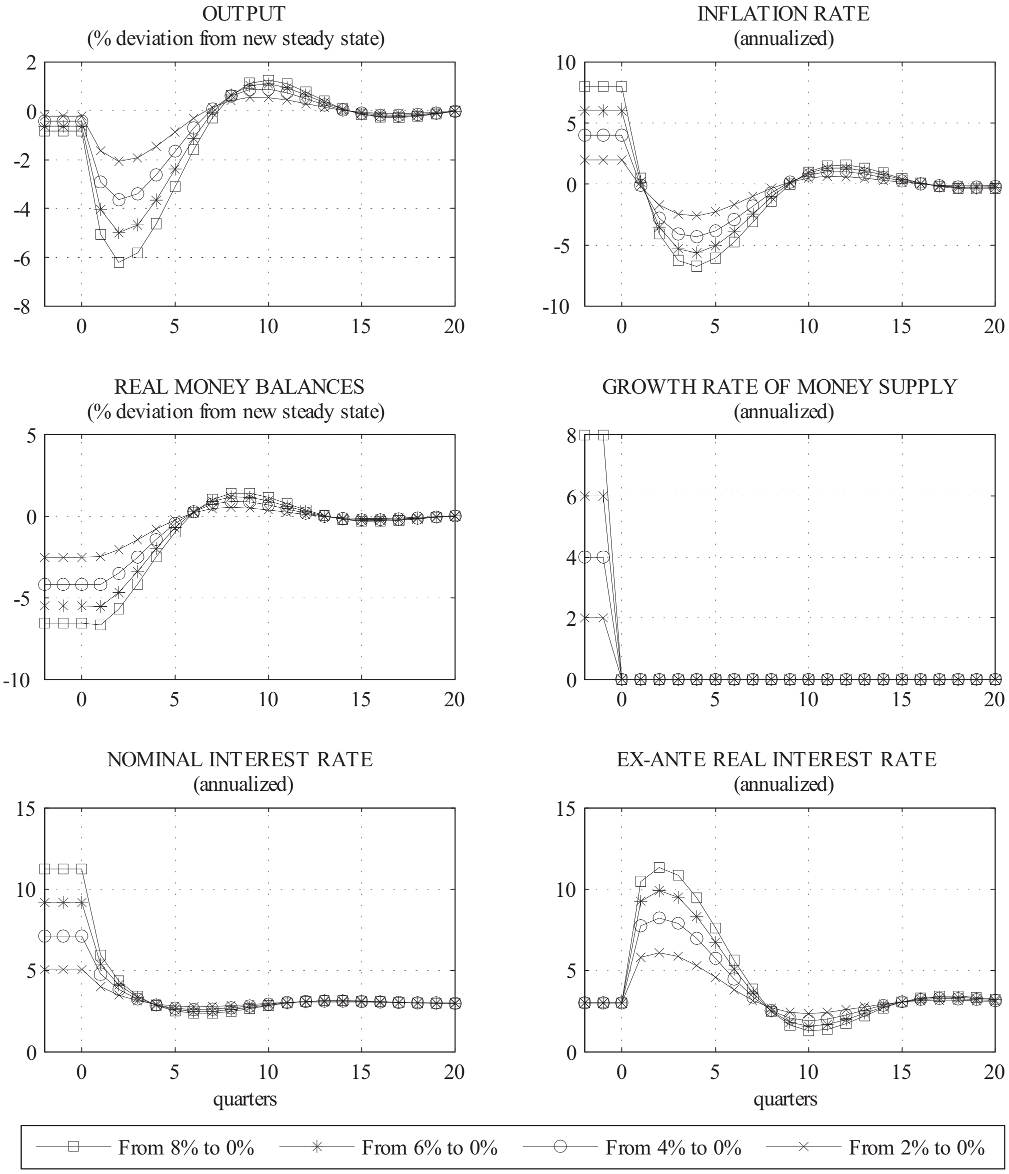

Figure 1: Cold-turkey disinflation under money supply rule.

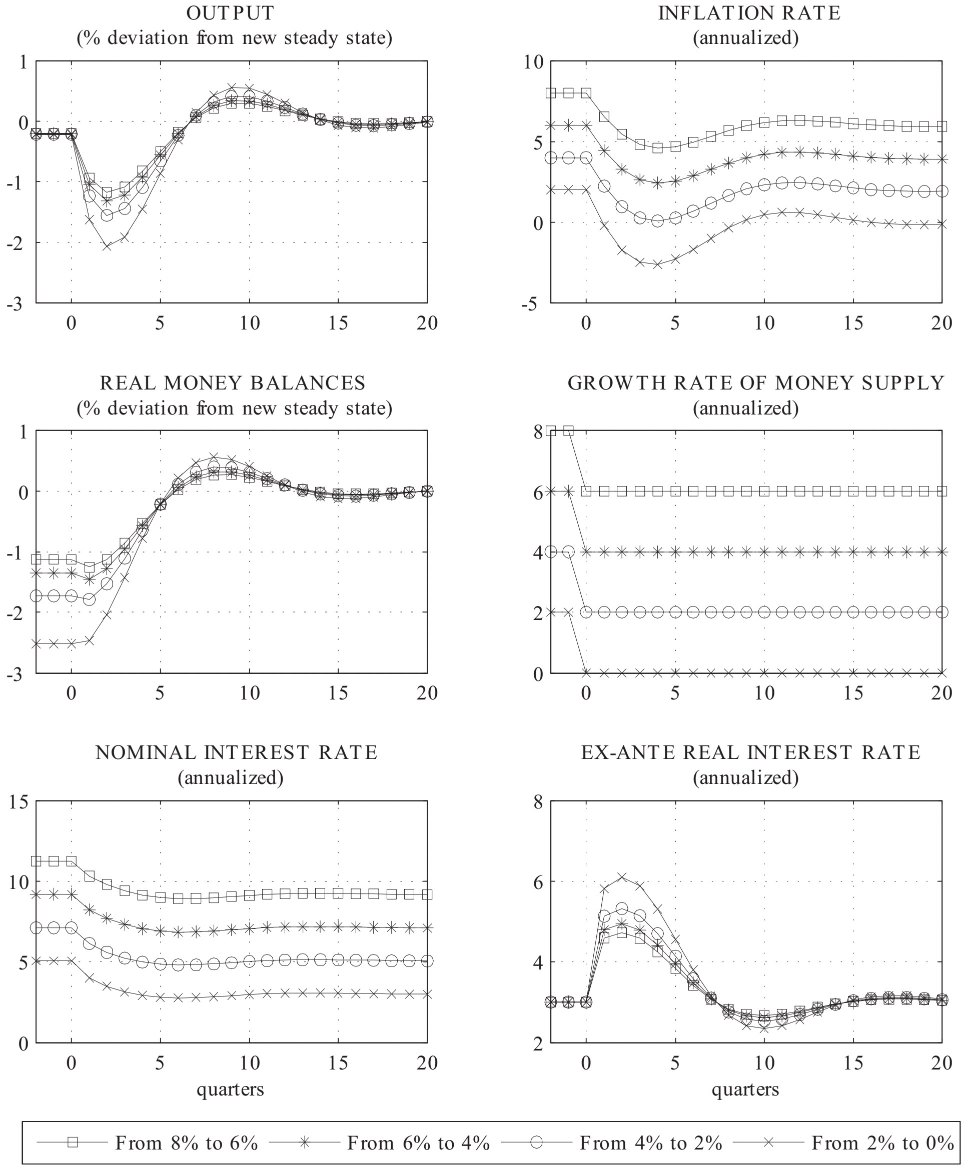

Figure 2: Cold-turkey disinflation under money supply rule for a fixed disinflation size: $\pi^*_{\text{old}} - \pi^*_{\text{new}} = 2\%$.

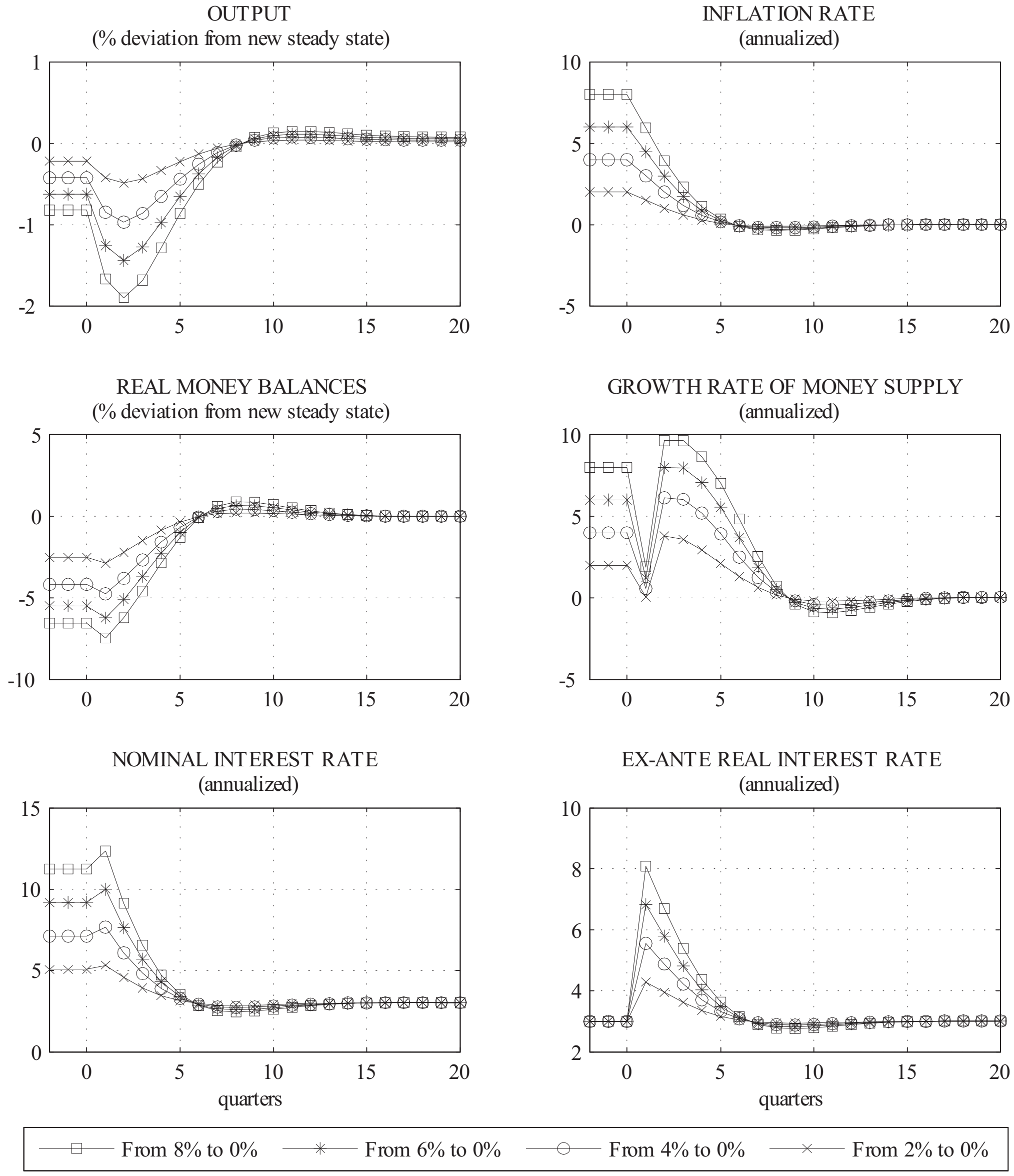

Figure 3: Cold-turkey disinflation under interest rate rule.

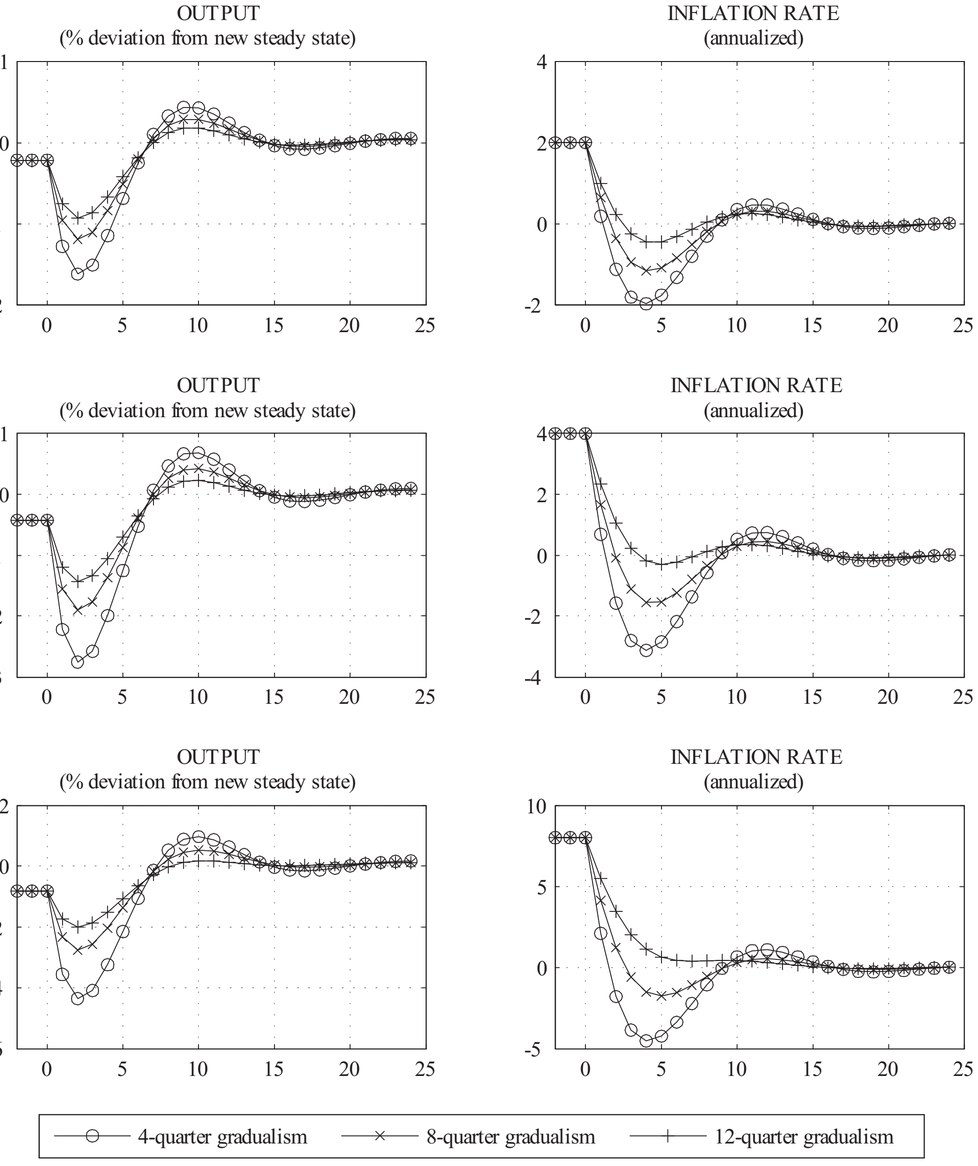

Figure 4: Gradual disinflation under money supply rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$. Bottom panels: $\pi^*_{\text{old}} = 8\%$.

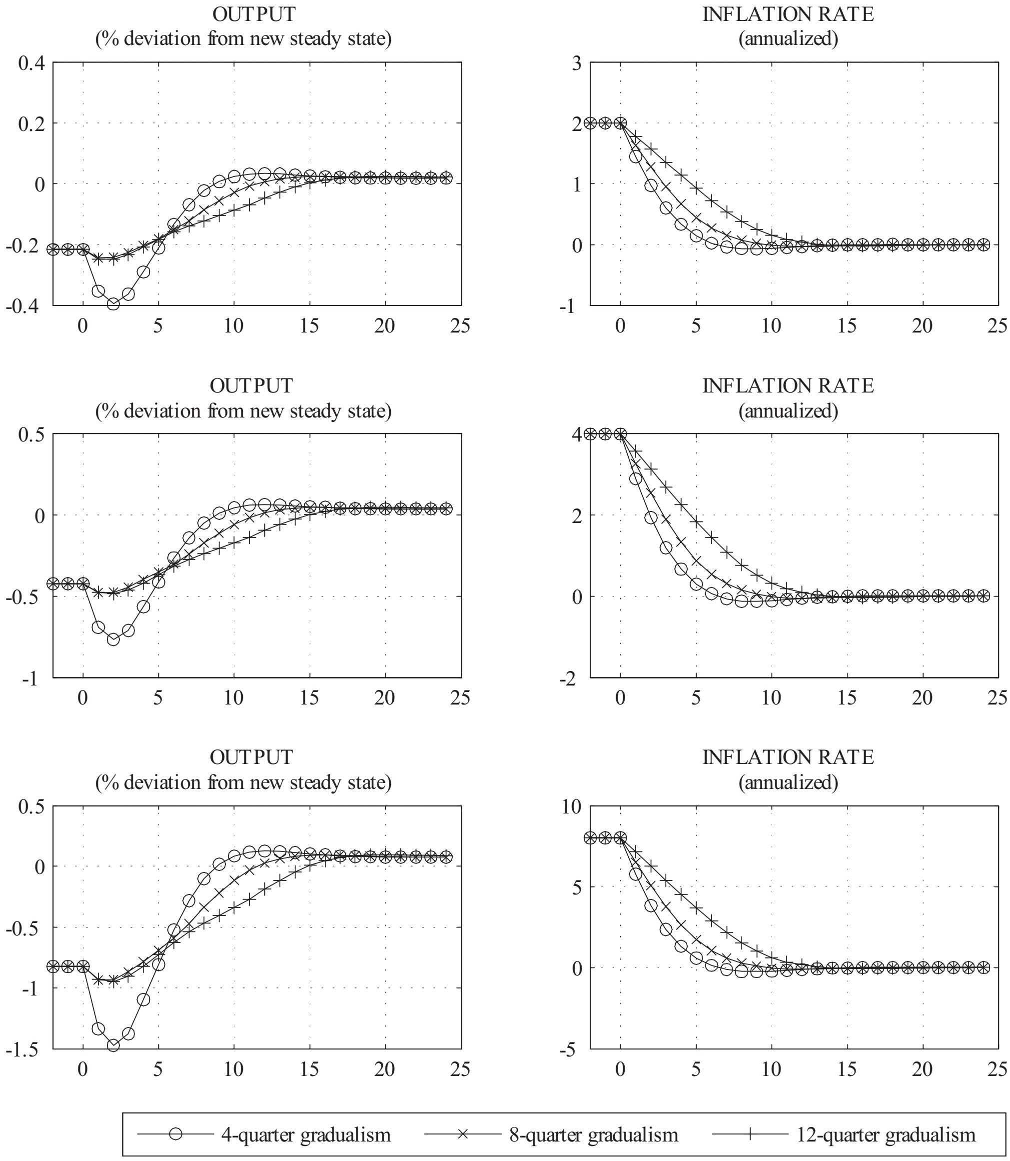

Figure 5: Gradual disinflation under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$. Transition paths are expressed in percentage deviations from the new steady-state.

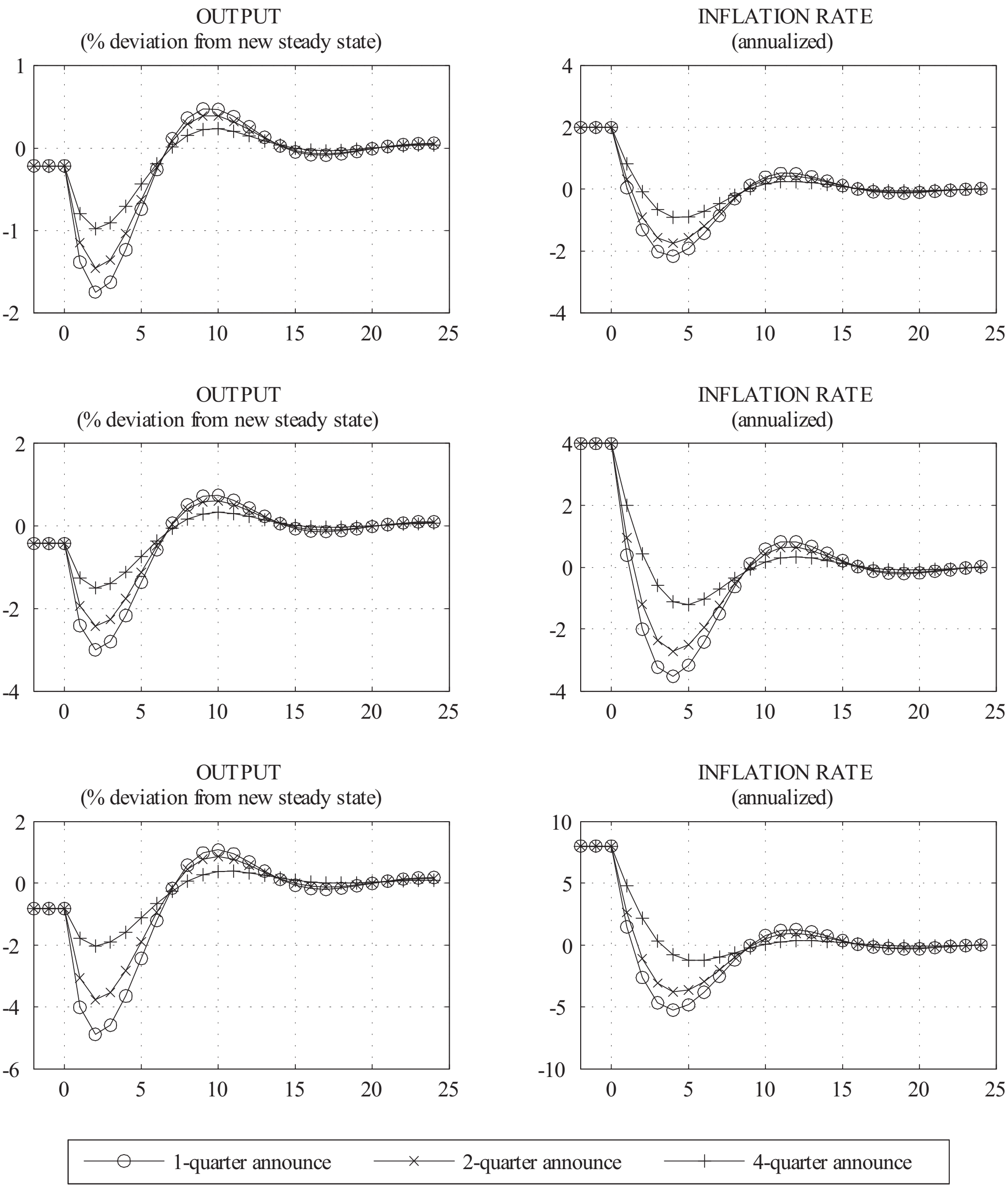

Figure 6: The effects of announcing future (cold-turkey) disinflations under money supply rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; and bottom panels: $\pi^*_{\text{old}} = 8\%$.

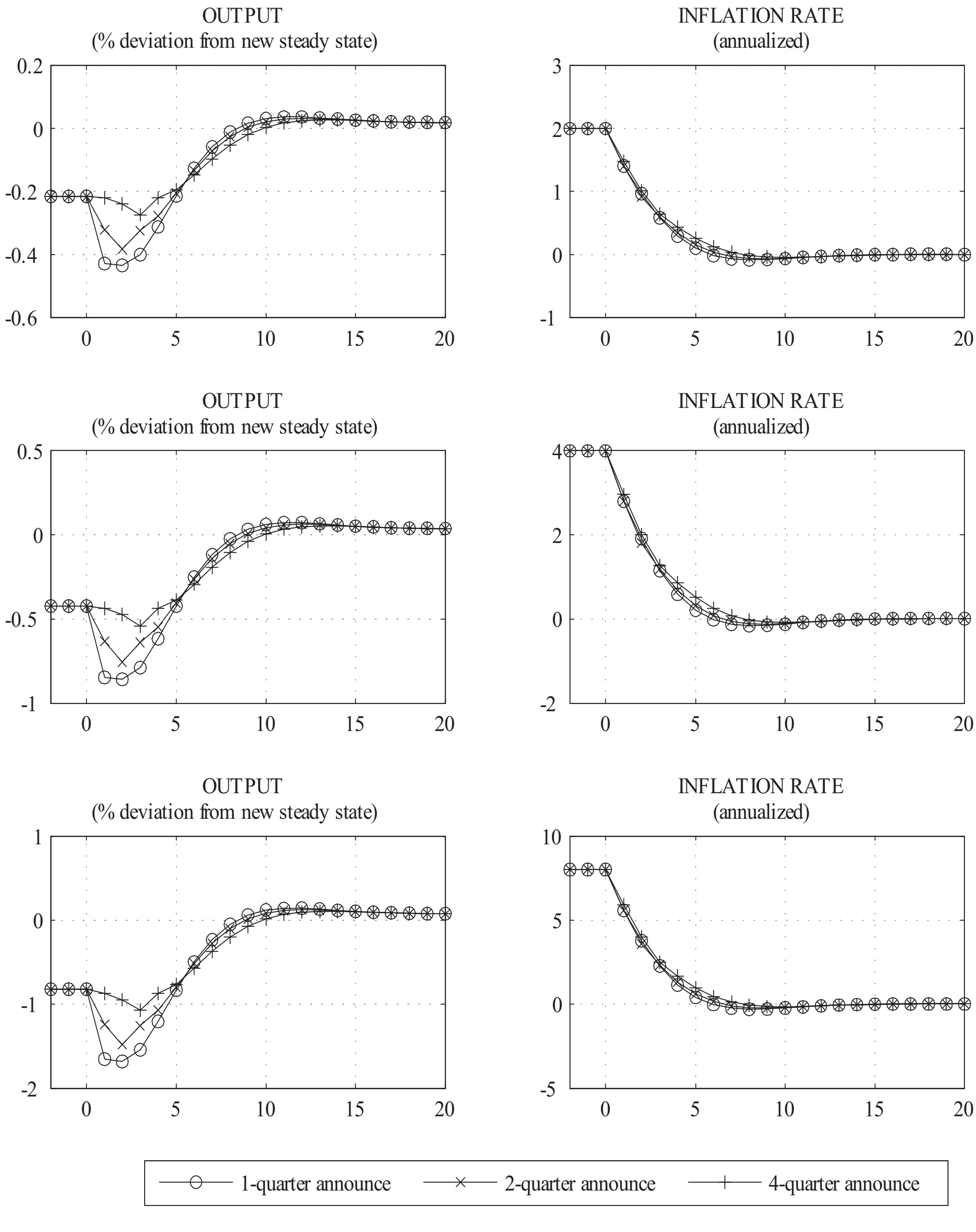

Figure 7: The effects of announcing future (cold turkey) disinflations under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; and bottom panels: $\pi^*_{\text{old}} = 8\%$.

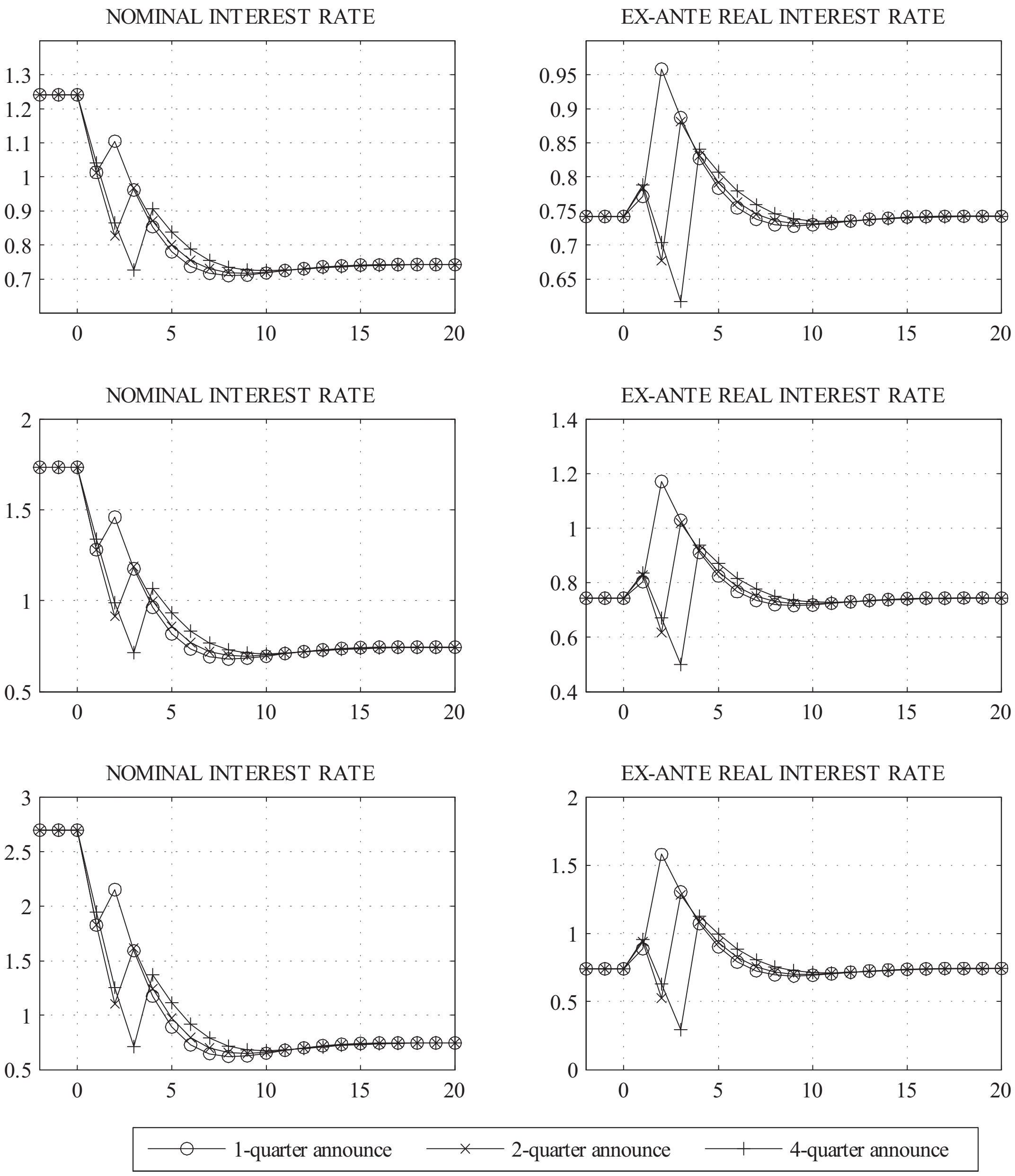

Figure 8: Anticipated disinflation under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$.

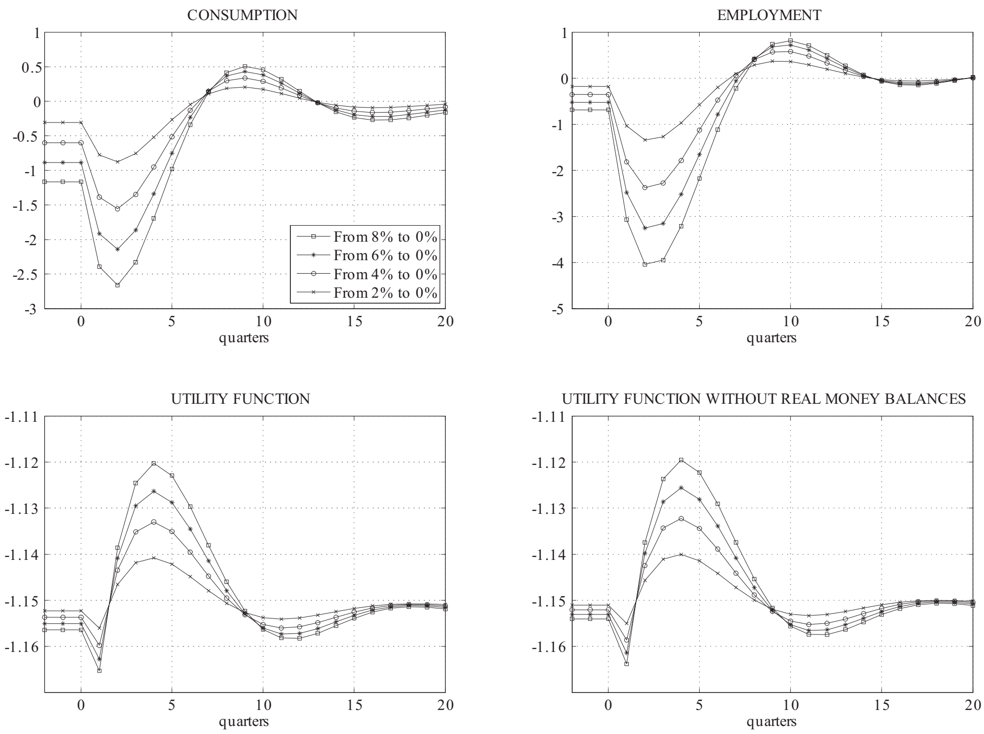

Figure 9: Cold-turkey disinflation under money supply rule. Consumption and employment paths are expressed in percentage deviations from the new steady-state. Utility function are expressed in levels.