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### Abstract

This paper uses a FAVAR model with external instruments to show that policy uncertainty shocks are recessionary and are associated with an increase in the exit of firms and a decrease in entry and in the stock price with total factor productivity rising in the medium run. To explain this result, we build a medium scale DSGE model featuring firm heterogeneity and endogenous firm entry and exit. These features are crucial in matching the empirical responses. Versions of the model with constant firms or constant firms' exit are unable to re-produce the FAVAR response of firms' entry and exit and suggest a much smaller effect of this shock on real activity.

Key words: Monetary policy uncertainty shocks, FAVAR, DSGE.

JEL codes: C5, E1, E5, E6

## 1 Introduction

Is monetary policy uncertainty important for business cycle fluctuations? Previous empirical studies find that policy uncertainty influences capital flows, the business cycle, and the speed of economic recovery (Mumtaz and Zanetti (2013), Mumtaz and Surico (2018), Bloom *et al.* (2018), Caggiano *et al.* (2020)). However, Born and Pfeifer (2014) claim that policy risk is unlikely to play a major role in business cycle fluctuations, with their DSGE model suggesting that policy uncertainty shocks are small and their impact is not sufficiently amplified.

In this paper, we re-visit this question and consider the role of firm dynamics in propagating the impact of policy uncertainty shocks. Using a FAVAR model, where the policy uncertainty shock is identified using an external instrument (a là Husted *et al.* (2019)), we show that this shock is recessionary. Moreover, in response to this shock firms' births decrease, whereas firms' exit increases. This evidence on establishment dynamics is robust both at the aggregate level, namely for establishments' birth and deaths in the total private sector, and at the industry level for the majority of the sectors considered. Also, the stock prices based on the S&P500 index decreases persistently. The utilization-adjusted TFP series reacts positively, at least in the medium-long run.

In the second part of the paper, we consider a medium-scale New Keynesian model extended by adding firm heterogeneity and endogenous firm entry and exit. In the intermediate sector, firms are heterogeneous in terms of their specific productivity. As in Rossi (2019) firms decide to produce as long as their specific productivity is above a cut-off level, which is determined by the level of productivity that makes the present discounted value of the stream of profits equal to the firms' liquidation value. The advantage of this framework is that firms' exit and average

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productivity evolve endogenously, bringing about endogenous TFP variations. During a recession, firms with a specific productivity below an endogenous threshold exit the market, so that the average productivity and the TFP increase. The opposite occurs in an expansion period. As in the seminal contribution by Bilbiie *et al.* (2012), firms enter the market up to the point where the expected discounted value of the future profits equals the sunk cost of entry. The investment in new firms is financed by households through the accumulation of shares in a portfolio of firms. This implies that the stock market price of investment in new firms fluctuates endogenously in response to shocks. Under this framework, we study the response to a monetary uncertainty shock implemented as an innovation to the time-varying volatility of the monetary policy shock. The main results of the theoretical model can be summarized as follows. First, as in the empirical evidence, the shock is recessionary, implying a prolonged fall in output, consumption, and investments in physical capital. Inflation and the policy rate decrease as well. Also, the stock price falls followed by a drop in investments in new firms and by an increase in the number of exiting firms that further amplifies the negative response of output. The recession improves the resource allocation by driving out less productive producers and increasing the TFP as in the FAVAR model for the medium-run.

To disentangle the role of the two margin of firm dynamics, namely the entry and exit, the baseline model is compared with two alternative models: a model with constant firms and a model with endogenous entry, but a constant exit rate. We show that the baseline model outperforms the two alternative models being more in line with the empirical evidence provided by the FAVAR model. By construction, the model with constant firms cannot replicate the dynamics of firms, furthermore it implies a lower reduction of output, consumption, investment in physical capital, and a mute response of the TFP and of the stock price. The model with endogenous entry, but a constant exit probability, shows a declining response of firm exit which is at odds with the empirical responses where exit is countercyclical. Also the fall in output is lower and the propagation of the shock is weaker than in the baseline model. Overall, both firms dynamics and firms heterogeneity are therefore crucial in the theoretical framework to replicate the qualitative results found in the FAVAR analysis.

The impact of firm dynamics on business cycle has been studied in many papers. The seminal paper by Bilbiie *et al.* (2012) in the DSGE literature shows that endogenous entry generates a new and potentially important endogenous propagation mechanism for real business cycle models. Among others, Jaimovich and Floetotto (2008), Lewis and Poilly (2012), Clementi and Palazzo (2016), provide evidence that the number of producers varies over the business cycle and that firms dynamics may play an important role in explaining business cycle statistics. Hamano and Zanetti (2014) and Casares *et al.* (2018) introduce endogenous firm entry and exit in a DSGE model, but consider different timing and exiting schemes. Further, while Hamano and Zanetti (2014) studies the effects of a negative technology shock in a simple RBC model, Casares *et al.* (2018) consider a medium scale model and estimates the effects of a set of level shocks on business cycle dynamics. Differently from this paper, in their paper firms exit at the end of the production period, implying that the average productivity remains exogenous and constant even in the short run. This prevents the TFP from varying along the business cycle. Closer to our theoretical framework is Rossi (2019), who however considers a simple small scale New Keynesian model with endogenous entry and exit interacting with banking frictions to study the effects of first moment shocks to the aggregate productivity level. Brand *et al.* (2019) instead study the effects on firm creation and destruction of second moment shocks, but they differ from our contribution along three lines. First, they provide an alternative way to formalize firm dynamics based on search frictions between entrepreneurs and banks. Second, the uncertainty shock they investigate concerns the dispersion of idiosyncratic firm productivity. Third, they do not provide evidence on firm dynamics at the industry level, but build up a theoretical model with search and monitoring costs in the credit market to study how the

higher dispersion in firm productivity affects the firm creation because of financial frictions.

Our paper makes two clear contributions. First, it extends the literature on policy uncertainty by considering the role of firm dynamics from an empirical and theoretical perspective. To the best of our knowledge, the role of firm dynamics in propagating policy uncertainty shocks has not been investigated in the existing literature. We show that this feature is a crucial component in amplifying the effect of this shock in DSGE models. From an econometric perspective, the paper proposes a FAVAR model that allows for mixed-frequency and missing data, allowing us to utilize series on aggregate and industry-specific firms' entry and exit which are available at a lower frequency and contain missing observations.

The remainder of the paper is organized as follows. Section 2 provides the empirical evidence by reporting the results implied by the estimation of the FAVAR model. Section 3 spells out the DSGE model economy, while Section 3.6 contains the main results of the theoretical model. Section 4 concludes. Technical details of the FAVAR estimation and of the robustness checks, as well as the full list of the DSGE model equations are left in the Technical Appendix.

## 2  Empirical analysis

We use a factor augmented VAR (FAVAR) to estimate the response to monetary policy uncertainty shocks for the US economy over the period 1985 to 2016. Relative to a small scale VAR, the FAVAR offers three key advantages. First, it allows the inclusion of data on sector-specific entry and exit, thus capturing the relationship between sectors. Second, the FAVAR can easily handle mixed frequencies and missing data allowing us to use monthly data on variables related to monetary policy uncertainty together with industry-specific data that is only available at a quarterly frequency. Finally, the use of a large data set makes it less likely that the model suffers from information insufficiency (see Forni and Gambetti (2014)).

The observation equation of the FAVAR model is defined as

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{1}$$

where $Z_t$ is the monetary policy uncertainty index built by Husted *et al.* (2019). $\tilde{X}_t$ is a $M \times 1$ vector of variables that include aggregate measures of macroeconomic and financial conditions provided by FRED-MD database (McCracken and Ng (2016)). $\tilde{X}_t$ also contains aggregate and sector-specific measures of firms' entry and exit provided by Bureau Labor Statistics-BED database. Details of the data used are in the Technical Appendix. $F_t$ denotes a $K \times 1$ matrix of unobserved factors while $\Lambda$ is a $M \times K$ matrix of factor loadings. Finally, $v_t$ is a $M \times 1$ matrix that holds the idiosyncratic components. We assume that each row of $v_t$ follows an $AR(q)$ process:

$$v_{it} = \sum_{p=1}^{P} \rho_{ip} v_{it-p} + e_{it}, \tag{2}$$

$$e_{it} \tilde{} N(0, r_i), R = diag\left([r_1, r_2, .., r_M]\right) \tag{3}$$

where $i = 1, 2, .., M$.

Collecting the factors in the $N \times 1$ vector $Y_t = \begin{pmatrix} Z_t \\ F_t \end{pmatrix}$, the transition equation can be described as:

$$Y_t = BX_t + u_t, \tag{4}$$

$$u_t \tilde{} N(0, \Sigma) \tag{5}$$

where $X_t = [Y'_{t-1}, .., Y'_{t-P}, 1]'$ is $(NP + 1) \times 1$ vector of regressors in each equation and $B$ denotes the $N \times (NP + 1)$ matrix of coefficients $B = [B_1, ..., B_P, c]$. The covariance matrix of the reduced form residuals $u_t$ is given by $\Sigma$. Note that the structural shocks are defined as $\varepsilon_t = A_0^{-1} u_t$ where $A_0 A'_0 = \Sigma$.

## 2.1 Temporal aggregation and missing data

The data on firms' entry and exit is only available at a quarterly frequency and also contains missing observations at the beginning of the sample period. For these series $(x_t)$ the observation equation is defined as:

$$\hat{x}_{jt} = \delta_j F_t + \hat{v}_{jt} \tag{6}$$

where $\hat{x}_{jt}$ denotes unobserved monthly growth rates of the jth series in $x_t$ and $\delta_j$ are the associated factor loadings. Over years where quarterly observations are available, we assume the following relationship between quarterly and monthly growth rates:

$$x_{jt}^Q = \sum_{j=0}^{2} \hat{x}_{jt} \tag{7}$$

In other words, the quarterly growth rates are assumed to be the sum of the unobserved monthly growth rates in that quarter. As explained below, we treat $\hat{x}_{jt}$ as additional unobserved states and add a step in our MCMC algorithm to draw from their conditional posterior distribution.

## 2.2 Identification

Following Husted *et al.* (2019), we employ an external instrument approach to identify $\varepsilon_t^{MPU}$, i.e. the monetary policy uncertainty shock, which is ordered first in $\varepsilon_t$ for convenience. As in Husted *et al.* (2019), the instrument is constructed by orthogonalizing the monetary policy volatility on FOMC meeting days with respect to observed monetary policy surprises. In details, we take as instrument the residual from the regression of the conditional volatility of 1-year swap rate 1-month ahead taken by Carlston and Ochoa (2016), over monetary policy surprises on FOMC meeting days. We consider the same three measures of monetary policy surprises of Rogers *et al.* (2018), which cover three components: target rate, forward guidance, and asset purchase.[1] The estimation is carried out using data on FOMC meeting days from October 2008 to December 2015, when all monetary policy surprises are available. The residual, $m_t$, from the regression, namely the instrument, can be thereby interpreted as the measure of monetary policy volatility on FOMC meeting days that is unexplained by the change in monetary policy itself. Moreover, the instrument is assumed to satisfy the following conditions:

$$E\left(m_t, \varepsilon_t^{MPU}\right) = \alpha, \alpha \neq 0 \tag{8}$$

$$E\left(m_t, \varepsilon_-\right) = 0 \tag{9}$$

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[1] We thank Marcelo Ochoa and John Rogers for sharing the data on respecitvely, the swaptions volatility and the three measures of monetary policy surprises.

That is, the instrument is assumed to be correlated with the monetary policy uncertainty shock $\varepsilon_t^{MPU}$ and uncorrelated with the remaining shocks $\varepsilon$. The instrument is incorporated into the FAVAR model via the following equation

$$m_t = bu_t + \hat{v}_t, \tag{10}$$

$$\hat{v}_t \~ N(0, \sigma^2) \tag{11}$$

Note that the coefficient $b$ can be written as the product $\alpha D_1$ where $D_1$ is the first column of $A_0^{-1}$. As shown in Bahaj (2019), $D_1$ can be recovered by using the fact that $D_1 \Sigma D_1' = 1$. Given a draw of $b$, this condition implies that $D_1 = \frac{1}{\sqrt{b \Sigma b'}} b$. The first column of $A_0$ (corresponding to the shock of interest $\varepsilon_t^{MPU}$) is then given by $A_{0(1)} = \Sigma D_1'$.

## 2.3  Estimation and specification

The FAVAR model is estimated using Bayesian methods. The priors and the Gibbs sampling algorithm is described in detail in the Technical Appendix. Here, we summarize the main steps. The Gibbs algorithm draws from the following conditional posterior distributions:

1. Conditional on the factors and the parameters of equation 10, the parameters of the VAR in equation 4 can be drawn using standard methods for Bayesian VARs once equation 4 is transformed to remove the correlation between the residual $u_t$ and the instrument $m_t$.

2. Conditional on the factors and the draw for missing data $\hat{x}_t$, the factor loadings and the parameters of the AR model for the idiosyncratic errors (equation 2) can be drawn using standard methods for Bayesian linear regressions. Similarly, these methods also apply when drawing from the conditional posterior of $b$ and $\sigma^2$ in equation 10.

3. Conditional on $\hat{x}_t$ and the remaining parameters, the model can be cast in to state-space form. The observation equation is given by equation 1, while equations 2, 4 and 10 form the transition equation. The factors can then be drawn from their conditional posterior distribution using the Carter and Kohn (1994) algorithm.

4. Conditional on the factors, factor loadings and the parameters of equation 2, missing data can be drawn for the $jth$ series in $\hat{x}_t$ by using the Carter and Kohn (1994) algorithm (see also Schorfheide and Song (2015)). The observation equation for the system is given by equation 7 when quarterly data is available. When data is missing, the observation equation is defined as $x_{jt}^Q = \tilde{u}_{jt}$ where $var(\tilde{u}_{jt})$ is calibrated to be a large number. Equations 1 and 2 can be used to form the transition equation.

The algorithm is run for 50,000 iterations with a burn-in of 25,000 iterations. Every fifth remaining draw is used to approximate the posterior distributions. The Technical Appendix presents evidence that is consistent with convergence.

In the benchmark case, we set the number of factors to 5. Following Bernanke *et al.* (2005), we show in the robustness analysis that the main results are similar when the number of factors is increased. In order to keep the number of unobserved states at a manageable level, the lag lengths in equation 4 and 2 are fixed at 6 and 1, respectively.

## 2.4  Empirical results

Figure 1. presents the impulse responses of key aggregate variables in the FAVAR model. The monetary policy uncertainty shock is normalized to increase the MPU index of Husted *et al.* (2019) by 100 percent points.
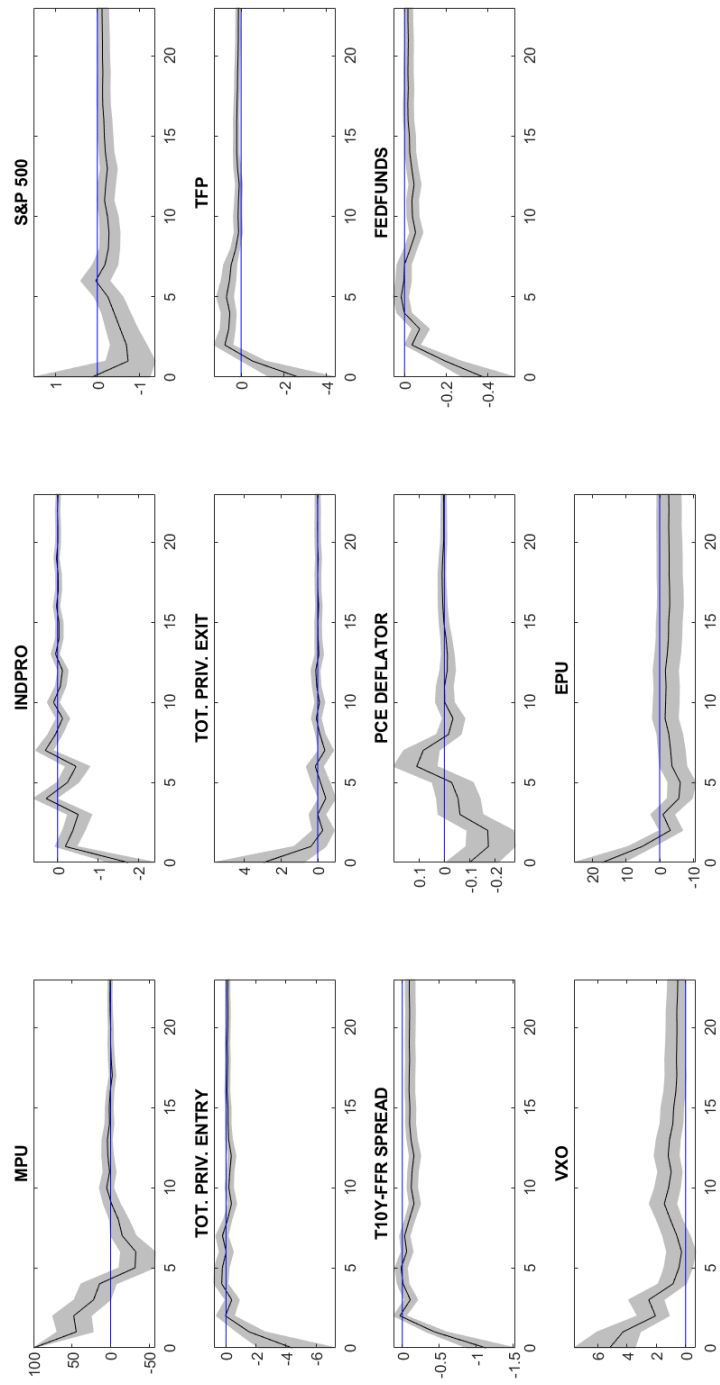
Figure 1: Impulse response to a monetary policy uncertainty shock. The solid line is the median. The shaded area is the 68% error band.
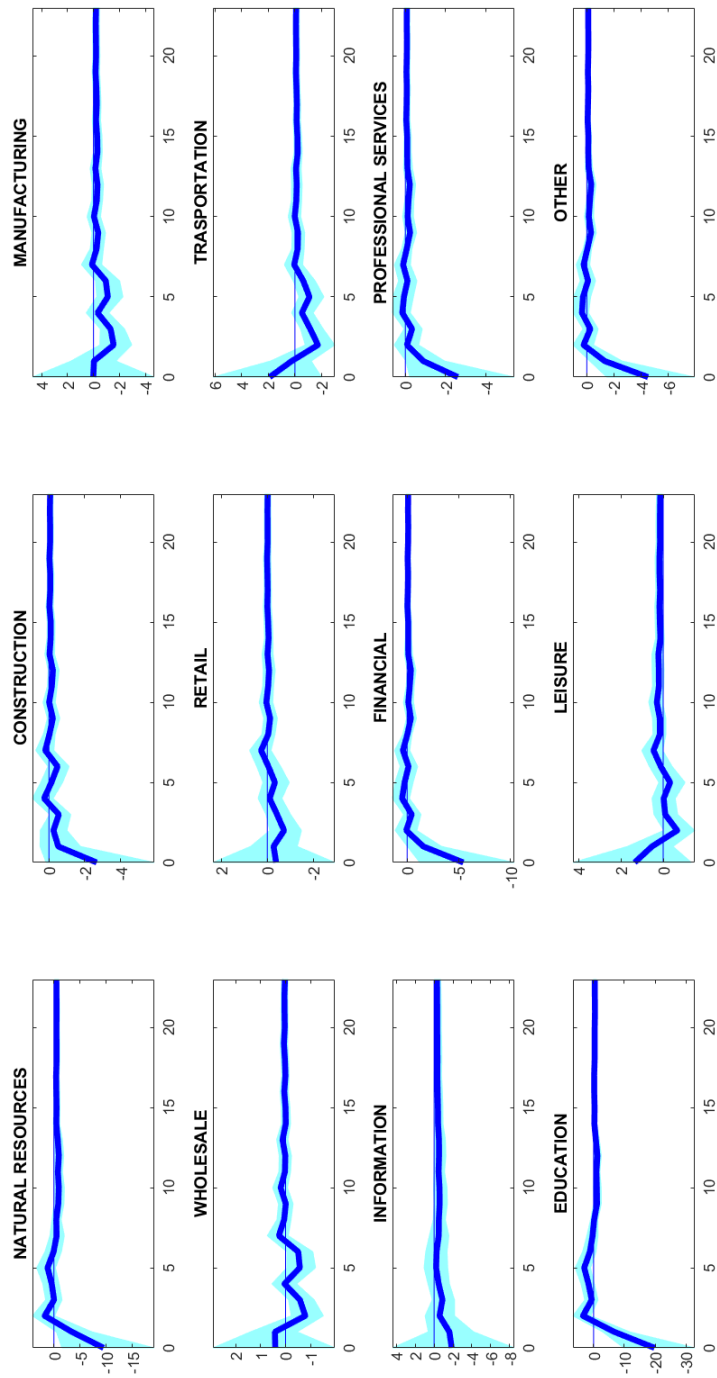
7

Figure 2: Impulse response of establishments' entry at the industry level. The solid line is the median. The shaded area is the 68% error band.
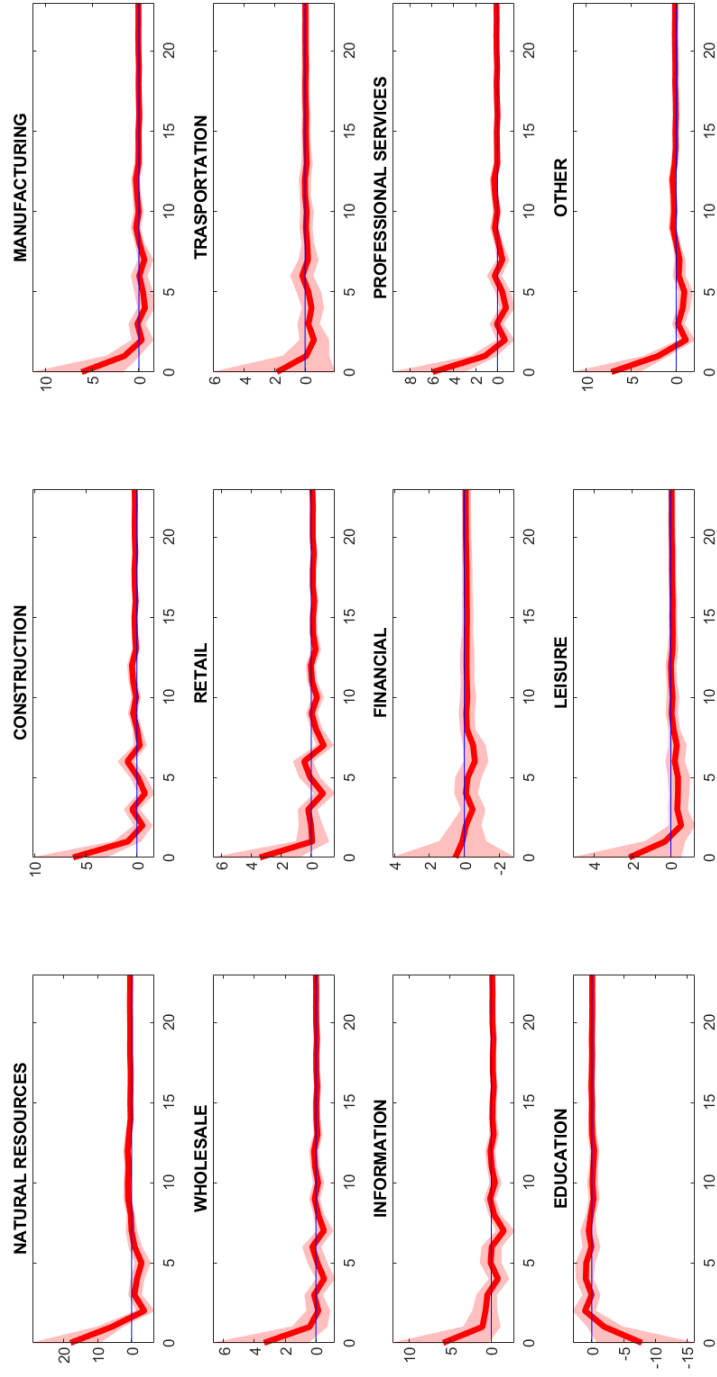
Figure 3: Impulse response of establishments' exit at the industry level. The solid line is the median. The shaded area is the 68% error band.

The monetary policy uncertainty shock implies a fall in the industrial production by around 2% on impact. Economic activity further declines in the following periods. A similar path is followed by CPI inflation and the Federal Fund rate making the innovation look like a recessionary demand shock. Also, the S&P 500 index contracts. Its fall is persistent and lasts for about three years after the shock. Establishments' entry and exit in the total private sector move in opposite directions. Firm entry reduces on impact -the median falls by around 4%- and remains negative in the subsequent periods. Firm exit increases, but its response is milder than for firm entry and lasts for fewer periods. The response of establishments' death is not systematically different from zero at 3-quarter horizon. Differently from other variables, the sign of the utilization-adjusted TFP changes from the short to the medium run. Although the TFP is initially decreasing, its response changes the sign after few periods and becomes positive at 2-year horizon. As the recession hits, the term premium reduces. The response of the volatility in the financial market -measured by the VXO index-, and of the uncertainty related to the economic policy -measured by the EPU index of Baker *et al.* (2016), are both positive and statistical significative. This also suggests an endogenous relationship between such broader measures of uncertainty and the monetary policy uncertainty.

Considering the data at the industry level, Figure 2 and 3 show the responses of establishments' entry and exit, respectively. Still with disaggrated data, the sign of the response to a monetary policy uncertainty shock is generally negative for entry and positive for exit. Results are particularly robust for the good-producing industries, namely Natural Resources and Mining, Construction, Manufacturing. For the Manufacturing industry, the response of establishments' entry is unprecisely estimated at impact, but it becomes negative after few periods. The rest of the sectors comprises the nine industries of the service-providing composite sector. For some of these industries -Financial services, Professional services, Education and Health services, Other services-establishments' entry declines since the impact of the shock, while for others -Wholesale, Retail, Trasportation- the confidence interval falls below the zero line after some periods. Establishments' exit in the service-providing industries are well-estimated. With few exceptions -Financial services, Education and Health services-, the response is firmly positive both at impact and thereafter confirming the evidence with the aggregate data.

## 2.5   Robustness

To validate the evidence of the FAVAR model, we consider a battery of robustness checks. A detailed description of the models and the data used for the robustness checks are left to the Technical Appendix, along with the plots of the impulse responses. Here, we sum up the robustness checks done and the findings as follow.

1. We estimate the benchmark FAVAR, but using four, six, seven factors. The responses of all variables considered are identical to the benchmark model with the only exception of firms exits whose response becomes imprecise when the number of factors is greater than the benchmark model.

2. We estimate the benchmark FAVAR using a different instrument to identify the monetary policy uncertainty shock -namely Carlston and Ochoa (2016)'s conditional volatility. Also in this case the results in the benchmark model are confirmed.

3. We estimate the benchmark FAVAR, but we identify the monetary policy uncertainty shock through a recursive scheme, that is via Cholesky decomposition taking Husted *et al.* (2019)'s MPU index as the most exogenous among the variables, or equivalently, considering the other series in the dataset as fast-moving variables that have a contemporaneous relationship with

the monetary policy uncertainty. Figures reported in the Appendix confirm that the dynamics triggered by the monetary policy uncertainty shock is consistent with the benchmark.

4. We check the evidence in a smaller model, that is in a mixed-frequency VAR model with monetary policy uncertainty, industrial production, CPI index, economic policy uncertainty, excess bond premium, 1-year Treasury bond rate, establishments' entry and exit in the total private sector, as endogenous variables.We use the external instrument used in the benchmark FAVAR for identifying the monetary policy uncertainty shock. The results of this check are also consistent with the benchmark model.

We can state that independently of the number of factors, of the measure of monetary uncertainty, and of the instrument used for the identification, the FAVAR estimation indicate that after a monetary policy uncertainty shock, the economy shrinks while the total factor productivity increases in the medium-run. The fall in the production is followed by a bust in the stock market and a decreasing firms' participation into the market. Firm establishments' entry and exit go downwardly and upwardly, respectively. Remarkably, the responses of aggregate firm entry and exit are in line with the benchmark also when we choose a recursive scheme to identify the monetary policy uncertainty shock. The impulse responses in the Proxy-VAR model strengthen the ones delivered by the FAVAR. The shock dampens industrial production, inflation, short interest rate, and fosters the economic policy uncertainty. Moreover, the negative pattern for firm entry and the positive one for firm exit in confirmed in a small-scale model. To address the empirical evidence, in the next section we build-up a medium scale model with heterogeneous firms and where both firms' creation and destruction are endogenous.

# 3   Theoretical Model

In this section, we summarize the theoretical framework of the baseline model considered all along the paper (labeled as *Baseline* henceforth). The Baseline model is a modified version of a standard medium scale model. The main ingredients of the medium scale model and its microfoundations are well know in the literature (Christiano *et al.* (2005), Smets and Wouters (2007)), and the details are not discussed here. We assume sticky nominal wages and prices a là Rotemberg (1982), adjustment costs and capacity utilization for capital, internal habit persistence. On the top of that we introduce firms heterogeneity and endogenous entry and exit dynamics in the intermediate sector. We model the sector as in Rossi (2019).

We now present a brief description of the Baseline model, underlying the main differences with respect to the standard medium scale model and the way in which monetary uncertainty shock is introduced. The full list of the equations characterizing the model is in the Technical Appendix.

The model consists of a closed economy composed by four agents: households, firms, a monetary authority, and a fiscal authority. In what follows, a description of the behavior of the four agents.

## 3.1   Households

Households consume a basket of differentiated retailer-goods, $C_t$ and their consumption is characterized by external habits. They supply labor, $L_t$, to intermediate-good producing firms, they save in the form of new risk free bonds, $B_{t+1}$, of physical capital, $K_{t+1}$, of a portfolio shares of incumbent firms, $x_{t+1}$, and of new entrants, $N_{t+1}^E$. The period utility of the household is defined

over the consumption bundle, $C_t$, and the labor bundle of services, $L_t$. It reads as follows:

$$U(C_t, L_t) = \frac{\left(C_t - h\overline{C}_{t-1}\right)^{1-\sigma_C}}{1-\sigma_C} \exp\left[\chi\frac{(\sigma_C - 1)(L_t)^{1+\sigma_L}}{1+\sigma_L}\right] \tag{12}$$

where $h$ measures the degree of external habits in consumption , $\overline{C}_{t-1}$ is the aggregate consumption, $\sigma_C$ defines the coefficient of the relative risk aversion that determines the constant intertemporal elasticity of substitution $(\frac{1}{\sigma_C})$, $\chi$ captures the relative weight assigned to labor and $\sigma_L > 0$ represents the inverse of the Frisch elasticity of labour supply.

Households own physical capital stocks, $K_t$, and leases capital services, $K_t^s$, to firms, as in Smets and Wouters (2007). Capital services are related to the physical capital according to the following relationship:

$$K_t^s = u_t K_t \tag{13}$$

The household budget constraint is the following

$$C_t + B_{t+1} + v_t(\tilde{z})x_{t+1} + I_t + FEX_t N_{t+1}^E + T_t \leq$$
$$w_t L_t + \left[r_t^K u_t - a(u_t)\right]K_t + \frac{1+r_{t-1}}{1+\pi_t}B_t+$$
$$+ \left[(1-\eta_t)(v_t(\tilde{z}) + j_t(\tilde{z})) + \eta_t lv_t\right]\left(x_t + N_t^E\right) \tag{14}$$

Households enter in the period $t$ earning the real gross income from labor, $w_t L_t$, the nominal return on bonds, $r_{t-1}B_t$, the real return of capital $\left[r_t^K u_t - a(u_t)\right]K_t$, where $r_t^K$ is the real rental rate of capital, and $a(u_t)$ is the adjustment cost of variable capital utilization $u_t$. During the period $t$, the households buy shares of incumbent firms, $x_{t+1}$ and of new entrants $N_{t+1}^E$. The households earn the firm value $v_t(\tilde{z})$ and the firm profit $j_t(\tilde{z})$ with a probability $(1 - \eta_t)$ measuring the survival rate of firms, and the liquidation value $lv_t$ with a probability $\eta_t$ measuring the exit rate of firms. $T_t$ is a lump sum transfer. The households spend all the earning to consume and save. The variable $FEX_t$ captures the cost of entry paid by the household for the new startup firms, which are defined as in Casares *et al.* (2018), as a combination of constant and variable costs,

$$FEX_t \equiv f^E + ec_t \tag{15}$$

where $f^E$ is the unit real cost of license fee paid to the fiscal authority to begin the production of a new variety, and $ec_t$ measures congestion externalities for start-up firms:

$$ec_t = \Theta^e E_t \left(\frac{N_{t+1}^E}{N_t}\right)^{\varsigma_e} \tag{16}$$

$\Theta > 0$ and $\varsigma > 1$. If a firm exits, a liquidation value is returned to households, which is a positive function of the fraction of the licence fee paid at entry, $f^E$, and a negative function of exit congestion externalities, $xc_t$:

$$lv_t = (1-\tau)f^E - xc_t \tag{17}$$

where, as in Casares *et al.* (2018), the parameter $\tau$ represents the fraction of license fees returning to the households and paid by the fiscal authority once a firm exits the market, while

$$xc_t = \Theta^x E_t \left(\frac{N_{t+1}^X}{N_t}\right)^{\varsigma_x} \tag{18}$$

represents exit congestion externalities.

The law of motion of the firms follows the standard time to build assumption as

$$N_{t+1} = (1 - \eta_t)\left(N_t + N_t^E\right) \ . \tag{19}$$

The stock of firms, $N_t$, is given by the sum of incumbent firms , $(1 - \eta_t) N_{t-1}$, and surviving new entrants, $(1 - \eta_t) N_{t-1}^E$. Firms separation rate depends on an endogenous probability of defaulting, $\eta_t$. Both new entrant and incumbents firms are subject to the same endogenous exit probability.

Households choose capital utilization and end up paying a quadratic cost for that utilization relative to its normalized steady state value, which is equal to 1,

$$a\left(u_t\right) = \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2} \left(u_t - 1\right)^2 \tag{20}$$

where $\gamma_1$ and $\gamma_2$ are the parameters governing the cost of utilization of capital.

Physical capital accumulates as follows:

$$K_{t+1} = \left(1 - \delta^K - S\left(\frac{I_t}{K_t}\right)\right) K_t + I_t - \tag{21}$$

where $\delta^K$ is the depreciation rate, and $S\left(\frac{I_t}{K_t}\right)$ are capital adjustment costs defined as in Hayashi (1982), as:

$$S\left(\frac{I_t}{K_t}\right) = \frac{\phi_K}{2}\left(\frac{I_t}{K_t} - \delta^K\right)^2 \tag{22}$$

The implied first order condition of the household problem are listed in the Technical Appendix. They are the households' labor supply, the households' investment choice, the Euler equation for consumption, for physical capital, for share holding, and the firm entry condition.

Households supply their homogenous labour to an intermediate labour union which differentiates the labour services and sets wages subject to Rotemberg (1982) adjustment costs. As for the FOCs of the household problem, the wage New-Keynesian Phillips curve (NKPC) resulting from the union problem is reported in the Technical Appendix.

## 3.2 Firms

As in Rossi (2019), the supply side of the economy consists of an intermediate and a retail sector. The intermediate sector is composed by a continuum of $N$ intermediate firms that compete under monopolistic competition and flexible prices to sell the intermediate goods to a continuum of measure 1 of retailers. Each $k \in (0, 1)$ retailer buys intermediate goods from the intermediate sector and differentiate them with a technology that transforms the intermediate goods into an aggregate industry good, $Y_t^I(k)$, solving a minimum expenditure problem. Retailers sell the differentiated industry goods to households, competing with other retailers under monopolistic competition. They face Rotemberg (1982) adjustment costs so that, due to the monopolistic competition structure, the second optimization problem gives rise to the price NKPC.

### 3.2.1 Intermediate Sector

Each firm in the intermediate sector produces a differentiated good under monopolistic competition and flexible prices.[2] Firms are heterogeneous in terms of their specific productivity, which is drawn from a Pareto distribution. In this context, the production function of firm $\iota$, with $\iota \in [1, N]$, is

$$y_{\iota,t} = z_{\iota,t} l_{\iota,t}^{1-\alpha} \left( k_{\iota,t}^s \right)^\alpha \tag{23}$$

where $l_{\iota,t}$ and $k_{\iota,t}^s$ are respectively, the amount of labor hours and capital services employed by firm $\iota$, while $z_{\iota,t}$ is a firm specific productivity, which is assumed to be Pareto distributed across firms, as in Ghironi and Melits (2005).

This sector is characterized by endogenous firms dynamics. The timing characterizing the dynamics of firms is the following. At the beginning of period, households invest in new firms until the entry condition is satisfied, that is until the average firms' value equals the entry costs,

$$v_t (\tilde{z}) = FEX_t \tag{24}$$

Then, both new entrants and incumbent firms draw their firms specific productivity from a Pareto distribution. The cumulative distribution function (CDF) of the Pareto implied for productivity $z_{\iota,t}$ is $G(z_{\iota,t}) = 1 - \left( \frac{z_{\min}}{z_{\iota,t}} \right)^\xi$, where $z_{\min}$ and $\xi$ are scaling parameters of the Pareto distribution.[3] After that, firms observe the aggregate shock and decide whether to produce or exit the market. Using this timing assumption, the decision of new entrants to exit the market is identical to the decision of incumbent firms. In particular, both new entrants and incumbent firms decide to produce as long as their specific productivity $z_{\iota,t}$ is above a cutoff level $\overline{z}_t$. The latter is the level of productivity that makes the sum of current and discounted future profits equal to the liquidation value, $lv_t$. Separated firms exit the market before starting the production. It follows that the average output and the average firms productivity depends on the cut-off level of productivity in the economy, $\overline{z}_t$, which is endogenously determined through the following exit condition:

$$v_t (\overline{z}_t) = j_t (\overline{z}_t) + \beta E_t \left[ \Lambda_{t,t+1} v_{t+1} (\overline{z}_{t+1}) \right] = lv_t, \tag{25}$$

with $\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \left( 1 - \eta_{t+1} \right)$, as the stochastic discount factor, $\lambda_t$ as the marginal utility of consumption at time $t$, $j_t (\overline{z}_t) = y_t (\overline{z}_t) - w_t l_{\overline{z},t} - r_t^K k_{\overline{z},t}^s$, as the current profits of the marginal firm with a productivity $z_{\iota,t} = \overline{z}_t$, $w_t l_{\overline{z},t}$ as the cost of labor of the marginal firm, $r_t^K k_{\overline{z},t}^s$ as the cost of capital services. The exit probability, $\eta_{t+1} = 1 - \left( \frac{z_{\min}}{\overline{z}_{t+1}} \right)^\xi$, is endogenously determined. As in Ghironi and Melits (2005), the lower bound productivity level, $z_{\min}$, is low enough relative to the production costs, so that $\overline{z}_t$ is above $z_{\min}$. In each period, this ensures the existence of an endogenously determined number of exiting firms. The number of firms with productivity levels between $z_{\min}$ and the cutoff level $\overline{z}_t$ are separated and exit the market without producing.

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[2]In this model sticky prices are in the final sector and not in the intermediate good sectors, where the firm dynamism is modeled. This is for technical reasons. To satisfy the Melitz (2003) theorem of price aggregation markups should be the same across firms. Yet, the main results are not affected by the sticky price assumption, since the stickiness in the final sector transmits to the intermediate sector.

[3]They represents respectively the lower bound and the shape parameter, which indexes the dispersion of productivity draws. As $\xi$ increases, the dispersion decreases and firm productivity levels are increasingly concentrated towards their lower bound $z_{\min}$.

### 3.2.2 Retailers

The retailer problem is split into two parts. First, each $k \in (0,1)$ retailer buys a fraction of the $N$ intermediate goods produced by the $N$ intermediate firms at the intermediate goods prices $p_{\iota,t}$, with $\iota \in [1, N]$. Retailers bundle the goods into an aggregate industry good, $Y_t^I(k)$, minimizing their expenditure according to a CES technology $Y_t^I(k) = \left( \int_{N_t} y_{\iota,t}^{\frac{\theta_p - 1}{\theta_p}} d\iota \right)^{\frac{\theta_p}{\theta_p - 1}}$, with $\theta_p > 1$, as the elasticity of substitution among the intermediate goods varieties. Retailer's minimum expenditure problem implies the following demand function for the intermediate good $i$:

$$y_{\iota,t} = \left( \frac{p_{\iota,t}}{P_t^I} \right)^{\theta_p} Y_t^I(k), \tag{26}$$

implying the intermediate sector price index as

$$P_t^I(k) = \left( \int_{N_t} p_{\iota,t}^{\theta_p - 1} d_i \right)^{\frac{1}{\theta_p - 1}}.$$

Second, each $k$ retailer competes with the others under monopolistic competition to sell its bundle, $Y_t^I(k)$, to the household at the price $P_t^R(k)$, which is a markup over the intermediate sector price index, $P_t^I(k)$. Retailers adjust prices according to the Rotemberg (1982) model. The retailer's optimal price decision rule implies the following standard NKPC:

$$1 = \frac{\theta_p}{\theta_p - 1} \rho_t^I - \frac{\phi_p}{\theta_p - 1} (\pi_t - 1) \pi_t + \frac{\phi_p}{2} (\pi_t - 1)^2 + \frac{\phi_p}{\theta_p - 1} E_t \left\{ \Lambda_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\} \tag{27}$$

with $\phi_p$ as the adjustment price parameter, and $\rho_t^I$ as the relative price $\frac{P_t^I(k)}{P_t}$. By symmetry among the retailers, it holds $Y^R(k) = Y_t$ and $P^R(k) = P_t$. Hence, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

## 3.3 Monetary and Fiscal Authority

**Monetary Authority**

To close the model we specify an equation for the Central Bank behavior. We simply assume that the monetary authority sets the nominal net interest rate $i_t$ following a standard Taylor-type rule given by

$$\left( \frac{1 + i_t}{1 + i} \right) = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{\phi_R} \left( \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{y_t}{y} \right)^{\phi_y} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_{dy}} \right)^{(1 - \phi_R)} \exp(\varepsilon_{m,t}),$$

where $\phi_\pi$, $\phi_y$, $\phi_{dy}$ being the elasticities of the nominal interest rate with respect to the deviation of the inflation and output from their long run target, and to the growth rate of output. The parameter $\phi_R$ is the interest rate smoothing parameter. We model the monetary uncertainty shocks by using the stochastic volatility approach proposed by Mumtaz and Zanetti (2013) and Born and Pfeifer (2014), that is by assuming time varying volatility of the innovation to the monetary shock. Specifically, the policy uncertainty shock enters into the economy through the monetary shock,

$\varepsilon_{m,t}$, that follows an AR(1) process,

$$\varepsilon_{m,t} = \rho_m \varepsilon_{m,t-1} + \exp(\sigma_{R,t}) u_{\varepsilon,t} \tag{28}$$

with

$$\sigma_{R,t} = \rho_\sigma \sigma_{R,t-1} + u_{\sigma,t} \tag{29}$$

where $u_{\sigma_{R,t}}$ is the Gaussian innovation to the monetary shock, i.e. the level shock, while $u_{\sigma,t}$ is the Gaussian innovation to the standard deviation, $\sigma_{R,t}$, of the monetary shock, i.e. the volatility shock.

### Fiscal Authority

The fiscal authority runs the following balanced budget:

$$T_t = f^E N_t^E - (1 - \tau) f^E N_t^X$$

where $T_t$ are lumps-sum transfers/taxes to the households, $f^E N_t^E$ are the revenues obtained from households in form of administrative fees for opening new startups, $(1 - \tau) f^E N_t^X$ is the expenditure in form of liquidation value paid to households as firms exit the market.

## 3.4 Aggregation and Market Clearings

The economy aggregate output is implied by the following

$$Y_t = N_t^{\frac{1}{\theta_p - 1}} \tilde{z}_t (L_t)^{1-\alpha} (K_t^s)^\alpha \tag{30}$$

while the resource constraint of the economy is given by,

$$Y_t = C_t + I_t + a(u) K_{t-1} + N_t^E ec_t + N_t^X xc_t + PAC_t + WAC_t \tag{31}$$

where

$$PAC_t = \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t \tag{32}$$

and

$$WAC_t = \frac{\phi_w}{2} \left( \frac{w_t}{w_{t-1}} \pi_t - 1 \right)^2 Y_t \tag{33}$$

are respectively the price and wage adjustment costs.

## 3.5 Calibration and Model Dynamics

Calibration is set on a quarterly basis. For comparability, we consider the same calibration for all DSGE specifications. The discount factor, $\beta$, is set at 0.99. The coefficient of the relative risk aversion, $\sigma_C$, is set to 1.5, while the inverse of Frisch elasticity of labor supply, $\sigma_L$, to 5. The habits persistence parameter is set to 0.6, in line with Boldrin *et al.* (2001). The steady state value of the exit probability $\eta$ is 0.0291 to match the U.S. empirical evidence for the period considered in the FAVAR model. Importantly, the steady state value of $\eta$ also determines the constant exit probability in the DSGE specification where firm exit is constant and exogenous determined. The parameter of the elasticity of substitution among intermediate goods, $\theta_p$, is set equal to 4.3, a value which is in line with Ghironi and Melits (2005) and Bilbiie *et al.* (2012). The shape parameter of the Pareto distribution $\xi$ is set equal to 6.51 to satisfy the steady state value of the exit rate. Also, it guarantees that the condition $\xi > \theta_p - 1$ is satisfied. The lower bound of productivity

distribution, $z_{\min}$, is equal to 1. The fix part of entry costs is calibrated to a have variable entry costs $ce_t$ due to congestion externality equal to 1.23% of the GDP in the steady state, as in Casares *et al.* (2018). The parameters of the entry and exit congestions externalities, $\varsigma_e$ and $\varsigma_x$, are set equal to 2 and 1, respectively. Once these two are calibrated the remaining parameters $\Theta^e$ and $\Theta^x$ are endogenously determined to satisfy the steady state entry rate. Finally the parameters $\tau$ is set equal to 0.75, so that only 25% of the fixed entry costs of the exiting firms are rebated to the households.

We set the Rotemberg parameter of price adjustment cost $\phi_p$ equal to 40 so that the slope of the Phillips curve in the model corresponds to that in a Calvo staggered price-setting model with four quarters of price contract duration. The parameters of the wage adjustment costs $\phi_w$ is set equal to 40 as in the good market. Also the wage markup is calibrated as in the good market, namely setting $\theta_w = 4.3$. The depreciation rate of the physical capital, $\delta_k$, is set equal to 0.02, the parameter measuring the elasticity of the capital utilization adjustment cost function, $\gamma_2$, is set to 0.54 as in Smets and Wouters (2007), while the capital adjustment costs parameter, $\phi_K$, is set to equal to 4.

We study a 100% increase to the volatility of the nominal interest rate, as in the FAVAR model. The persistence parameter of the uncertainty shock is calibrated following Leduc and Liu (2016). Our evidence suggests that the effects of the uncertainty shock falls gradually to about 47.7% of its impact value after one quarter. This observation suggests that, if the shock is approximated by an AR(1) process, as in our model, the persistence parameter is about 0.8314 at quarterly frequency. Thus, we set $\rho_\sigma = 0.8314$.

Finally, we set the persistence parameter in the monetary policy shock, $\rho_m$, equal to 0.5, and a Taylor rule, with $\phi_R = 0.75$, $\phi_\pi = 2.5$, $\phi_y = 0.01$, $\phi_{dy} = 0.05$. This rule guarantees the uniqueness of the equilibrium. Further, these parameters are in the range of the values estimated for the U.S. economy.[4]

## 3.6 IRFs to Monetary Uncertainty Shocks

We now show the responses to a monetary uncertainty shock in the DSGE model. To examine the dynamic effects of the uncertainty shock, we solve the model using third-order approximations to the equilibrium conditions around the steady state. We follow the procedure suggested by s Fern ndez Villaverde *et al.* (2011) to compute the impulse responses in deviation from the stochastic steady state.

We carried out the impulse response analysis as follows. First, we compare the performance of the Baseline model with two alternative models: i) a medium scale model with constant firms (labeled as *No Firms*); ii) a model with endogenous entry, but exogenous and constant exit probability, $\eta$, (labeled as *Exo Exit*). Second, we investigate the responses by letting the degrees of price stickiness to vary. Third, we hold fixed the degree of price stickiness, but study the responses by changing the persistence of the monetary policy shock.

Figure 4. shows the responses to a monetary uncertainty shock of three DSGE specifications: Baseline, Exo Exit, No Firms. The comparison allows us to investigate the relevance of firm dynamics and firm heterogeneity in explaining the propagation of the shock. In Exo Exit, the heterogeneity in firm productivity does not play a role as the probability of defaulting is constant and does not depend on the idiosyncratic level of productivity each firm draws at the beginning of the period. In No Firms, only the intensive margin of the investment, namely the one in physical

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[4]See for example Smets and Wouters (2007). The qualitative results and the comparison with the exogenous exit model and with the model with no firm dynamics are not qualitatively altered by the choice of the Taylor rule.
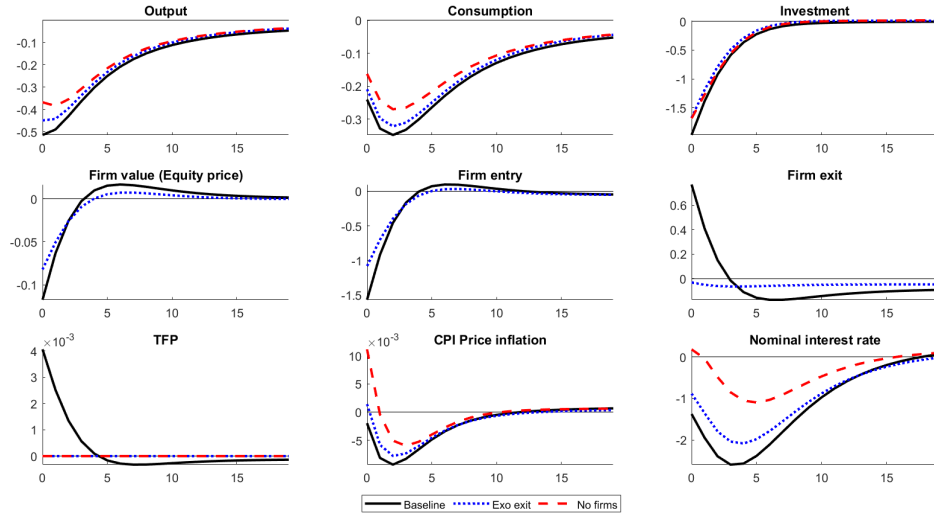
Figure 4: IRFs to 100% increase to the volatility of the nominal interest rate under the benchmark calibration. Baseline model versus the model with exogenous exit (Exo Exit) and the model without entry and exit (No Firms).

capital, is allowed to respond, while the extensive margin, namely the investment in new firms, is neglected.

In all specifications considered, an increase in the monetary policy volatility is followed by a reduction in output, consumption and investment in physical capital. However, thanks to the dynamics of firms, the magnitude of this drop is the largest in the baseline model. Remarkably, in line with the evidence for the aggregate and industry level data in the FAVAR, the Baseline model is the only specification showing a contemporaneous decrease in firm entry and an increase in firm exit. The decline in entry is driven by the strong reduction in the firm value, that corresponds to the price of firms stocks. In both Baseline and Exo Exit model, the firm value falls mimicking the S&P 500 index in the FAVAR, but in Baseline this contraction is heavier. The firm value is equal to the present discounted value of the stream of expected future profits. As firm profits decline, in Baseline model the threshold value of productivity $\overline{z}_t$ increases, and the exit probability rises as well. It follows that in Baseline model, firm exit increases after the shock. Things are different for Exo Exit and No firms models. In the former, entry declines after the shock, but less than in Baseline model. This occurs because the exit rate $\eta$, which affects firms' stochastic discount factor, remains constant along the business cycle. Instead, in Baseline model the exit probability increases on impact reducing the stochastic discount factor that makes the reduction of the average firm value, larger. As a consequence, firm entry falls less in Exo Exit than in Baseline model. In Exo Exit, the dynamics of exiting firms is proportional to the stock of firms participating the market, as in Bilbiie *et al.* (2012). As the number of firms falls because of decreasing entry, the number of exiting firms reduces proportionally and, at odds with the evidence found in the FAVAR, becomes procyclical with respect to output. In Exo Exit, firm exit indeed reduces on impact and remains below zero for several periods before returning to its long run level. This helps to make the effects of the shock milder than in Baseline model, so that output shrinks less. In No Firms, the number of firms is fixed and the shock cannot propagate at all through the extensive margin of investment. The recession is then shown to be even less severe. A lower contraction in real variables have

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consequences on the nominal side of the economy in both Exo Exit and No Firms. In contrast with the evidence of the FAVAR, the price inflation increases on impact for both the Exo Exit and the No Firms model. Although the inflation falls below zero after a few periods for both models, under NO firms the rise at impact is such that the policy rate is increasing after the shock, at odds with the pattern of the federal fund rate in the FAVAR.

Furthermore, neither No Firms nor Exo Exit model are able to replicate the dynamics of the TFP in the data. The model response of the TFP is indeed mute, as long as either no firms exit the market or firms exit the market exogenously, and not because of their low level of idiosyncratic productivity. In both cases, firm average productivity and the aggregate TFP remain constant. In Baseline model, the TFP reacts instead positively. Though in the data the TFP shows a different in sign on impact, the response reads however positive in the medium run. The differences at a very short horizon can be justified by the fact that the creative destruction mechanism we emphasize is only one of the possible mechanism affecting the TFP. Another possible channel might be the dynamics of the labor market and more in particular, of the unemployment. During a recession, unemployment increases with lags and thereby, total hours worked decrease with delays as well. The same occurs for the stock of capital. While output reacts immediately, sluggish adjustments in the productive factors can justify the initial reduction of the TFP. However, as soon as unemployment increases and firms with lower productivity are pushed out of the market, the TFP increases and, as the response in the FAVAR shows, remains positive in the medium term. All in all, this confirms that the response in Baseline model are the most in accordance with the ones obtained in the FAVAR.

## 3.7   Robustness

With the FAVAR model we have identified the uncertainty shock and computed its persistence, that we have taken to calibrate the process of the monetary policy volatility (29). For the rest of the parameters, we followed the related literature. Specifically, in the benchmark calibration, we imposed a degree of rigidity in price adjustment, i.e $\phi_p = 40$, that corresponds to that in a Calvo staggered price-setting model with four quarters of price contract duration, and a persistence of the monetary shock, $\rho_m$, equal to 0.5. To better understand the role played by nominal persistence in price adjustment and monetary shocks, in this section we run robustness checks considering alternative values for $\phi_p$ and $\rho_m$.

**Price stickiness.**   Figure 5 compares the responses of the Baseline model when $\phi_p$ is set to 10, 40, 70, and $\rho_m$ to 0.5. At the first glance, the results of the Baseline model are highly robust to different costs for firms in adjusting prices. Higher costs in changing prices makes the effects of an uncertainty shock stronger and lasting for much more periods. The fall in output, consumption, investment more than doubles when the price contract duration in the corresponding Calvo pricing scheme rises from two ($\phi_p = 10$) to four ($\phi_p = 40$) quarters. The impact to firm flows and aggregate productivity is enhanced by the same size. A similar change is obtained when the price contract duration is even higher, i.e. more than five quarter ($\phi_p = 70$). In this case, however, the response of inflation changes the sign becoming positive. As explained by Born and Pfeifer (2014), when uncertainty increases, it might be convenient for firms to increase the selling prices because of the convexity of the marginal profit curve. This assumption implies that keeping prices high is more profitable for firms when the uncertainty about future outcomes is increased (*inverse Oi–Hartman–Abel effect* in Born and Pfeifer (2014)). The response of inflation in Figure 5, indicates that this
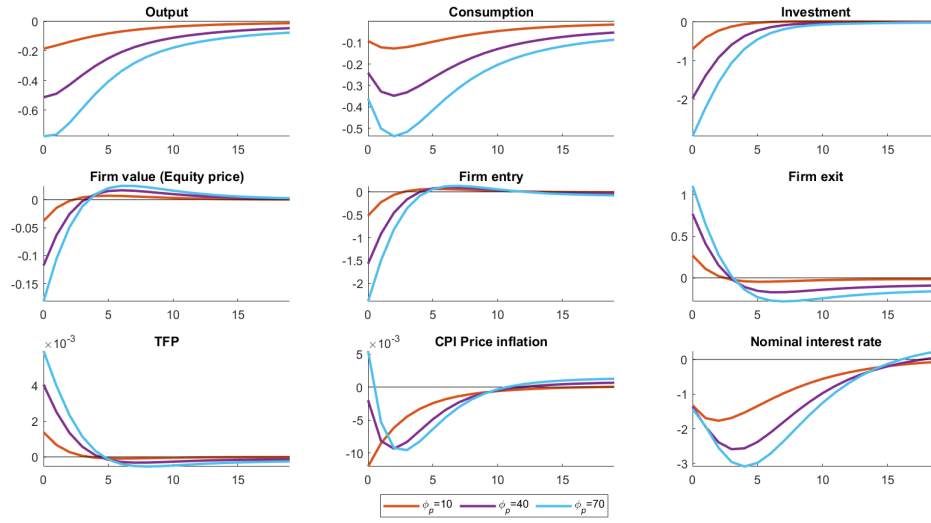
Figure 5: IRFs to 100% increase to the volatility of the nominal interest rate under different degrees of the persistence of the price stickiness degree.
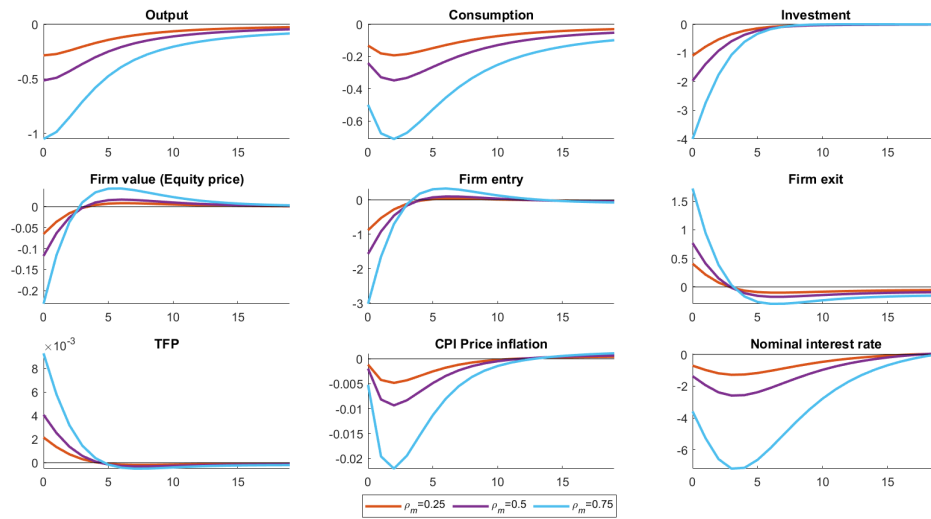


Figure 6: IRFs to 100% increase to the volatility of the nominal interest rate under different degrees of the persistence of the monetary polick shock.

effect prevails in our model when it is more costly for firms to update prices, or alternately, the duration for price contracts they face is longer.

**Monetary shock persistence.** Figure 6 compares the responses of the Baseline model when $\rho_m$ is set to 0.25, 0.5, 0.75, and $\phi_p$ to 40. Although the monetary policy uncertainty shock is a second moment shock, the persistence of the monetary policy shock, i.e. a first moment shock, matters for the transmission of the former to the economy. Equation (28) clarifies the relationship between the volatility shock, $\sigma_{R,t}$, and the level shock, $\varepsilon_{m,t}$. As for the degree of price stickiness, changing the persistence of the monetary policy shock does not alter qualitatively the responses to the monetary policy uncertainty shock. However, the plots in Figure 6 suggest that the impact is stronger when the persistence of the monetary shock increases. For some variables, the effects also last for much more periods when the persistence is higher. The differences in the propagation of the shock are huge in output and consumption, among the real variables, and inflation and policy rate, among the nominal variables. For all, the impact is magnified as the persistence of the monetary shock raises.

# 4 Conclusion

In this paper we use a FAVAR model to show that a policy uncertainty shock is associated with a negative response of output and inflation, declining stock prices, a lower entrance of new firms and an increased firms' exit. Further, the utility-adjusted TFP increases persistently in the medium- and long-run. To explain these results, we provide a medium scale DSGE model with heterogenous firms and endogenous firm dynamics. Unlike the standard DSGE model, the extended model can match the response for firms' entry and exit obtained from the FAVAR. The proposed model suggests a larger response of real activity to the policy uncertainty shock highlighting the role of firm dynamics in propagating the shock. Furthermore, thanks to the presence of firms heterogeneity and endogenous firms default, a monetary uncertainty shock improves resource allocation by driving out less productive producers and increasing TFP.

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# Technical Appendix:
# Monetary Policy Uncertainty and Firm Dynamics

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## 1 Estimation

The FAVAR model is defined by the following equations

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{1}$$

$$Y_t = BX_t + u_t \tag{2}$$

$$m_t = bu_t + \hat{v}_t \tag{3}$$

$$v_{it} = \rho_i v_{it-1} + e_{it} \tag{4}$$

where $Z_t$ is the monetary policy uncertainty index built by Husted *et al.* (2019). $\tilde{X}_t$ is a $M \times 1$ vector of variables that include aggregate measures of macroeconomic and financial conditions. $F_t$ denotes a $K \times 1$ matrix of unobserved factors while $\Lambda$ is a $M \times K$ matrix of factor loadings. $X_t = [Y'_{t-1}, .., Y'_{t-P}, 1]'$ is $(NP+1) \times 1$ vector of regressors in each equation and $B$ denotes the $N \times (NP+1)$ matrix of coefficients $B = [B_1, ..., B_P, c]$ The disturbances of the model are defined as:

$$\begin{pmatrix} u_t \\ \hat{v}_t \\ e_t \end{pmatrix} \tilde{} N \left( 0, \begin{pmatrix} \Sigma & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & R \end{pmatrix} \right) \tag{5}$$

where $e_t = [e_{1t}, e_{2t}, .., e_{Mt}]$.

Note that $cov \begin{pmatrix} u_t \\ m_t \end{pmatrix} = \begin{pmatrix} \Sigma & \Sigma b' \\ b\Sigma & b\Sigma b' + \sigma^2 \end{pmatrix}$. Partition this covariance matrix as $\begin{pmatrix} \Omega_u & \Omega_{um} \\ \Omega'_{um} & \Omega_m \end{pmatrix}$. Then using the rules of normal conditional distributions, the conditional mean and variance is given by

$$E(u_t|m_t) = \Omega_{um}\Omega_m^{-1}m'_t = \mu_t \tag{6}$$

$$var(u_t|m_t) = \Omega_u - \Omega_{um}\Omega_m^{-1}\Omega'_{um} = \tilde{\Omega} \tag{7}$$

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The expressions in equations 6 and 7 are used in the Gibbs sampling algorithm described below. Note, however, that the instrument is not available for the full sample and equations 6 and 7 apply over the periods when the instrument is observed.

As mentioned in the main text, the data on entry and exit is available at a quarterly frequency only. Denote this quarterly data as $x_t^Q$ and the corresponding unobserved monthly data as $\hat{x}_t$. Then $\tilde{X}_t = \begin{pmatrix} \tilde{X}_t^M \\ \hat{x}_t \end{pmatrix}$ where $\tilde{X}_t^M$ denotes monthly series taken from FRED-MD. We treat $\hat{x}_t$ as unobserved states and draw them in an additional step in the Gibbs sampler. During periods when quarterly data on the $jth$ series in $x_t$ is observed, we assume that:

$$x_{jt}^Q = \sum_{j=0}^{2} \hat{x}_{jt} \tag{8}$$

That is, quarterly growth rates are the sum of unobserved monthly growth rates. In contrast, when $x_{jt}^Q = nan$, then:

$$x_{jt}^Q = \tilde{u}_{jt}$$

where $var(\tilde{u}_{jt})$ is calibrated to be a large number.

## 1.1 Priors

1. Factor loadings $\Lambda$. We obtain an initial estimate of the factors $F_t$ using an EM algorithm $\left(F_t^{PC}\right)$. Using this estimate we obtain an OLS estimate of the factor loadings $\Lambda_{ols}$. Denote the factor loading for the $ith$ series in $\tilde{X}_t$ as $\Lambda_i$. The prior for $\Lambda_i$ is assumed to be $N(\Lambda_{i,0}, V_\Lambda)$ where $V_\Lambda$ is set as a diagonal matrix with diagonal elements equal to 0.1 and $\Lambda_{i,0}$ equals $\Lambda_{ols}$ for the ith series.

2. Factors $F_t$. The initial values for the factors are assumed to be normal with mean $F_{0\backslash 0}$ and variance $P_{0\backslash 0}$. $F_{0\backslash 0}$ is assumed to be the initial value of $F_t^{PC}$ and $P_{0\backslash 0}$ is set equal to an identity matrix.

3. Equation for Idiosyncratic errors. We use a normal prior for $\rho_i : N(\rho_{i0}, V_{\rho i})$. The prior for $r_i$ is inverse Gamma: $IG(r_{i0}, T_0)$. We set $\rho_{i0} = 0$ and $V_{\rho i} = 10$. The scale parameter and degrees of freedom for the inverse Gamma prior are 0.00001 and 1, respectively.

4. VAR parameters. We use a natural conjugate prior implemented via dummy observations (see Banbura $et\ al.$ (2010)):

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\gamma_1 \sigma_1 ... \gamma_N \sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ .............. \\ diag(\sigma_1 ... \sigma_N) \\ .............. \\ 0_{1 \times N} \end{pmatrix}, and\ X_{D,1} = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1 ... \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP+1} \\ .............. \\ 0_{1 \times NP} & I_1 \times c \end{pmatrix} \tag{9}$$

where $\gamma_1$ to $\gamma_N$ denotes the prior mean for the coefficients on the first lag, $\tau$ is the tightness of the prior on the VAR coefficients, $c$ is the tightness of the prior on the constant terms and $N$ is the number of endogenous variables, i.e. the columns of $Y_t$. In our application, the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated

for each endogenous variable. We use principal component estimates of the factors $F_t^{PC}$ for this purpose. We set $\tau = 1$. The scaling factors $\sigma_i$ are set using the standard deviation of the error terms from these preliminary AR(1) regressions. Finally we set $c = 1/10000$ in our implementation indicating a flat prior on the constant. We also introduce a prior on the sum of the lagged dependent variables by adding the following dummy observations:

$$Y_{D,2} = \frac{diag\left(\gamma_1\mu_1...\gamma_N\mu_N\right)}{\lambda}, \; X_{D,2} = \left(\begin{array}{cc} \frac{(1_{1\times P})\otimes diag(\gamma_1\mu_1...\gamma_N\mu_N)}{\lambda} & 0_{N\times 1} \end{array}\right) \tag{10}$$

where $\mu_i$ denotes the sample means of the endogenous variables calculated using $F_t^{PC}$. The prior tightness is set as $\lambda = 10\tau$.

5. Instrument equation. The prior for $b$ is normal $N\left(b_0, V_0\right)$. The prior for $\sigma^2$ is inverse Gamma with mean $\sigma_0$ and standard deviation $v_0$. $b_0$ is set equal to the OLS estimate from the regression $m_t = b_{ols}\hat{u}_t + \hat{v}_t$ where $\hat{u}_t$ are the residuals obtained by estimating a VAR using $\left(\begin{array}{c} Z_t \\ F_t^{PC} \end{array}\right)$. $\sigma_0$ is set equal to the variance of $\hat{v}_t$. $V_0$ is set to an identity matrix while $v_0 = 1$.

## 1.2   Gibbs sampling algorithm

The symbol $\Theta$ denotes all other parameters and states. The Gibbs sampling algorithm samples from the following conditional posterior distributions:

1. $G\left(\tilde{B}\backslash\Theta\right)$. $\tilde{B}$ denoted the VAR coefficients in vectorised form: $\tilde{B} = vec(B)$. Over the period where $m_t$ is available, the VAR model in equation 2 can be written as

$$Y_t^* = BX_t + u_t^*$$

where $Y_t^* = Y_t - \mu_t$ and $var\left(u_t^*\right) = \tilde{\Omega}$. Note that when $m_t$ is unavailable, $\mu_t = 0$ and $var\left(u_t^*\right) = \Sigma$. This transformed model is a VAR with heteroscedasticity. The conditional posterior distribution is normal with mean $M^*$ and variance $V^*$ where

$$V^* = \left(\sum_{t=1}^{T}\left(R_t^{-1}\otimes X_tX_t'\right) + S_0^{-1}\right)^{-1}$$

$$M^* = V^*\left(vec\left(\sum_{t=1}^{T}\left(X_tY_t^{*\prime}R_t^{-1}\right)\right) + S_0^{-1}\tilde{B}_0'\right)$$

where $R_t = \tilde{\Omega}$ over periods $m_t$ is available and $R_t = \Sigma$, otherwise. The prior for the VAR coefficients based on dummy observations is $N\left(\tilde{B}_0, S_0\right)$.

2. $G\left(\Sigma\backslash\Theta\right)$. The conditional posterior is inverse Wishart:

$$IW\left(u_t'u_t + \Sigma_0, T + T_D\right)$$

where $\Sigma_0$ and $T_D$ represent the prior scale matrix and degrees of freedom based on the dummy observations specified above.

3. $p\left(b, \sigma^2|\Theta\right)$. Equation 3 is a standard linear regression, so specifying a conditional Normal-Gamma prior delivers a Normal-Gamma posterior. Particularly, we first draw $p\left(\sigma^2|\Theta\right)$. Assuming an inverse-Gamma prior, this conditional posterior is also inverse-Gamma. As the

3

prior is parameterised in terms of mean $\sigma_0$ and standard deviation $v_0$, it is convenient to draw the precision $\frac{1}{\sigma^2}$ using Gamma distribution. Note that $\frac{1}{\sigma^2} \sim \mathcal{G}(a, b)$ where $a = \frac{\nu_1}{2}$, $b = \frac{2}{s_1}$. The parameters of this Gamma density are given by $\nu_1 = \nu_0 + T$ and $s_1 = s_0 + \hat{v}_t'\hat{v}_t$ where $\hat{v}_t = m_t - bu_t$. $s_0$ can be calculated as $2\sigma_0 \left(1 + \frac{\sigma_0^2}{v_0^2}\right)$ while $\nu_0 = 2\left(2 + \frac{\sigma_0^2}{v_0^2}\right)$. Moreover, the posterior for $b$ is also conditional Normal $p(b|\Theta) \sim \mathcal{N}(\tilde{\beta}, \tilde{V}^{-1})$, where $\tilde{\beta} = \tilde{V}^{-1}\left[u_t'm_t + V_0^{-1}b_0\right]$ and $\tilde{V} = V_0 + \frac{1}{\sigma^2}u_t'u_t$.

4. $H(\Lambda|\Theta)$. Given the factors $F_t$ and a draw of the monthly observations $\hat{x}_t$, the observation equation is set of $M$ independent linear regressions with serial correlation

$$\tilde{X}_{it} = F_t\Lambda_i' + v_{it}$$

where $\Lambda_i$ denotes the ith row of the factor loading matrix. The serial correlation can be dealt with via a GLS transformation of the variables:

$$\bar{X}_{it} = \bar{F}_t\Lambda_i' + e_{it}$$

where $\bar{X}_{it} = \tilde{X}_{it} - \rho_i\tilde{X}_{it-1}$ and $\bar{F}_{kt} = F_{kt} - \rho_iF_{kt-1}$. The conditional posterior is normal $\mathcal{N}(M, V)$:

$$V = \left(V_\Lambda^{-1} + \frac{1}{r_i}\bar{F}_t'\bar{F}_t\right)^{-1}$$

$$M = V\left(V_\Lambda^{-1}\Lambda_{i,0} + \frac{1}{r_i}\bar{F}_t'\bar{X}_{it}\right)$$

To account for rotational indeterminacy the top $K \times K$ block of $\Lambda$ is set to an identity matrix.

5. $H(r_i|\Theta)$. The conditional posterior for $r_i$ is $IG(T_0 + T, e_{it}'e_{it} + r_{i0})$ where $T$ is the sample size.

6. $H(\rho|\Theta)$. Given a draw of the factors, the AR coefficients are drawn for each $i$ independently. The conditional posterior is normal $\mathcal{N}(m, v)$

$$v = \left(V_{\rho i}^{-1} + \frac{1}{r_i}x_{it}'x_{it}\right)^{-1}$$

$$m = V\left(V_{\rho i}^{-1}\rho_{i0} + \frac{1}{r_i}x_{it}'y_{it}\right)$$

where $y_{it} = v_{it}$ and $x_{it} = v_{it-1}$.

7. $H(F_t|\Theta)$. To draw the factors, we write the model in state-space form taking into account the covariance between $m_t$ and $u_t$ and the serial correlation in the idiosyncratic components.

The observation equation is defined as:

$$
\underbrace{\begin{pmatrix} Z_t \\ \bar{X}_t \end{pmatrix}}_{x_t} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & \Lambda & 0 & \tilde{\Lambda}_1 & \cdots & 0 & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} Z_t \\ F_t \\ \cdot \\ \cdot \\ \cdot \\ Z_{t-P} \\ F_{t-P} \end{pmatrix}}_{f_t} + \underbrace{\begin{pmatrix} 0 \\ e_t \end{pmatrix}}_{V_t}
$$

where $\bar{X}_t = \begin{pmatrix} \tilde{X}_{1t} - \rho_1 \tilde{X}_{1t-1} \\ \cdot \\ \cdot \\ \tilde{X}_{Mt} - \rho_M \tilde{X}_{Mt-1} \end{pmatrix}$ and recall that $\tilde{X}_t$ contains data at the monthly frequency

$\tilde{X}_t = \begin{pmatrix} \tilde{X}_t^M \\ \hat{x}_t \end{pmatrix}$. The blocks of the $H$ matrix contain the factor loadings multiplied by the

negative of the corresponding serial correlation coefficient. For example $\tilde{\Lambda}_1 = \begin{pmatrix} -\Lambda_1 \rho_1 \\ \cdot \\ \cdot \\ -\Lambda_M \rho_M \end{pmatrix}$

where $\Lambda_i$ denotes the factor loadings for the $ith$ variable $X_{it}$. Finally, the variance of $V_t$ is $R = diag\left([0, r_1, .., r_M]\right)$. The transition equation is defined as:

$$
f_t - \mu_t^* = \mu + \tilde{B} f_{t-1} + U_t^*
$$

where $\tilde{B} = \begin{pmatrix} B_1 & \cdot & \cdot & B_P \\ & I_{N(P-1) \times NP} & \end{pmatrix}, \mu = \begin{pmatrix} c \\ 0_{N(P-1)} \end{pmatrix}, U_t = \begin{pmatrix} u_t^* \\ 0_{N(P-1)} \end{pmatrix}, \mu_t^* = \begin{pmatrix} \mu_t \\ 0_{N(P-1)} \end{pmatrix}$.

The non-zero block of $cov\left(U_t\right)$ is given by $\tilde{\Omega}$ when the instrument is non-missing. When the instrument is missing $u_t^* = u_t$ and the covariance is $\Sigma$. In other words, the structure of the transition equation accounts for the relationship between the instrument and the reduced form residuals where relevant. Given this Gaussian linear state-space, the state vector can be drawn from the normal distribution using the Carter and Kohn (1994) algorithm.

8. $H\left(\hat{x}_t | \Theta\right)$. Conditional on the remaining parameters, an independent state-space model applies for each quarterly series with missing observations. The observation equation is:

$$
x_{jt}^Q = \begin{pmatrix} 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}_{jt} \\ \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ v_{jt} \end{pmatrix} \text{ if } x_{jt}^Q \neq nan
$$

$$
x_{jt}^Q = \tilde{u}_{jt} \text{ if } x_{jt}^Q = nan
$$

where $var\left(\tilde{u}_{jt}\right) = 1e10$. With the assumption of one lag in equation 4, the transition equation is:

$$
\begin{pmatrix} \hat{x}_{jt} \\ \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ v_{jt} \end{pmatrix} = \begin{pmatrix} F_t \Lambda_i' \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \rho_i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho_i \end{pmatrix} \begin{pmatrix} \hat{x}_{jt-1} \\ \hat{x}_{jt-2} \\ \hat{x}_{jt-3} \\ v_{jt-1} \end{pmatrix} + \begin{pmatrix} e_{jt} \\ 0 \\ 0 \\ e_{jt} \end{pmatrix}
$$

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where $var \begin{pmatrix} e_{jt} \\ 0 \\ 0 \\ e_{jt} \end{pmatrix} = \begin{pmatrix} r_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_j \end{pmatrix}.$

To test the algorithm and computer code we carry out a simple Monte Carlo experiment. Artificial data is generated from the following model:

$$\begin{pmatrix} Z_t \\ \tilde{X}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{11}$$

$$Y_t = BX_t + u_t \tag{12}$$

$$m_t = bu_t + \hat{v}_t \tag{13}$$

$$v_{it} = \rho_i v_{it-1} + e_{it} \tag{14}$$

where $\tilde{X}_t$ contains 50 series where 10 are subject to temporal aggregation. We assume two factors with $\Lambda\tilde{\ }N(0,1)$. To calibrate $B$ and $var(u_t)$ we use OLS estimates from a VAR model that includes GDP growth, inflation and corporate bond spread for the US. We assume that the VAR has three lags. $b = \begin{pmatrix} 1.1 \\ -1.1 \\ 2 \end{pmatrix}$ and $var(\hat{v}_t) = 0.1$. Finally, $\rho_i\tilde{\ }U(0,1)$ while $var(e_{it})$ is set as the exponential of a draw from the standard normal distribution. We draw 340 observations, discarding the first 100. The experiment is repeated 100 times.

Figure 1.2 compares the true impulse response to the shock to $Z_t$ with that obtained over the 100 Monte Carlo replications. The estimates suggest that the algorithm performs well and is able to recover the true impulse responses.

## 1.3 Convergence

Figure 1 shows that the inefficiency factors are fairly low. This provides evidence in favour of convergence.
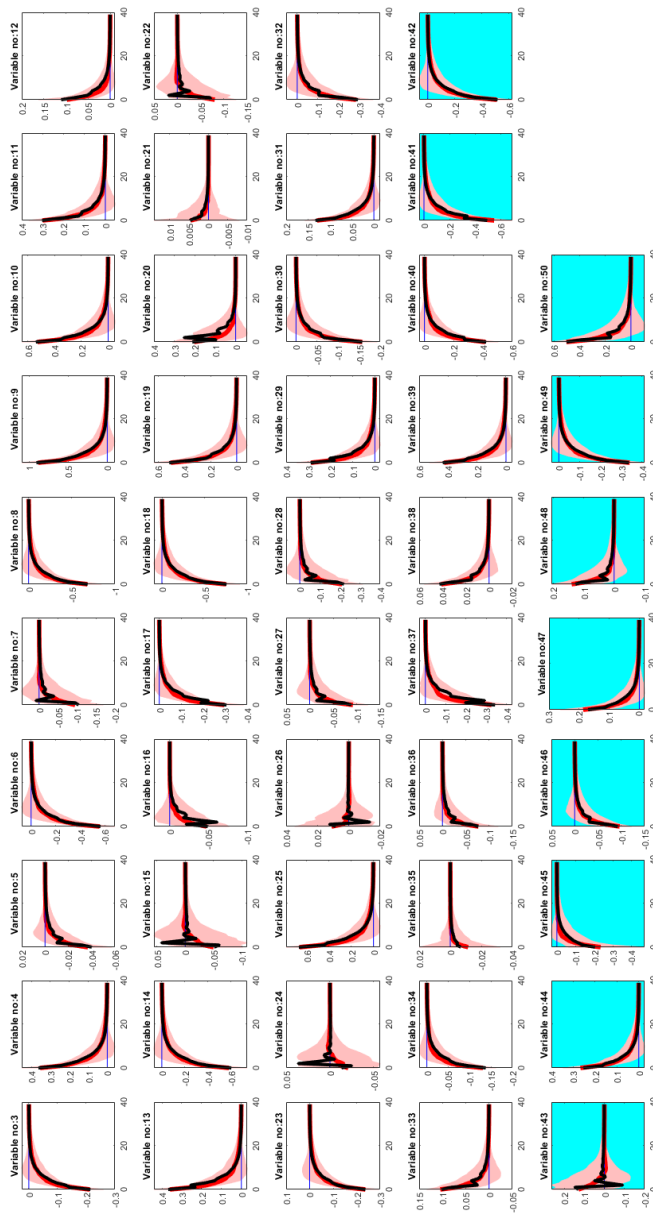
Figure 1.2. The black line is the true response. The red line and shaded area represent the median and 68% interval estimated using 100 Monte-Carlo replications. The plots highlighted with blue show series subject to temporal aggregation.

## 1.4 Data

The list of monthly series used in the FAVAR model are shown in Table 1-3. Quarterly series follow in Table 4- FRED-MD refers to Federal Reserve monthly database for macroeconomic research (https://research.stlouisfed.org/wp/more/2015-012), BLS-BED refers to Business Employment Dynamics database provided by the U.S, Bureau of Labor Statistics (https://www.bls.gov/bdm/).

| | Variable | Source | | Variable | Source |
|---|---|---|---|---|---|
| 1) | Real Personal Income | FRED-MD | 30) | All Employees: Goods-Producing Industries | FRED-MD |
| 2) | Real personal income ex transfer receipts | FRED-MD | 31) | All Employees: Mining and Logging | FRED-MD |
| 3) | IP Index | FRED-MD | 32) | All Employees: Construction | FRED-MD |
| 4) | IP: Final Products and Nonindustrial Supplies | FRED-MD | 33) | All Employees: Manufacturing | FRED-MD |
| 5) | IP: Final Products (Market Group) | FRED-MD | 34) | All Employees: Durable goods | FRED-MD |
| 6) | IP: Consumer Goods | FRED-MD | 35) | All Employees: Nondurable goods | FRED-MD |
| 7) | IP: Durable Consumer Goods | FRED-MD | 36) | All Employees: Service-Providing Industries | FRED-MD |
| 8) | IP: Nondurable Consumer Goods | FRED-MD | 37) | All Employees: Trade, Transportation & Utilities | FRED-MD |
| 9) | IP: Business Equipment | FRED-MD | 38) | All Employees: Wholesale Trade | FRED-MD |
| 10) | IP: Materials | FRED-MD | 39) | All Employees: Retail Trade | FRED-MD |
| 11) | IP: Durable Materials | FRED-MD | 40) | All Employees: Financial Activities | FRED-MD |
| 12) | IP: Nondurable Materials | FRED-MD | 41) | All Employees: Government | FRED-MD |
| 13) | IP: Manufacturing (SIC) | FRED-MD | 42) | Avg Weekly Hours : Goods-Producing | FRED-MD |
| 14) | IP: Residential Utilities | FRED-MD | 43) | Avg Weekly Overtime Hours : Manufacturing | FRED-MD |
| 15) | IP: Fuels | FRED-MD | 44) | Avg Weekly Hours : Manufacturing | FRED-MD |
| 16) | Capacity Utilization: Manufacturing | FRED-MD | 45) | Avg Hourly Earnings : Goods-Producing | FRED-MD |
| 17) | HWI Help-Wanted Index for United States | FRED-MD | 46) | Avg Hourly Earnings : Construction | FRED-MD |
| 18) | Ratio of Help Wanted/No. Unemployed | FRED-MD | 47) | Avg Hourly Earnings : Manufacturing | FRED-MD |
| 19) | Civilian Labor Force | FRED-MD | 48) | Housing Starts: Total New Privately Owned | FRED-MD |
| 20) | Civilian Employment | FRED-MD | 49) | Housing Starts, Northeast | FRED-MD |
| 21) | Civilian Unemployment Rate | FRED-MD | 50) | Housing Starts, Midwest | FRED-MD |
| 22) | Average Duration of Unemployment (Weeks) | FRED-MD | 51) | Housing Starts, South | FRED-MD |
| 23) | Civilians Unemployed - Less Than 5 Weeks | FRED-MD | 52) | Housing Starts, West | FRED-MD |
| 24) | Civilians Unemployed for 5-14 Weeks | FRED-MD | 53) | New Private Housing Permits (SAAR) | FRED-MD |
| 25) | Civilians Unemployed - 15 Weeks & Over | FRED-MD | 54) | New Private Housing Permits, Northeast (SAAR) | FRED-MD |
| 26) | Civilians Unemployed for 15-26 Weeks | FRED-MD | 55) | New Private Housing Permits, Midwest (SAAR) | FRED-MD |
| 27) | Civilians Unemployed for 27 Weeks and Over | FRED-MD | 56) | New Private Housing Permits, West (SAAR) | FRED-MD |
| 28) | Initial Claims | FRED-MD | 57) | New Private Housing Permits, South (SAAR) | FRED-MD |
| 29) | All Employees: Total nonfarm | FRED-MD | 58) | Real personal consumption expenditures | FRED-MD |

Table 1: List of monthly series in the FAVAR model

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| # | Variable | Source | # | Variable | Source |
|---|---|---|---|---|---|
| 59) | Retail and Food Services Sales | FRED-MD | 88) | 10-Year Treasury Rate | FRED-MD |
| 60) | Real Manu. and Trade Industries Sales | FRED-MD | 89) | Moody's Seasoned Aaa Corporate Bond Yield | FRED-MD |
| 61) | New Orders for Consumer Goods | FRED-MD | 90) | Moody's Seasoned Baa Corporate Bond Yield | FRED-MD |
| 62) | New Orders for Durable Goods | FRED-MD | 91) | 30-Year Fixed Rate Mortgage Avg in the U. S. | FRED-MD |
| 63) | New Orders for Nondefense Capital Goods | FRED-MD | 92) | 3-Month Commercial Paper Minus FEDFUNDS | FRED-MD |
| 64) | Unfilled Orders for Durable Goods | FRED-MD | 93) | 3-Month Treasury C Minus FEDFUNDS | FRED-MD |
| 65) | Total Business Inventories | FRED-MD | 94) | 6-Month Treasury C Minus FEDFUNDS | FRED-MD |
| 66) | Total Business: Inventories to Sales Ratio | FRED-MD | 95) | 1-Year Treasury C Minus FEDFUNDS | FRED-MD |
| 67) | Consumer Sentiment Index | FRED-MD | 96) | 5-Year Treasury C Minus FEDFUNDS | FRED-MD |
| 68) | M1 Money Stock | FRED-MD | 97) | 10-Year Treasury C Minus FEDFUNDS | FRED-MD |
| 69) | M2 Money Stock | FRED-MD | 98) | Moody's Aaa Corp. Bond Minus FEDFUNDS | FRED-MD |
| 70) | Real M2 Money Stock | FRED-MD | 99) | Moody's Baa Corp. Bond Minus FEDFUNDS | FRED-MD |
| 71) | St. Louis Adjusted Monetary Base | FRED-MD | 100) | Trade Weighted U.S. $ Inx: Major Currencies | FRED-MD |
| 72) | Total Reserves of Depository Institutions | FRED-MD | 101) | Switzerland / U.S. Foreign Exchange Rate | FRED-MD |
| 73) | Reserves Of Depository Institutions | FRED-MD | 102) | PPI: Finished Goods | FRED-MD |
| 74) | Commercial and Industrial Loans | FRED-MD | 103) | PPI: Finished Consumer Goods | FRED-MD |
| 75) | Real Estate Loans at All Commercial Banks | FRED-MD | 104) | PPI: Intermediate Materials | FRED-MD |
| 76) | Total Nonrevolving Credit | FRED-MD | 105) | PPI: Crude Materials | FRED-MD |
| 77) | Nonrevolving cons. credit to Personal Income | FRED-MD | 106) | Crude Oil, spliced WTI and Cushing | FRED-MD |
| 78) | MZM Money Stock | FRED-MD | 107) | PPI: Metals and metal products | FRED-MD |
| 79) | Consumer Motor Vehicle Loans Outst. | FRED-MD | 108) | CPI : All Items | FRED-MD |
| 80) | Total Consumer Loans and Leases Outst. | FRED-MD | 109) | CPI : Apparel | FRED-MD |
| 81) | Securities in Bank Credit at All Comm. Banks | FRED-MD | 110) | CPI : Transportation | FRED-MD |
| 82) | Effective Federal Funds Rate | FRED-MD | 111) | CPI : Medical Care | FRED-MD |
| 83) | CP3Mx 3-Month AA Fin. Comm. Paper Rate | FRED-MD | 112) | CPI : Commodities | FRED-MD |
| 84) | TB3MS 3-Month Treasury Bill | FRED-MD | 113) | CPI : Durables | FRED-MD |
| 85) | 6-Month Treasury Bill | FRED-MD | 114) | CPI : Services | FRED-MD |
| 86) | 1-Year Treasury Rate | FRED-MD | 115) | CPI : All Items Less Food | FRED-MD |
| 87) | 5-Year Treasury Rate | FRED-MD | 116) | CPI : All items less shelter | FRED-MD |

Table 2: List of monthly series in the FAVAR model

| | Variable | Source | | Variable | Source |
|---|---|---|---|---|---|
| 117) | CPI : All items less medical care | FRED-MD | 125) | S&P's Comp. Common Stock: Price-Earn. Ratio | FRED-MD |
| 118) | Personal Cons. Expend.: Chain Index | FRED-MD | 126) | VXO | FRED-MD |
| 119) | Personal Cons. Exp: Durable goods | FRED-MD | 127) | CP3MxMTB3MS | FRED-MD |
| 120) | Personal Cons. Exp: Nond. goods | FRED-MD | 128) | Moody's Seasoned Baa Corp. Bond Yield Relative to Yield on 10-Year Treasury | FRED-MD |
| 121) | Personal Cons. Exp: Services | FRED-MD | 129) | MRTM10 | FRED-MD |
| 122) | S&P's Common Stock Price Inx: Comp. | FRED-MD | 130) | Monetary Policy Uncertainty | Husted *et al.* (2019) |
| 123) | S&P's Common Stock Price Inx: Indust. | FRED-MD | 131) | Economic Policy Uncertainty | Baker *et al.* (2016) |
| 124) | S&P's Comp. Common Stock: Div. Yield | FRED-MD | 132) | Excess Bond Premium | Gilchrist and Zakrajšek (2012) |

Table 3: List of monthly series in the FAVAR model

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| | Variable | | Source | | Variable | | Source |
|---|---|---|---|---|---|---|---|
| 133) | Utilization-adjusted TFP | | ? | 149) | Total private | Establishm. deaths, s.a. | BLS-BDS |
| 134) | Total private | Establishm. births, s.a | BLS-BED | 150) | Goods-producing | Establishm. deaths, s.a. | BLS-BDS |
| 135) | Goods-producing | Establishm. births, s.a | BLS-BED | 151) | Natural resources | Establishm. deaths, s.a. | BLS-BDS |
| 136) | Natural resources | Establishm. births, s.a | BLS-BED | 152) | Construction | Establishm. deaths, s.a. | BLS-BDS |
| 137) | Construction | Establishm. births, s.a | BLS-BED | 153) | Manufacturing | Establishm. deaths, s.a. | BLS-BDS |
| 138) | Manufacturing | Establishm. births, s.a | BLS-BED | 154) | Service-providing | Establishm. deaths, s.a. | BLS-BDS |
| 139) | Service-providing | Establishm. births, s.a | BLS-BED | 155) | Wholesale trade | Establishm. deaths, s.a. | BLS-BDS |
| 140) | Wholesale trade | Establishm. births, s.a | BLS-BED | 156) | Retail trade | Establishm. deaths, s.a. | BLS-BDS |
| 141) | Retail trade | Establishm. births, s.a | BLS-BED | 157) | Transportation | Establishm. deaths, s.a. | BLS-BDS |
| 142) | Transportation | Establishm. births, s.a | BLS-BED | 158) | Information | Establishm. deaths, s.a. | BLS-BDS |
| 143) | Information | Establishm. births, s.a | BLS-BED | 159) | Financial activities | Establishm. deaths, s.a. | BLS-BDS |
| 144) | Financial activities | Establishm. births, s.a | BLS-BED | 160) | Professional services | Establishm. deaths, s.a. | BLS-BDS |
| 145) | Professional services | Establishm. births, s.a | BLS-BED | 161) | Education services | Establishm. deaths, s.a. | BLS-BDS |
| 146) | Education services | Establishm. births, s.a | BLS-BED | 162) | Leisure and hospitality | Establishm. deaths, s.a. | BLS-BDS |
| 147) | Leisure and hospitality | Establishm. births, s.a | BLS-BED | 163) | Other services | Establishm. deaths, s.a. | BLS-BDS |
| 148) | Other services | Establishm. births, s.a | BLS-BED | | | | |

Table 4: List of quarterly series in the FAVAR model

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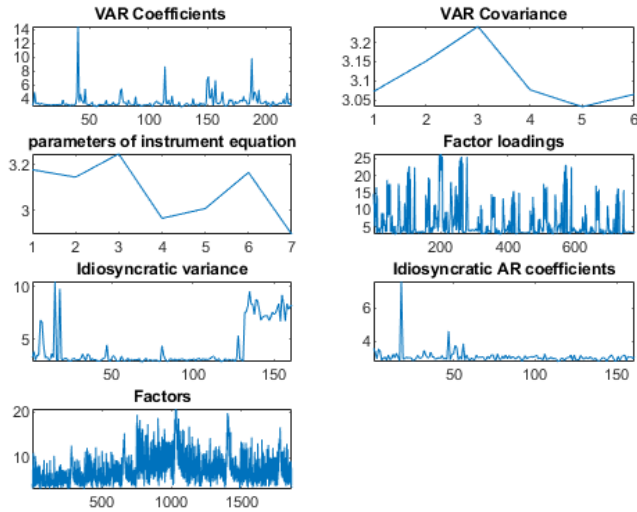
Figure 1: Figure 1. Inefficiency factors.

# 2 Robustness

We check the robustness of the evidence in the benchmark FAVAR model along two lines. First, we run a FAVAR model as the benchmark, but considering several specifications, which differ over the number of factors, the measure of monetary uncertainty, the instrument to identify the shock, and the identification strategy itself. Second, we run a smaller model, i.e. a Proxy-VAR, that, however, take a mixed-frequency dataset and an instrumental approach for the shock identification as for the benchmark FAVAR. We provide details of both kind of robustness in the following.

## 2.1 FAVAR model

In this section, we provide the evidence in response to the monetary policy uncertainty shock under different specifications of the FAVAR model. First, we run the FAVAR assuming different number of factors -they are five in the benchmark. Figures 2-4 illustrate the responses of FAVAR models, which are identical to the benchmark described in Section 2. of the main text, but with four, six, seven factors, respectively. In all three cases, the shock clearly implies a contraction in output, inflation and short interest rate. Entering firms at the aggregate level decrease as well, while the capacity-adjusted TFP rises at the medium horizon. Exiting firms seem to be more sensitive to the persistence of the shock. When the shock hitting the monetary uncertainty is persistent -about 1-year long when the factors are set to four-, the response of aggregate firms' exit is positive and long-lasting. Instead, when the estimated shock is short-living -about a half year as for FAVAR with six and seven factors-, its response is more ambiguous. Notably, this is not the case for aggregate firms' entry, which declines independently of the persistence of the shock.

Second, we run FAVAR models with five factors as the benchmark, but taking either a different measure of monetary policy uncertainty or a different instrument to identify the shock to the monetary policy uncertainty. Figure 5 shows the FAVAR responses, when we consider Baker *et al.* (2016)'s monetary policy uncertainty index instead of Husted *et al.* (2019)'s index. The responses are basically identical to the benchmark FAVAR. This is not surprising since the two measures of
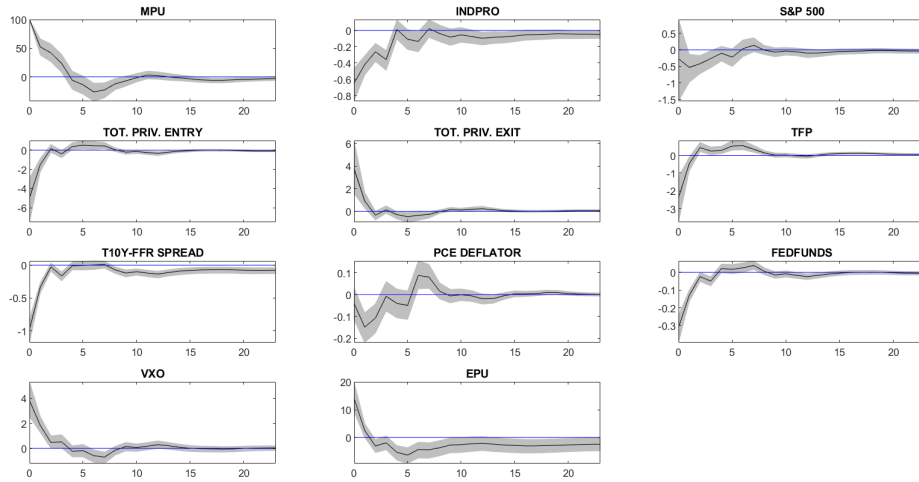
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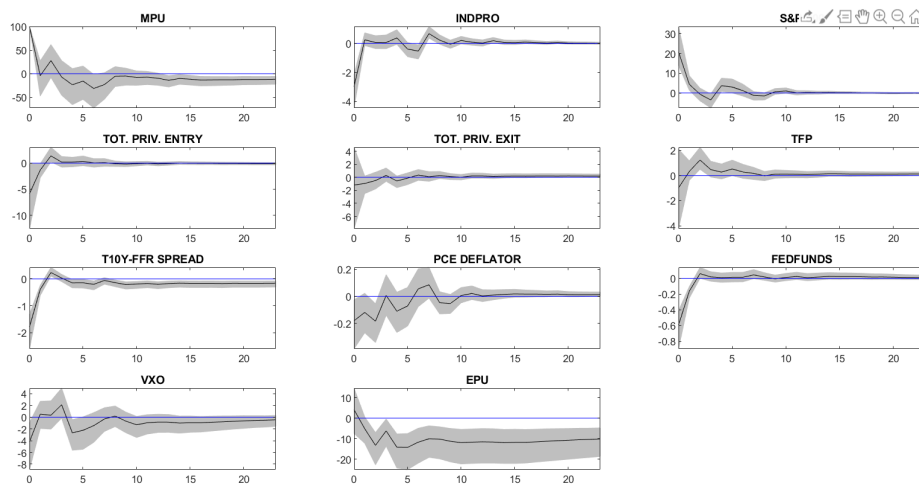Figure 2: Impulse responses of the FAVAR model with 4 factors.



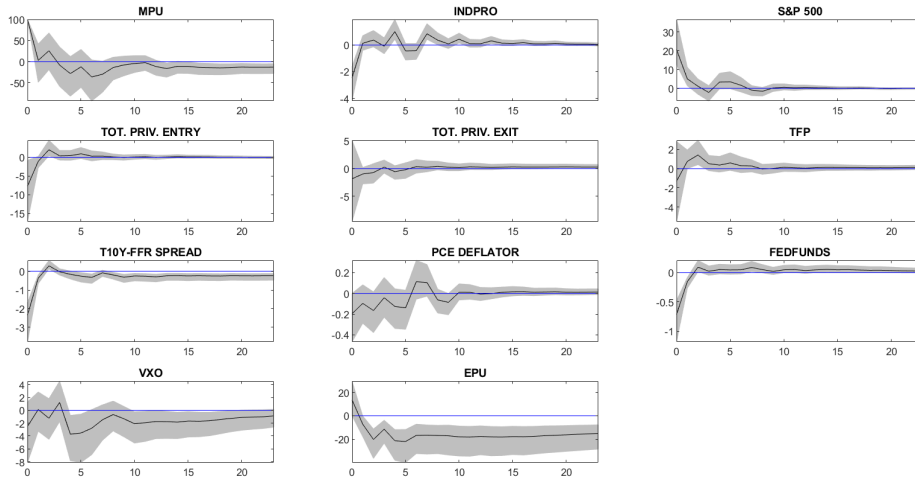Figure 3: Impulse responses of the FAVAR with 6 factors.

Figure 4: Impulse responses of the FAVAR with 7 factors.

monetary policy uncertainty are positively correlated along the sample period 1985:m1-2016:m6. The correlation is slightly lower than 0.6.[1] Figure 6 shows the FAVAR responses, when we use Carlston and Ochoa (2016)'s conditional volatility of the 1-year swap rate at a the 1-month horizon as the instrument, without orthogonalizing it to monetary policy surprises. This accomodates those concerns highligthed in the monetary policy literature (Ramey (2016)) about the feasibility of identifying true shocks (surprises) in the monetary policy, since the latter has been conducted more systematically in the last decades. Taking the volatility as the instrument does not alter however our results. The responses in Figure 6 are very close to the benchmark FAVAR for all variables.

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[1] Baker and co-authors, through policyuncertainty.com, provide two updated indices of monetary policy uncertainty for the U.S. The indices use the same criteria to identify articles about monetary policy uncertainty, but differ in the set of newspapers covered. The index we consider is built up with articles from a balanced panel of 10 major national and regional U.S. newspapers. The other index draws articles from hundreds of U.S. newspapers covered by Access World News.
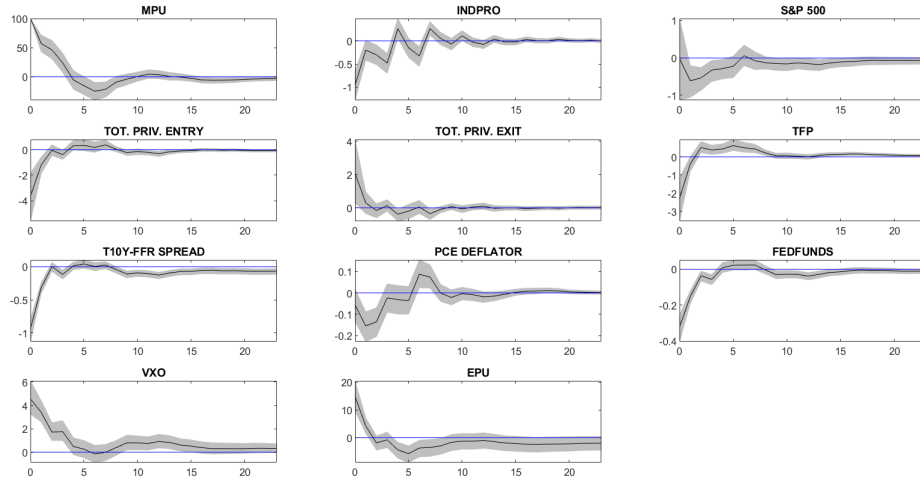
Figure 5: Impulse responses of the FAVAR model with Baker *et al.* (2016)'s index as a measure of monetary uncertainty.
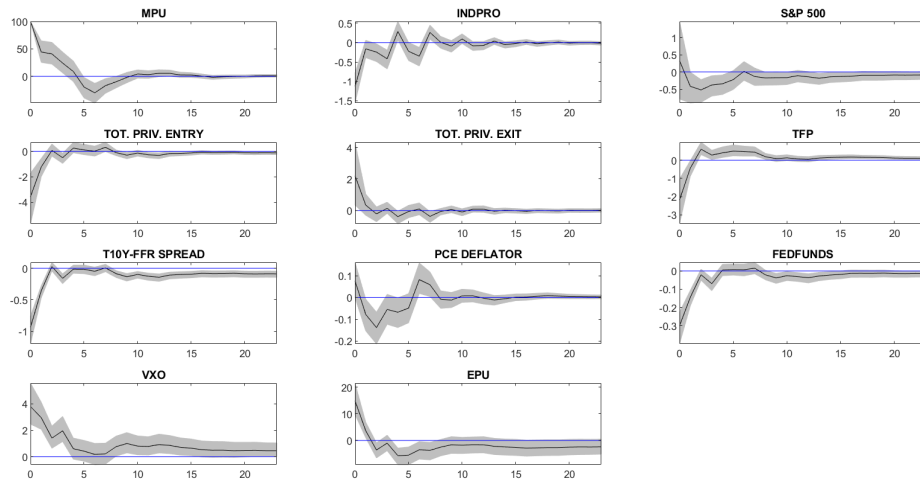


Figure 6: Impulse responses of the FAVAR model with Carlston and Ochoa (2016)'s conditional volatility as an instrument to identify the monetary policy uncertainy shock.
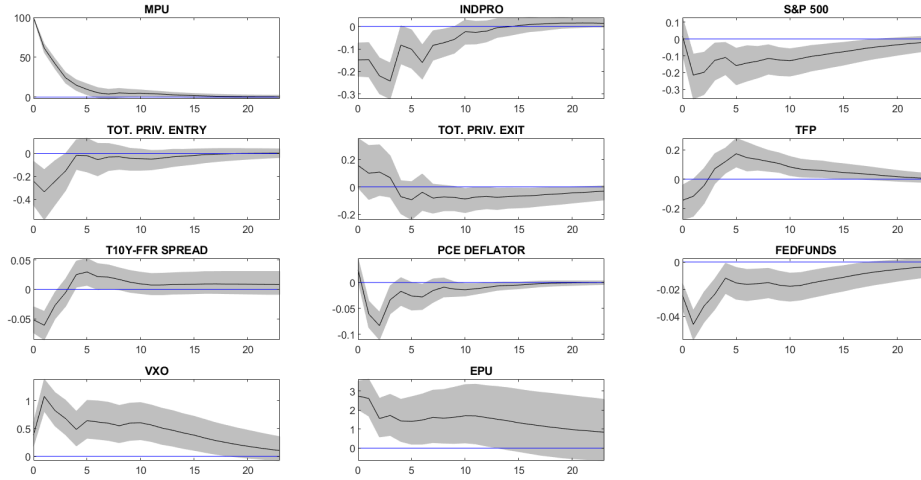
Figure 7: Impulse responses of the FAVAR model in which the monetary policy uncertainty shock is identified through a recursive scheme.

As the last check for the FAVAR model, we examine if the evidence is robust to a different strategy for identifying the monetary policy uncertainty shock. Thus, we estimate a FAVAR as the benchmark, but using a recursive scheme for identifying the shock. We do not impose any zero restrictions to the impact response of the shock. Equivalently, we assume that all macroeconomic and financial series in the dataset are fast-moving in the sense that have a contemporaneous relationship with the monetary policy uncertainty. The plots in Figure 7 confirm that the dynamics triggered by the monetary policy uncertainty shock is consistent with the benchmark. The fall in real and nominal variables are significative and long–lasting. At 1-year horizon, the industrial production is about 1.5% lower, whereas prices are reduced by around 0.3% . The nominal interest rate reduces to alleviate the effects of the recession, as expected. Still for the firm flows, the aggregate productivity, and the stock prices index the evidence is very close to the benchmark FAVAR. Some differences are only in the responses of the term spread and of the policy uncertainty index. The former declines on impact as in the benchmark, but the response reverts back only after few periods. Instead, the rise in the EPU index is persistent and not short-lived as in the benchmark. Figures 8-9 show the responses of establishments' entry and exit, respectively, for industry-level data. Still for disaggregated series, the dynamics is consistent with the benchmark FAVAR. Although there is some uncertainty in the estimates at very few periods after the shocks, for most of the industries the response of entry is negative and that of exit is positive as for the aggregate data.

## 2.2 Proxy-VAR model

We test the evidence from the FAVAR also considering a smaller model, namely a Proxy-VAR, that incorporates less information than in the FAVAR, but reconciles our exercise with a large strand of the literature on uncertainty shocks. We estimate the VAR via Bayesian technique employing the same external instrument to identify the monetary policy uncertainty shock of the benchmark FAVAR. Our VAR model includes mixed-frequency series as endogenous variables. Taking the sample period spanning from 1985:m1 to 2016:m6, we consider the log of the monetary policy uncertainty (source: Husted *et al.* (2019)), the log of the industrial production index (source: FRED-
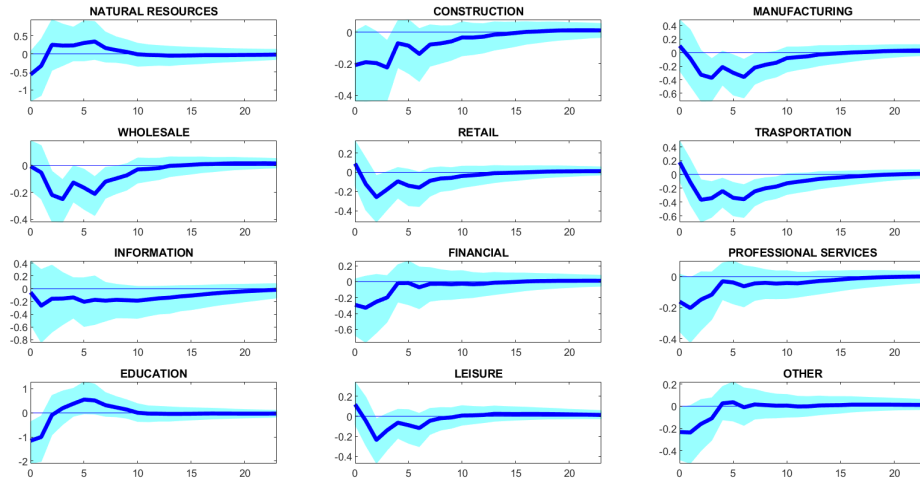
Figure 8: Impulse response of establishments' entry at the industry level of the FAVAR model in which the monetary policy uncertainty shock is identified through a recursive scheme. The solid line is the median. The shaded area is the 68% error band.
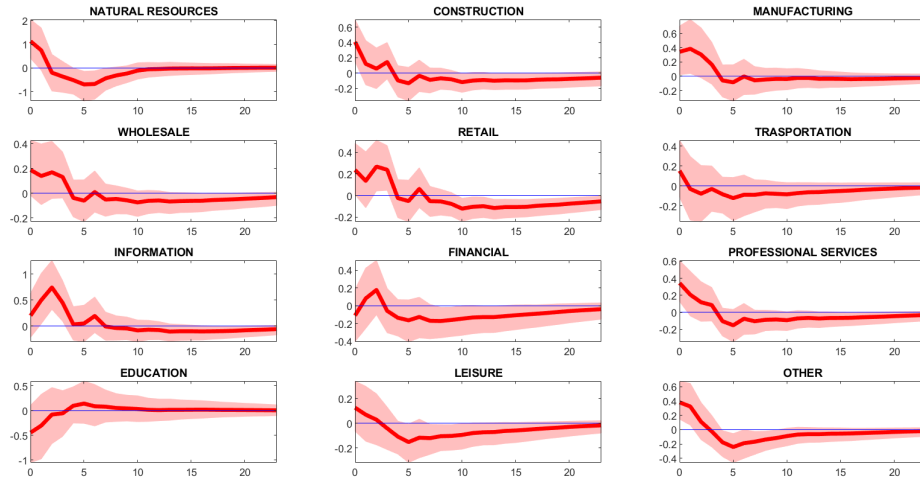


Figure 9: Impulse response of establishments' exit at the industry level of the FAVAR model in which the monetary policy uncertainty shock is identified through a recursive scheme. The solid line is the median. The shaded area is the 68% error band.
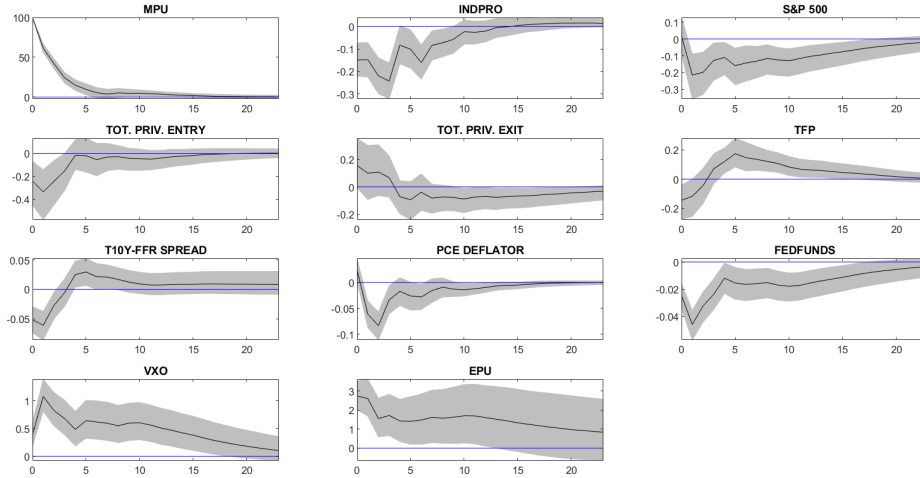
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Figure 10: Impulse responses of the Proxy-VAR model with linear trend.

MD), the log of the CPI - All Items index (source: FRED-MD), the economic policy uncertainty index (source: Baker *et al.* (2016)), the excess bond premium (source: Gilchrist and Zakrajšek (2012)), the 1-year Treasury bond rate (source: FRED-MD), the log of total private establishments' birth (source: BLS-BED), the log of total private establishments' death (source: BLS-BED). We use a natural conjugate prior for the VAR parameters implemented via dummy observations (see Banbura *et al.* (2010)). We set the lag order to six and test versions of VAR with a linear trend and a quadratic trend. To shock identification is achieved following the same approach of Bahaj (2019). Figures 10-11 illustrate the dynamic responses of the Proxy-VAR with linear and quadratic trend, respectively. The differences between the specifications are minor. In response to a monetary policy uncertainty shocks normalized to increase the MPU index of Husted *et al.* (2019) by 100 percent points, industrial production drops by about 2% at impact, in line with the evidence in the FAVAR. The CPI index does not react immediately to the shock, but then it falls substantially. The median response is around -1.5% after two years. As in Gertler and Karadi (2015), we consider the one-year government bond rate as the indicator of monetary policy. To respond to the contraction in both real and nominal variables, the monetary policy becomes accommodative and the short-interest rate plunges more than 2% at two quarter horizon. During the recession, both the measure of economic policy uncertainty and of credit spread tilt, although their effects fade away rapidly, significantly before than for the other variables. Lastly, the VAR-implied responses of firm entry and exit at the aggregate level are consistent with the FAVAR. Although, in a small-scale model the response of entering firms is less precisely estimated, at the impact the drop of the median is about 5% as in the FAVAR. Exiting firms instead increase after the shock, and the confidence interval are much closer than in the large-scale model. However, the response in the VAR is significantly higher than in the FAVAR. Thus, according to the VAR, firm flows are such that firm exit is more cyclical than firm entry. This contrasts the claim that for a firm is more likely to decide to not enter the market rather than exit the market. Adding more data information to the model, the outcome changes and the volatility of the firm flows is reshuffled. In the FAVAR, firm entry is shown to be more responsive, as it reduces at impact by 5%, instead of around 2.5% of firm exit. This is moreover in line with the previuos literature (Lee and Mukoyama (2008)) on the cyclicality of entry and exit.
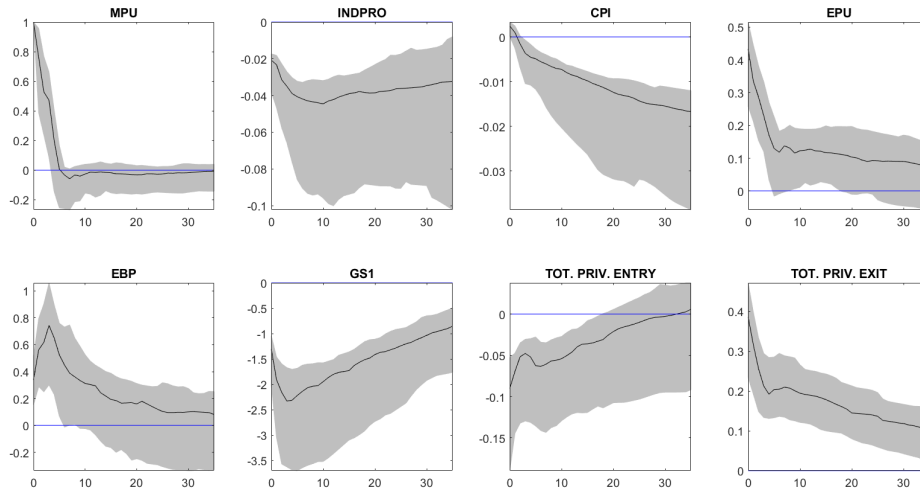
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Figure 11: Impulse responses of the Proxy-VAR model with quadratic trend.

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# 3 DSGE model: list of equations

The system of non-linear equation is summarized in Table 5-7.

| | Description | Equations |
|---|---|---|
| 1) | Marginal utility of consumption | $\lambda_t = (C_t - hC_{t-1})^{-\sigma} \exp\left(\chi \frac{(\sigma_C - 1)(L_t)^{1+\sigma_L}}{1+\sigma_L}\right),$ |
| 2) | Marginal rate of substitution | $mrs_t = \chi \left(C_t - hC_{t-1}\right)(L_t)^{\sigma_L},$ |
| 3) | Law of motion of capital | $K_{t+1} = \left(1 - \delta^K - \frac{\phi_K}{2}\left(\frac{I_t}{K_t} - 1\right)^2\right)K_t + I_t,$ |
| 4) | Euler equation | $\lambda_t = \beta E_t \left[\lambda_{t+1}(1 + r_t)\right],$ |
| 5) | Euler equation for incumbent firm | $v_t\left(\widetilde{z}_t\right) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\left(1-\eta_{t+1}\right)\left[v_{t+1}\left(\widetilde{z}_t\right) + j_{t+1}\left(\widetilde{z}_t\right)\right] + \eta_{t+1}[v_{t+1}]\right],$ |
| 6) | Euler equation for entrant firm | $v_t\left(\widetilde{z}_t\right) = f^e + ec_t,$ |
| 7) | Euler equation for capital | $\Psi_t = \beta E_t\left[\lambda_{t+1}\left(r_{t+1}^K u_{t+1} - a(u_{t+1})\right) + \Psi_{t+1}\left(1 - \delta_K\right) - \Psi_{t+1}\left(\frac{\phi_K}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta^K\right)^2\right) - \phi_K\left(\frac{I_{t+1}}{K_{t+1}} - \delta^K\right)\frac{I_{t+1}}{K_{t+1}}\right],$ |
| 8) | Euler equation for investments | $1 = q_t\left(1 - \phi_K\left(\frac{I_t}{K_t} - \delta^K\right)\right),$ |
| 9) | Tobin q | $q_t = \frac{\Psi_t}{\lambda_t},$ |
| 10) | Foc variable capital utilization | $r_t^k = \gamma_1 + \gamma_2\left(u_t - 1\right),$ |
| 11) | Variable capital utilization adj. costs | $a(u_t) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2,$ |
| 12) | Law of motion of firms | $N_{t+1} = \left(1 - \eta_t\right)\left(N_t + N_{t+1}^e\right),$ |
| 13) | Wage NKPC | $1 = \frac{\theta_w}{\theta_w - 1}\frac{mrs_t}{w_t} - \frac{\phi_w}{\theta_w - 1}\left(\pi_t^w - 1\right)\pi_t\frac{Y_t}{w_{t-1}L_t} + \frac{\phi_w}{\theta_w - 1}E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\pi_{t+1}^w - 1\right)\pi_{t+1}^w\frac{Y_{t+1}}{w_tL_t}\right],$ |
| 14) | Wage inflation | $\pi_t^w = \frac{w_t}{w_{t-1}}\pi_t,$ |
| 15) | Wage adj. costs | $WAC_t = \frac{\phi_w}{2}\left(\pi_t^w - 1\right)^2 Y_t,$ |

Table 5: System of non-linear equations

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| | Description | Equations |
|---|---|---|
| 16) | Price NKPC | $1 = \frac{\theta_p}{\theta_p-1}\rho_t^I - \frac{\phi_p}{\theta_p-1}(\pi_t-1)\pi_t + \frac{\phi_p}{2}(\pi_t-1)^2 + \frac{\phi_p}{\theta_p-1}E_t\left[\Lambda_{t,t+1}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]$, |
| 17) | Love of variety equation | $\rho_t(\widetilde{z}_t) = N_t^{\frac{1}{\theta_p-1}}\rho_t^I$, |
| 18) | Stochastic discount factor | $\Lambda_{t,t+1} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\eta_{t+1})\right\}$, |
| 19) | Entry congestion externalities | $ec_t = \Theta^e\left(\frac{N_{t+1}^E}{N_t}\right)^{\varsigma_e}$, |
| 20) | Exit congestion externalities | $xc_t = \Theta^x\left(\frac{N_t^X}{N_t}\right)^{\varsigma_x}$, |
| 21) | Exiting firms | $N_t^X = \eta_t\left(N_t + N_{t+1}^E\right)$, |
| 22) | Liquidation value | $lv_t = (1-\tau)f^E - xc_t$, |
| 23) | Exit probability | $\eta_t = 1 - \left(\frac{z_{\min}}{\widetilde{z}_t}\right)^{\xi}$, |
| 24) | Average productivity | $\widetilde{z}_t = \left(\frac{\xi}{\xi+1-\theta_p}\right)^{\frac{1}{\theta_p-1}}\bar{z}_t$, |
| 25) | Value of the marginal firm | $v_t(\bar{z}_t) = j_t(\bar{z}_t) + \beta E_t\left[\Lambda_{t,t+1}v_{t+1}(\bar{z}_{t+1})\right]$, |
| 26) | Exit condition | $v_t(\bar{z}_t) = lv_t$, |
| 27) | Profits of the average firm | $j_t(\bar{z}_t) = \left(\frac{\theta_p}{\theta_p-1}-1\right)mc_t(\widetilde{z}_t)\left(\frac{\widetilde{z}_t}{\bar{z}_t}\right)^{1-\theta_p}y_t(\widetilde{z}_t)$, |
| 28) | Optimal price of the average firm | $\rho_t(\widetilde{z}_t) = \mu(\widetilde{z}_t)mc_t(\widetilde{z}_t)$, |
| 29) | Mark-up of the average firm | $\mu(\widetilde{z}_t) = \frac{\theta_p}{\theta_p-1}$, |
| 30) | Profits of the average firm | $j_t(\widetilde{z}_t) = N_t^{-1}\left[Y_t - w_tL_t - r_t^kK_t^s - \frac{\phi_l}{2}(L_t - L_{t-1})^2\right]$, |

Table 6: System of non-linear equations

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Table 7: System of non-linear equations

| | Description | Equations |
|---|---|---|
| 31) | Labor demand | $w_t = \tilde{z}_t mc_t(\tilde{\tilde{z}}_t)(1-\alpha)\left(\frac{L_t}{K_t^s}\right)^{-\alpha} - \phi_l(L_t - L_{t-1}) + E_t\left[\Lambda_{t,t+1}\phi_l(L_{t+1} - L_t)\right],$ |
| 32) | Capital service demand | $r_t^K = \tilde{z}_t mc_t(\tilde{\tilde{z}}_t)\alpha\left(\frac{L_t}{K_t^s}\right)^{1-\alpha},$ |
| 33) | Output of the average firm | $y_t(\tilde{\tilde{z}}_t) = N_t^{-1}\tilde{\tilde{z}}_t(L_t)^{1-\alpha}(K_t^s)^\alpha,$ |
| 34) | Capital-capital service relation | $K_t^s = u_t K_t,$ |
| 35) | Aggregate output | $Y_t = N_t^{\frac{1}{\theta_p - 1}}\tilde{\tilde{z}}_t(L_t)^{1-\alpha}(K_t^s)^\alpha,$ |
| 36) | Aggregate resource constraint | $Y_t = C_t + I_t + a(u_t)K_t + N_t^E ec_t + N_t^X xc_t + PAC_t + WAC_t,$ |
| 37) | Price adj. costs | $PAC_t = \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t,$ |
| 38) | Aggregate resource constraint | $Y_t = C_t + I_t + a(u_t)K_t + N_t^E ec_t + N_t^X xc_t + PAC_t + WAC_t,$ |
| 39) | Government budget constraint | $T_t = f^E N_t^E - (1-\tau)f^E N_t^X,$ |
| 40) | Taylor rule | $\left(\frac{1+i_t}{1+i}\right) = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_R}\left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y}\right)^{\phi_y}\left(\frac{y_t}{y_{t-1}}\right)^{\phi_{dy}}\right)^{(1-\phi_R)}\exp(\varepsilon_{m,t}),$ |
| 41) | Fisher equation | $1 + i_t = (1+r)E_t\left[\pi_{t+1}\right],$ |
| 42) | Monetary shock | $\varepsilon_{m,t} = \rho_{m,t}\varepsilon_{m,t-1} + \exp(\sigma_{R,t})u_{\varepsilon,t},$ |
| 43) | Uncertainty shock | $\sigma_{R,t} = \rho_\sigma\sigma_{R,t-1} + u_{\sigma,t},$ |

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