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ABSTRACT

Interconnectedness between economic institution and sectors, already recognised as a trigger of the great financial crisis in 2008-2009, is assuming growing importance in financial systems. In this paper we study contagion effects between corporate sectors using financial network models, in which the significant links are identified through conditional independence testing. While the existing financial network literature is mostly focused on Gaussian processes, our approach is based on discrete data. We indeed test dependence in the conditional mean (and volatility) of default counts in different economic sector estimated from Poisson autoregressive models, and in its shocks. Our empirical application to Italian corporate defaults in the 1996-2018 period reveals evidence of a high inter-sector vulnerability, especially at the onset of the global financial crisis in 2008 and in the following years. Many contagion effects between corporate sectors are indeed found in the shock component of the default count dynamics.

KEYWORDS

Financial networks; Inter-sector contagion; Poisson autoregressive models; Vector autoregressive models; Conditional Granger causality; PC-algorithm

JEL: C01, C32, C58, G21, G32

1. Introduction

Credit risk arising from interconnectedness between organisations and/or individuals, often referred to as *contagion*, is known as an important source of financial risk. Contagion channels originate from the inter-linkages between companies, sectors, and countries. In particular, corporate default dependency can be conditional on the business cycle and, more in general, the economic environment where all firms in a system operate. The investors' risk perception is another possible source of inter-linkages between companies and sectors. For example, companies can be impacted by news concerning other firms in the same market. Direct linkages could instead arise from trading and legal relationships, making the default of a company more probable due to the bankruptcy of connected companies. Studying interconnections and their changes allows policy makers and regulators to assess vulnerabilities for risk transmission among corporate and financial institution, and banks and other lending institutions to improve default prediction and credit risk assessment.

Trying to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, able to combine the rich structure of financial networks (see e.g. Mantegna (1999); Lorenz et al. (2009); Battiston et al. (2012)) with a more parsimonious approach that estimates contagion effects from the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derived contagion measures based on Granger causality tests and variance decompositions. More recently, Ahelegbey et al. (2016) and Giudici and Spelta (2016) extended the methodology introducing stochastic correlation networks. So far, stochastic correlation network models have been based on Gaussian processes, a natural approach when data are market prices. However, when studying contagion using default data, modelling default counts through discrete processes is a more natural choice, for mainly two reasons. First, in many applications, defaults are rare events, making not suitable to apply Gaussian models. Secondly, the number of borrowers in a given institution or system - which is used as the denominator of default rates - is usually more stable and easily known in advance than the numerator: the number of defaults at the end of a given period, which can be modelled using count processes.

Several works in the econometric literature have indeed applied discrete data models to analyse possible dependence between default events conditional on macroeconomic and/or financial factors affecting the overall probability of corporate default. For example, Lando and Nielsen (2010) modelled default times through Poisson processes with macroeconomic and firm-specific covariates entering the default intensities. The idea of distinguishing between latent and exogenous effects is also present in Agosto et al. (2016), who applied Poisson autoregressive models with exogenous covariates to the US default count time series. Recently, Escribano and Maggi (2019) applied a multivariate Poisson model to default counts among large US companies, estimating the propensity of several economic sectors to affect or being affected by others.

In this paper we combine the network approach with the default count analysis. More specifically, we estimate pairwise conditional dependencies between corporate sectors based on their default count dynamics, modelled through Poisson autoregressive processes. Operationally, this translates into directed networks where significant edges are identified through conditional Granger causality testing and the PC algorithm applied to the mean and the shocks of the monthly number of defaults.

The paper is organised as follows. Section 2 describes the default count data employed in our application, motivating our modelling approach. The proposed network models are presented in Section 3. Section 4 shows the main empirical findings derived from the network model application. Section 5 contains some concluding remarks.

2. Data

In this work, we focus on the default dynamics of one of the countries which most suffered the recent financial crisis: Italy.

In its statistical database¹, Bank of Italy provides data relative to transitions to *bad loans* among Italian banks' credit exposures. Bad loans are exposures to insolvent debtors that the bank believes it is not possible to recover and must then report as losses in its balance sheet. Specifically, for each quarter it is provided the number of loans that were not classified as *bad* at the beginning of the period and turned out

¹<http://www.bancaditalia.it/statistiche>.

to be so at the end. The data include information on the sector to which the debtors belong.

In our analysis we consider the bad loans flow as a proxy of quarterly default counts. In order to exclude small-sized missed payments, we only consider the class of exposures greater than 500k euro. Using the classification provided in the dataset, we divide corporates into the following sectors: Commerce, Energy, Manufacturing, Primary, Real Estate (including both constructions and real estate corporates), Transports, Information and Communication Technologies (ICT), Services. The data employed in our study cover the period from March 1996 to June 2018. Figure 1 plots the default count time series, showing an increase in level and variability starting from 2008, with the mean returning close to the pre-crisis level after 2015. All the eight series, for which descriptive statistics are provided in Table 1, show indeed overdispersion, that is their variance is greater than the mean. This confirms the empirical evidence that defaults tend to cluster in time and show peaks during economic downturns.

Sector	ID	Mean	Std. dev.	Min	Max
Commerce	CMM	235.5	124.8	100	560
Energy	ENG	9.4	7.3	1	33
Manufacturing	MNF	312.7	126.1	157	634
Primary	PRM	62.8	34.9	17	153
Real Estate	RES	484.2	346.5	115	1354
Transports	TSP	37.9	23.9	5	93
ICT	ICT	19.4	8.7	6	41
Services	SRV	75.6	42.9	22	181

Table 1. Summary statistics of the default count time series in the Italian credit system (March 1996 - June 2018) by economic sector.

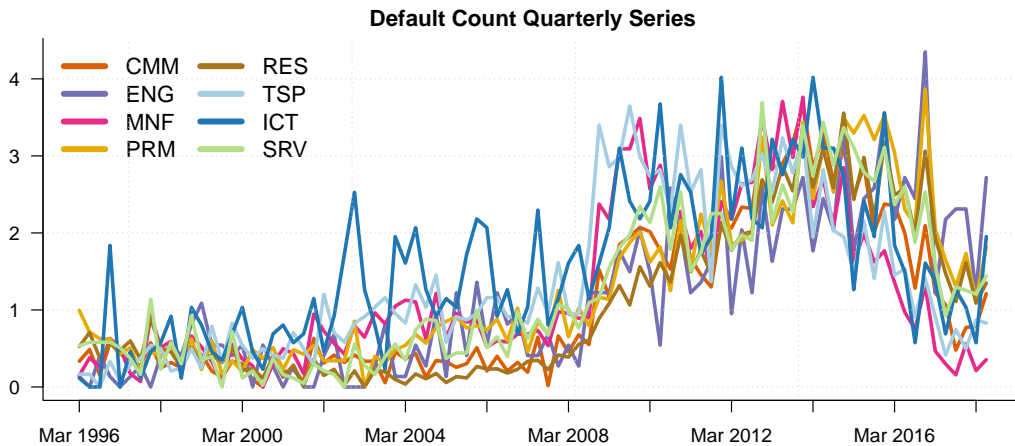


Figure 1. Normalised quarterly default counts in the Italian credit system (March 1996 - June 2018) by economic sector.

It is worth remarking that the data considered are driven not only by firm-specific and systematic default risk dynamics, but also by the behaviour of the lending banks, who decide when an exposure should be considered as a bad loan based on both financial considerations and regulatory constraints (for example, asset quality review processes performed by banking supervisors). Thus, the non-performing loan flow re-

flects not only the solvency dynamics of the debtors, but also the level of trust of the credit system in the borrowing sector, affected by overall and sector-specific downturns strongly connected to the increase in default probability.

3. Model

3.1. Poisson autoregressive model

Our strategy for modelling inter-sector contagion effects starts with the estimation of univariate models on the count time series. In particular, we resort to an autoregressive model for counts allowing for the overdispersion found in our data. We then assume the conditional distribution of the default counts in economic sector i at time t to be Poisson distributed with a log-linear autoregressive intensity:

$$Y_{it}|\mathcal{F}_{t-1} \sim \text{Poisson}(\lambda_{it})$$

$$\log(\lambda_{it}) = \omega_i + \sum_{j=1}^p \alpha_{ij} \log(1 + y_{it-j}) + \sum_{j=1}^q \beta_{ij} \log(\lambda_{it-j}) \quad (1)$$

where

$\omega_i \in \mathbb{R}$, $y_{ij} \in \mathbb{N}$, $\alpha_i \in \mathbb{R}^p$, $\beta_i \in \mathbb{R}^q$, $\forall i = 1, \dots, k$ with $k = 8$ sectors.

Considering lags of $\log(1 + y_{it-j})$, rather than lagged log counts directly, allows to deal with possible zero values in the time series. Note that in Model 1, whose properties were studied by Fokianos and Tjøstheim (2011), both the α_i and the β_i coefficients express dependence of the expected number of defaults to past default counts. The inclusion of lagged default intensities (the β component) is analogous to moving from an ARCH to a GARCH specification when modelling Gaussian processes, and allows to capture long memory effects in count data. The advantage of a log-linear intensity specification, rather than the linear one commonly known as integer-valued GARCH (see, e.g., Ferland et al. (2006)) is that it allows for negative dependence. In our data, the number of defaults could be negatively correlated to their lagged values for either a mean-reversion effect after a peak or because banks have already classified many non-performing loans as defaults in the previous quarters, due to balance sheet and/or regulatory constraints.

For each of the considered sectors, we estimate Model 1 by maximum likelihood (see Fokianos and Tjøstheim (2011) for details on the inference theory). While we fix the number of q lags at 1, as commonly done in the GARCH model applications, for each equation we choose the number of p lags between 1 and 4 which minimises the Akaike Information Criteria (AIC). In all cases apart from the energy sector, for which the preferred specification includes 2 lags of the past default counts, the best model turns out to be the one with $p = 1$ and $q = 1$.

As it can be seen in Table 2, the estimated autoregressive coefficients are significant in all sectors and their sum is close to unity, indicating a high level of persistence in the series. Thus, for each $i = 1, \dots, 8$ sector, we obtain an intensity process $\hat{\lambda}_{ij}$, that is the estimated mean and volatility of the quarterly default counts, conditional on their past. The eight default intensity series are plotted in Figure 2.

Sector	ω	α_1	α_2	β
CMM	0.157 (0.116)	0.559*** (0.054)		0.413*** (0.054)
ENG	0.041 (0.083)	0.074*** (0.108)	0.341*** (0.133)	0.568*** (0.089)
MNF	0.247 (0.182)	0.606*** (0.061)		0.351*** (0.054)
PRM	0.059 (0.081)	0.414*** (0.054)		0.573*** (0.063)
RES	0.127 (0.104)	0.489*** (0.045)		0.493*** (0.048)
TSP	0.144 (0.097)	0.451*** (0.072)		0.511*** (0.079)
ICT	0.141 (0.093)	0.239*** (0.052)		0.714*** (0.066)
SRV	0.108 (0.085)	0.415*** (0.052)		0.562*** (0.051)

Table 2. Parameter estimates of the Poisson autoregressive models applied to the default count quarterly series (standard errors in brackets; (***) denoting significance at the 0.001 level).

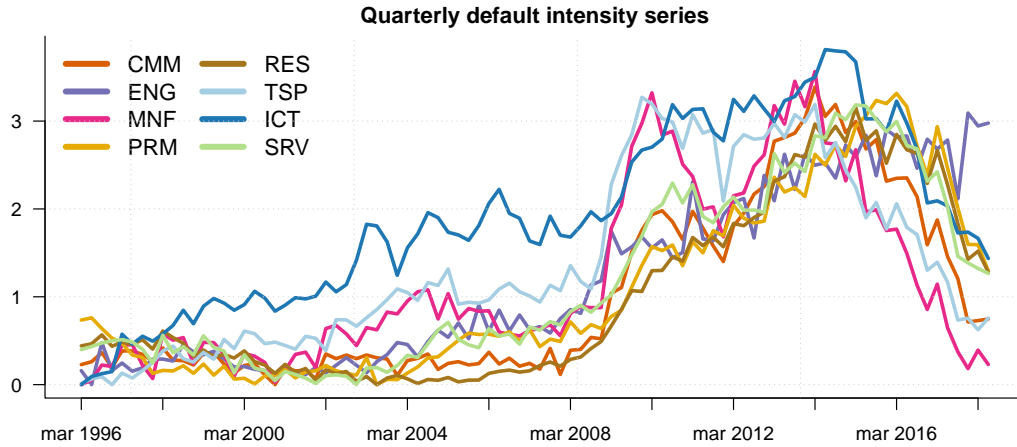


Figure 2. Normalised quarterly default intensities in the Italian credit system (March 1996 - June 2018) by economic sector, estimated through Poisson autoregressive models.

The estimated default intensities are the expected number of defaults in a given quarter. In the framework of model (1), the unexpected component is instead the difference between the observed counts y_{it} and the estimated intensity $\hat{\lambda}_{it}$. Specifically, for each sector i we calculate the so-called Pearson residuals which are usually considered in generalized linear models (see Gourieroux et al. (1987) and Kedem and Fokianos (2002)):

$$e_{it} = \frac{y_{it} - \hat{\lambda}_{it}}{\sqrt{\hat{\lambda}_{it}}} \quad (2)$$

In the following sections, we estimate inter-sector linkages through testing dependencies in both the intensity ($\hat{\lambda}_{it}$) and the shock (e_{it}) processes.

3.2. Network model

We use the estimates obtained from application of the log-linear Poisson model to build two networks, where the nodes are the considered sectors and the directed edges represent significant dependencies between the default counts of different sectors.

The first network is made up of the links between the default intensities estimated in Section 3.1 and represents lagged effects between sectors. Each intensity series is indeed the expected number of defaults conditional on past counts.

We then build a network based on the shock component of the default count processes, estimated through residuals of the Poisson model, to take contemporaneous effects into account.

In order to identify the inter-sector significant links in the first network, we employ conditional Granger causality testing developed by Granger (1969), while we use the PC algorithm by Spirtes et al. (2000) to build the second network. The estimation procedure for the two networks is detailed in Sections 3.2.1 and 3.2.2.

3.2.1. Network model of lagged effects

We use the default intensities estimated in Section 3.1 to investigate interdependencies in the conditional mean (and variance) of different economic sectors by testing Granger causality.

Before proceeding with the multivariate analysis, we pre-process the data as follows. First, we convert the quarterly estimates to monthly observations by applying the R Spline function, which implements the cubic interpolation method by Forsythe et al. (1977). This makes available longer time series, thus providing more stable estimates. Figure 3 shows the monthly default count intensities.

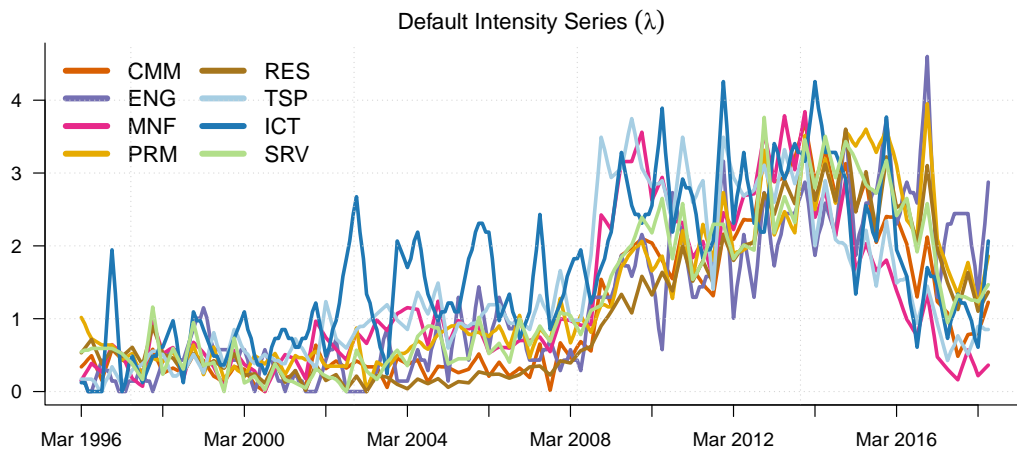


Figure 3. Normalised monthly default intensity estimated through Poisson autoregressive models applied to the Italian corporate default counts (March 1996 - June 2018) by economic sector.

It can be seen from Figure 3 that there is evidence of non-stationarity in the series. This is confirmed by Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. Thus, we take first-order differ-

ences of each series and repeat the stationarity tests, whose p-values are reported in Table 3. At a 1% significance level, the null hypothesis of unit root cannot be accepted in the ADF test and the null hypothesis of stationarity is not rejected with the KPSS test. Note that the ICT sector series could be considered as non-stationary if using a 5% significance level. Though, increasing the differencing order would pose an issue of interpretation of the estimated relationships. By taking first differences we are actually considering the expected changes in the monthly number of defaults, conditional on their past. The differenced intensity series $\Delta\hat{\lambda}_{it} = \hat{\lambda}_{it} - \hat{\lambda}_{it-1}$ are plotted in Figure 4.

Sector	ADF	KPSS
CMM	0.01	0.09
ENG	0.01	0.10
MNF	0.01	0.09
PRM	0.01	0.10
RES	0.01	0.07
TSP	0.01	0.05
ICT	0.01	0.03
SRV	0.01	0.10

Table 3. P-values of the ADF and KPSS tests performed on the first-order differenced default intensity series.

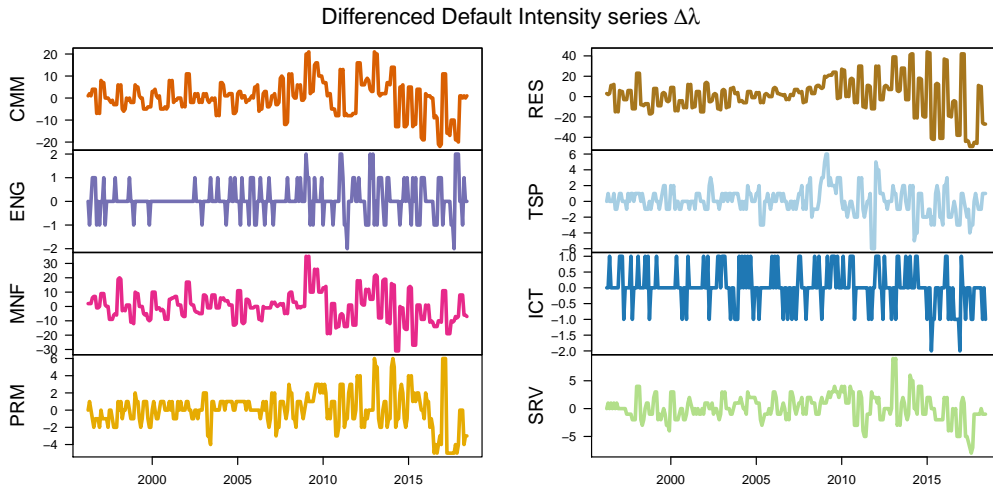


Figure 4. Differenced monthly default intensities coming from the application of Poisson autoregressive models to the Italian corporate default counts (March 1996 - June 2018) by economic sector.

To perform the conditional Granger causality test on the estimated changes in default intensity, we consider the following 8-variate Vector AutoRegressive (VAR) specification:

$$\Delta\lambda_t = \Phi_0 + \sum_{k=1}^p \Phi_k \Delta\lambda_{t-k} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma) \quad (3)$$

where the lag order p is chosen through model selection based on the Bayesian Information Criteria (BIC). In this setting, the hypothesis that a change of default intensity in sector l does not Granger cause a change of default intensity in sector i can be verified by testing the restriction $\phi_{il,j}$ for $j = 1, \dots, p$.

Once the test is performed for each of the ϕ_{il} coefficients, we are able to build the A_λ adjacency matrix with binary entries such that

$$A_{\lambda,il} = \begin{cases} 0 & \text{if } \Delta\lambda_{lt-j} \text{ does not Granger cause } \Delta\lambda_{it} \\ 1 & \text{if } \Delta\lambda_{lt-j} \text{ Granger causes } \Delta\lambda_{it} \end{cases} \quad (4)$$

This defines a directed network where an edge from node l to node i is drawn if changes in the expected number of defaults in sector i are conditionally dependent on changes in the expected number of defaults in sector l .

3.2.2. Network model of contemporaneous effects

The previous network is based on lagged inter-sector effects in the conditional mean (and volatility) of the default count process. However, credit risk contagion can spread through shocks in the default dynamics, generating contemporaneous systemic effects. We thus consider conditional inter-sector dependencies between the residuals of the log-linear models calculated in Section 3.1 (see equation 2), corresponding to the random (Poisson) part of the default count dynamics.

As previously done for the estimated default intensity (see Section 3.2.1), we transform the quarterly residual series into monthly data. The obtained Pearson residual monthly series are plotted in Figure 5.

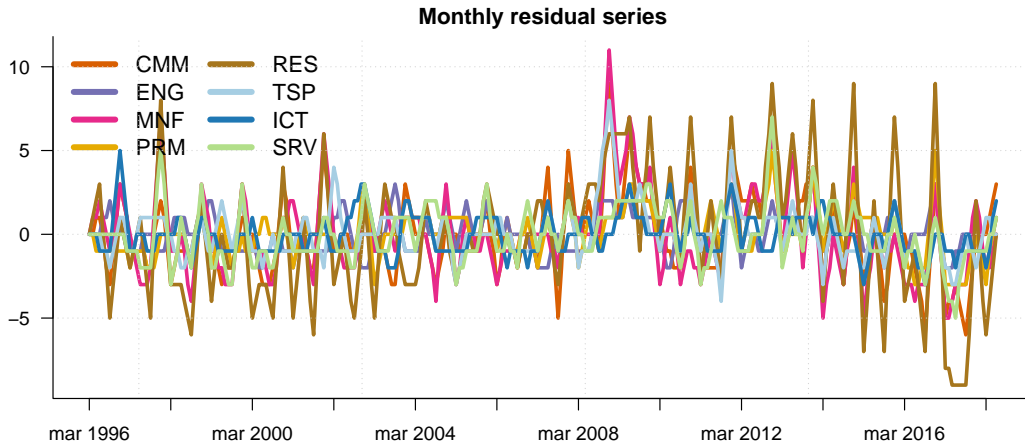


Figure 5. Monthly Pearson residuals coming from the application of a dynamic log-linear Poisson model to the Italian corporate default counts (March 1996 - June 2018) by economic sector.

In order to estimate directed links between sectors based on the estimated shocks, we employ the PC algorithm. The PC algorithm is a constrained-based network inference developed by Spirtes et al. (2000) for learning partially directed networks. The algorithm is designed to test a set of conditional independence and dependence statements using statistical tests. The most widely used conditional independence test for such networks is the Fisher's z -statistic. See Spirtes et al. (2000) for details on implementation of the PC algorithm.

We thus build an adjacency matrix called A_e with binary entries such that

$$A_{e,il} = \begin{cases} 0 & \text{if } e_{it} \text{ does not depend on } e_{lt} \\ 1 & \text{if } e_{it} \text{ does depend on } e_{lt} \end{cases} \quad (5)$$

This way we are able to draw a directed network where an edge from node l to node i means that shocks in the number of defaults in sector i are conditionally dependent on shocks in the number of defaults in sector l .

4. Application

In this section we show the empirical findings obtained from application of the network models described in Sections 3.2.1 and 3.2.2.

In particular, we perform the network estimation in two subsamples: March 1996 - December 2007 and January 2008 - June 2018. The cut-off point is chosen so that the sub-periods can be interpreted as the *pre-crisis* and the *post-crisis* sample respectively, and corresponds to a possible structural change in the corporate default count series. Note that the second period contains not only the global financial crisis, but also the sovereign one in the Euro area, which caused liquidity distress in many financial system, including the Italian one.

4.1. Inter-sector network of lagged effects

To provide a first insight of the inter-sector relationships, Figures 6 and 7 plot the network structure in the two non-rolling subsamples based on the VAR specification (3). To aid interpretation, we distinguish the links using the signs of estimated coefficients: positive dependencies are depicted in green and negative in red. While the positive links can be interpreted as possible contagion channels between sectors, interpretation of the negative links is less straight-forward. Negative correlation could be due to the fact that default peaks in some sectors run out more quickly than in others. Another possible explanation could be linked to lenders' perception of risk, encouraging banks to reduce their exposure in some sectors and increase their investments in others. The edge thickness represents instead the magnitude of the estimated relationships, measured by the coefficient values.

It can be seen from Figure 6 and 7 that the networks based on lagged effects in the conditional mean are sparse. In the first subsample (1996-2007) the only positive edge is directed from the commerce sector to ICT, while a negative link is present between Energy and the commerce sector. As it can be seen from Figure 7, the period during and following the global financial crisis shows instead three positive connections, all involving the energy sector, which turns out to affect Primary, Real Estate and Commerce.

4.1.1. Including exogenous covariates

The estimated links in the default intensity process are few and not stable over the sample, thus they could potentially change when exogenous variables, reflecting the business cycle and the financial context, are considered. Furthermore, it is interestingly to assess whether exogenous covariates can significantly predict the expected number

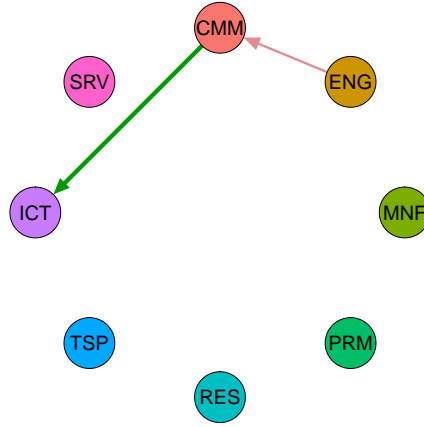


Figure 6. Inter-sector network based on the monthly expected number of defaults (March 1996 - December 2007).

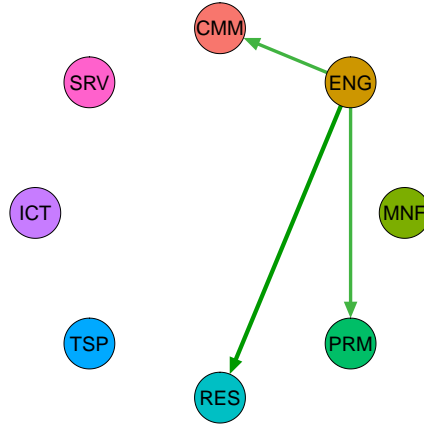


Figure 7. Inter-sector network based on the monthly expected number of defaults (January 2008 - June 2018).

of defaults.

We thus repeat the network estimation based on a Vector AutoRegression with eXogenous covariates (VARX) specification:

$$\Delta\lambda_t = \Phi_0 + \sum_{k=1}^p \Phi_k \Delta\lambda_{t-k} + \sum_{k=1}^p \Psi_k X_{t-k} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma) \quad (6)$$

In particular, we include the following variables in the set X of exogenous covariates, collected on monthly basis:

- the growth rate of the gross domestic product from the same quarter one-year ago (GDP), to control for mid-term fluctuations in the Italian economy;
- the growth rate of Industrial Production change (IPR), to consider the effect of short-term changes in the economic conditions;

- the growth rate of the national currency-to-dollar exchange rate (EXC), to control for variations in the trading market due to the exchange dynamics;
- the change in 10-year Treasury bond yield (YIE), to consider the domestic credit market fluctuations;
- the change in unemployment rate (UEM), to take the dynamic of the labour market conditions into account;
- the realized volatility of Eurostoxx 50 index (REV), reflecting the European financial market turmoil.

While the first five indicators were downloaded from the Federal Reserve Bank of St. Louis website², the proxy of monthly realized volatility for the Eurostoxx 50 index was calculated using the daily squared returns ($RV_t = \sum_{i=1}^{n_t} r_{it}^2$, with r_{it} denoting the i -th daily return on the index in month t and n_t indicating the number of trading days in month t).

The considered exogenous series are shown in Figure 8.

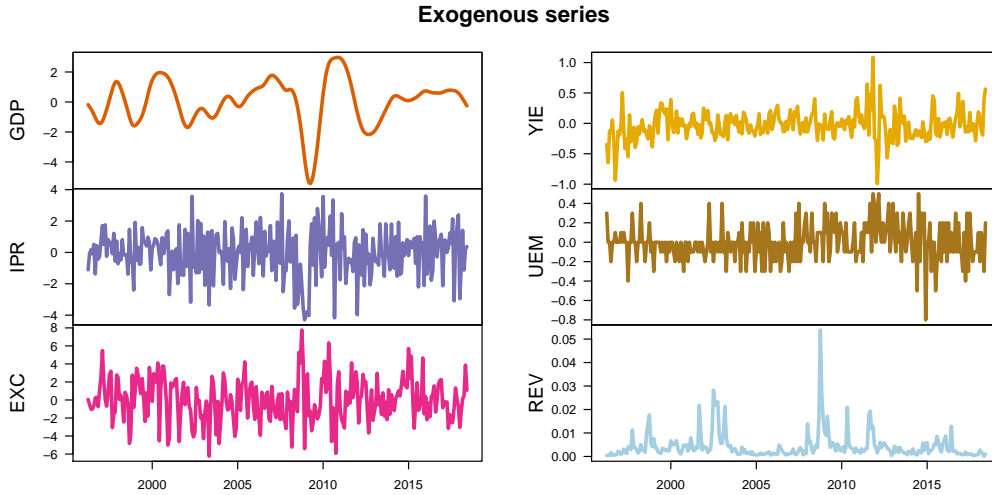


Figure 8. Monthly time series of macroeconomic and financial exogenous covariates.

Figures 9 and 10 show then the corporate sector network conditional on the exogenous processes listed above. In the first sub-period, the positive link between Commerce and ICT remains significant, while, in the second sub-sample, the energy sector positively affects the real estate only. Similarly to the findings of Agosto et al. (2016) and Escribano and Maggi (2019), significant exogenous covariates are few, conditional on the autoregressive effects. Among the included variables, in the pre-crisis period a negative relationship between realized volatility and the Real Estate sector is found. The negative correlation between the stock and the real estate market in the non-crisis periods has been empirically found in several works, such as Quan and Sheridan (1997), and, as expected, is not significant in the crisis subsample, when the collapse of the real estate market was the main cause of the financial market turmoil. In the sub-period from 2008 on, the GDP growth rate turns out to significantly predict the default count dynamics, showing negative links with Commerce and Manufacturing.

²<http://www.stlouisfed.org>.

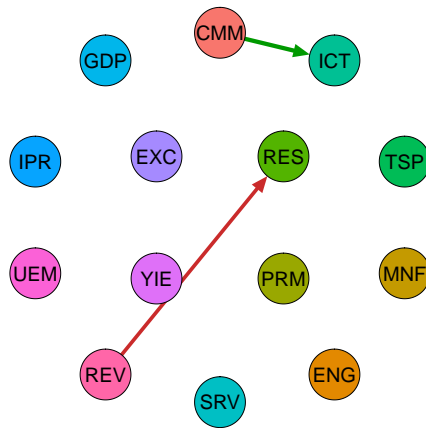


Figure 9. Inter-sector network based on the monthly expected number of defaults (March 1996 - December 2007).

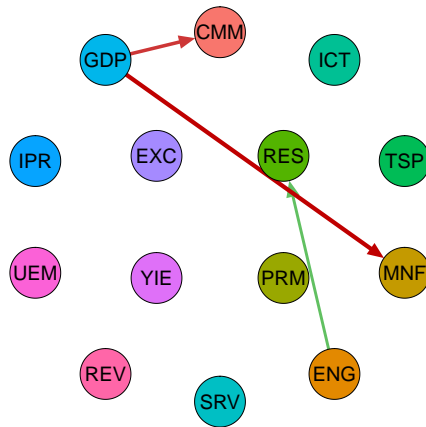


Figure 10. Inter-sector network based on the monthly expected number of defaults (January 2008 - June 2018).

4.2. *Inter-sector network of contemporaneous effects*

The networks of connections in default count shocks are shown in Figures 11 and 12 for the pre-crisis and post-crisis periods respectively. Again, the edge colour indicates the sign of relationships, while the thickness depends on their magnitude, here measured by conditional dependencies estimated through the PC algorithm, as explained in Section 3.2.2.

Many connections - all with positive sign - can be found in both sub-periods. This indicates that shocks in the default dynamics of different sectors are highly and positively correlated, acting as possible contagion channels.

In both periods, Commerce appears as highly *infective*, and its mutual relationship with the manufacturing sector is in line with the economic intuition. The impact of Commerce on ICT, already found in the lagged effects network, is confirmed. It can be noticed from Figure 12 that the period from the financial crisis on is characterised by a larger number of inter-sector links and, as expected, a higher centrality of the real

estate sector, which is strongly *infected*. Transports is highly affected by other sectors' default shocks.

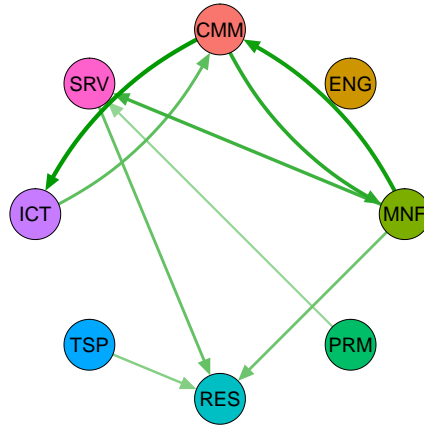


Figure 11. Inter-sector network based on shocks to the monthly number of defaults (March 1996 - December 2007).

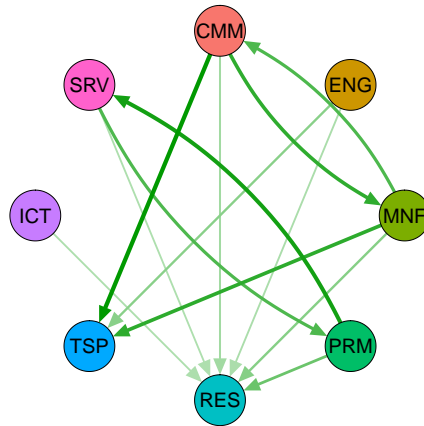


Figure 12. Inter-sector network based on shocks to the monthly number of defaults (January 2008 - June 2018).

To take the network evolution over the full sample into account, we also perform a rolling windows estimation procedure, where we fix the window size at 5 years. Figure 13 shows the network made up of persistent edges, whose mean weight over the windows is larger than the first quartile of the weight distribution.

Comparing Figure 13 with Figures 11 and 12, we notice that some links are stable over the sample. In particular, the mutual relationship between Commerce and Manufacturing, as well as the one between Primary and Services, is persistent. Also the vulnerability of Real Estate, affected by shocks in commerce and manufacturing sectors, is stable. Furthermore, a negative dependence of ICT on the energy sector, which was not significant in the two sub-periods estimation, emerges. A more detailed analysis, which we do not show for brevity, revealed that this latter relationship has

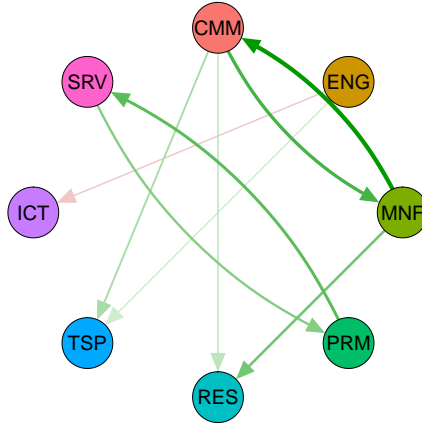


Figure 13. Inter-sector network based on shocks to the monthly number of defaults: persistent links over the full sample (March 1996 - June 2018).

highest magnitude in the period from 1999 to 2003, when the ICT sector suffered the effects of the new economy bubble.

To provide a further insight of interconnections between shocks to the number of defaults in the corporate sectors, Tables from 4 to 7 report centrality measures for the estimated shock-based networks. In particular, we consider network degree, hub and authority measures in the two analysed sub-periods (1996-2007, 2008-2018) and in the full sample (rolling windows estimation). These metrics are briefly defined in Appendix and allow to assess the position of sectors in the networks.

Note that, in our application, out-degree measures the extent to which shocks in the default risk of a sector affects the others. According to these metric (see Table 4), the most central sectors are Manufacturing and Commercial.

In-degree measures instead how much a sector is infected by default risk peaks in the others. Note from Table 5 that Real Estate, followed by Transports, clearly dominates in terms of in-degree, especially during the crisis period.

We finally look at the hub and authority metrics, measuring centrality of a node in terms of importance of vertices pointing towards and pointed by that node respectively. Commerce and Manufacturing are the sectors with largest hub centrality, while Real Estate has high authority, especially during the crisis subsample.

	1996-2007	2008-2018	1996-2018
CMM	2.0	3.0	3.0
ENG	0.0	2.0	2.0
MNF	3.0	3.0	2.0
PRM	1.0	2.0	1.0
RES	0.0	0.0	0.0
TSP	1.0	0.0	0.0
ICT	1.0	1.0	0.0
SRV	1.0	2.0	1.0

Table 4. Out-degree centralities in the estimated shock-based networks.

	1996-2007	2008-2018	1996-2018
CMM	2.0	1.0	1.0
ENG	0.0	0.0	0.0
MNF	1.0	1.0	1.0
PRM	0.0	1.0	1.0
RES	3.0	6.0	2.0
TSP	0.0	3.0	2.0
ICT	1.0	0.0	1.0
SRV	2.0	1.0	1.0

Table 5. In-degree centralities in the estimated shock-based networks.

	1996-2007	2008-2018	1996-2018
CMM	0.0	0.7	0.2
ENG	0.0	0.3	0.0
MNF	0.9	0.6	1.0
PRM	0.1	0.2	0.0
RES	0.0	0.0	0.0
TSP	0.2	0.0	0.0
ICT	0.3	0.1	0.0
SRV	0.2	0.1	0.0

Table 6. Hub centralities in the estimated shock-based networks.

4.3. Network density

Through the rolling windows estimation we are able to represent and compare the density of estimated networks over the whole period. Figure 14 and 15 show the normalized density of unweighted and weighted networks respectively. Both series are obtained through estimation in overlapping 5-years samples.

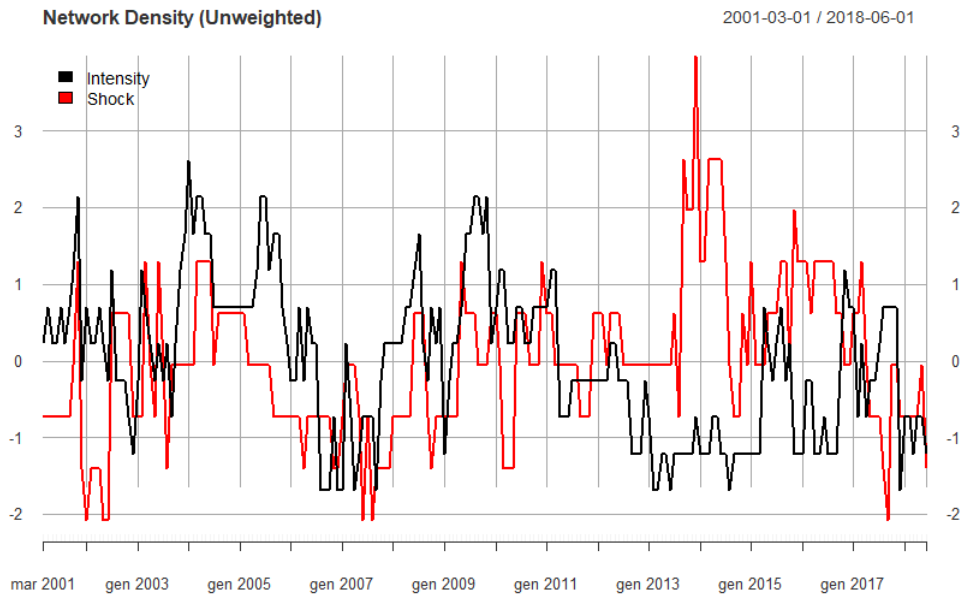


Figure 14. Density of the rolling window estimated unweighted networks.

	1996-2007	2008-2018	1996-2018
CMM	0.5	0.2	0.3
ENG	0.0	0.0	0.0
MNF	0.0	0.2	0.3
PRM	0.0	0.1	0.0
RES	0.7	0.8	0.6
TSP	0.0	0.5	0.7
ICT	0.0	0.0	0.2
SRV	0.5	0.1	0.0

Table 7. Authority centralities in the estimated shock-based networks.

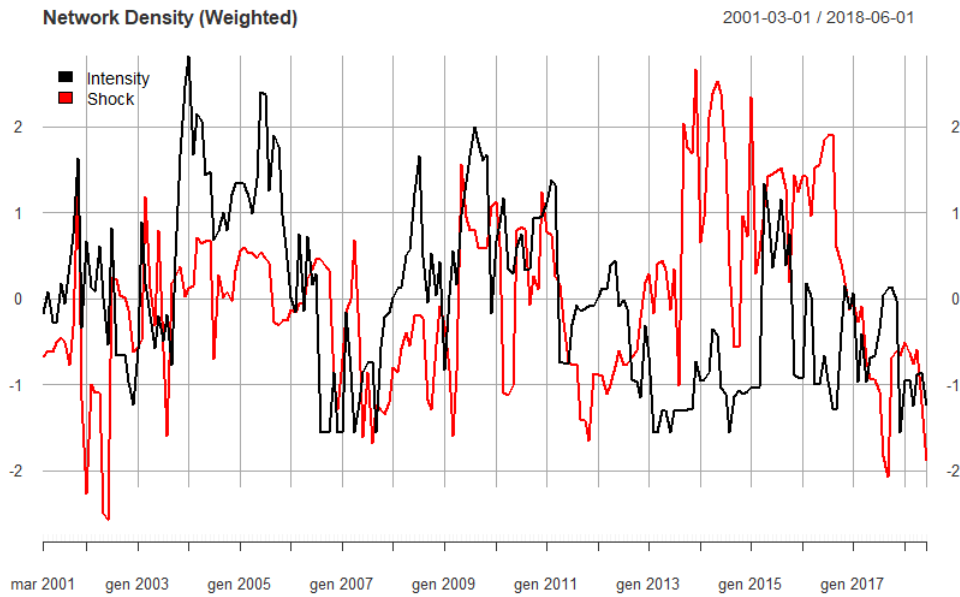


Figure 15. Density of the rolling window estimated weighted networks.

Looking at Figure 14, showing the unweighted network density for each window end date, the peak in the number of inter-sector links is found in the default count shocks in the 2009-2014 period, including both the spread of the financial crisis to the real economy and the sovereign crisis, when the density of the shock-based network is instead relatively flat. However, a spike in the density of the lagged effects network can be seen in the 1999-2004 window, which includes the dot.com bubble and the market turmoil following the 2001 terrorist attack. This peak is the highest when looking at the weighted density (Figure 14). Another spike in the default intensity network can be seen in the window ending in 2009, corresponding to the onset of the financial crisis.

Overall, our findings reveal that default risk contagion spreads mainly through shocks, but inter-sector links can also be found in the systematic part of the default count process and are relevant during stable periods and non-systemic crises.

5. Conclusions

In this paper we have considered inter-sector contagion effects in corporate default counts, combining Poisson autoregressive models with the financial network approach.

We estimated directed links between corporate sectors based on their default dynamics, considering both lagged and contemporaneous effects.

In particular, we built two networks: one based on dynamic effects in the conditional mean (and volatility) of the default count process and one made up of inter-linkages between the shocks. Both networks identified possible contagion channels through which credit risk propagates during macroeconomic and financial stress periods.

Focusing on Italian corporate default counts in eight economic sectors from 1996 to 2018 we found evidence of a high inter-sector vulnerability especially in the global financial crisis period and in the following years. Most significant links between corporate sectors were found in the default count shocks, while the mean-based network based is quite sparse and its density is higher out of the systemic crisis periods.

Our empirical findings also show that Real Estate, Manufacturing and Commerce are the most central sectors in possible contagion paths.

Future research may consider the application of the proposed model to other contexts and data, like defaults of corporate bond issuers, and on countries characterised by high levels of corporate defaults.

Furthermore, possible feedback effects of the default dynamics on macroeconomic and financial variables could be considered.

Appendix A. Network centrality measures

A.1. In and out-degree

Let G be an n -node weighted network graph where $G(ij)$ measures the presence or absence of a link from node- j to node- i with a weight value. The in-degree of node- i , denoted by D_i^{in} , and out-degree of node- j , by D_j^{out} , both defined by

$$D_i^{in} = \sum_j \mathbb{1}(|G(ij)| > 0), \quad D_j^{out} = \sum_i \mathbb{1}(|G(ij)| > 0) \quad (\text{A1})$$

where $\mathbb{1}(|G(ij)| > 0)$ is the indicator function, i.e., unity if the absolute value $|G(ij)| > 0$ and zero otherwise. Thus, D_i^{in} counts the number of links directed towards node- i , while D_j^{out} is the number of links going out of node- j .

A.2. Hub and authority

We can also calculate centrality measures that take into account the importance of neighbourhood of a node in a network graph. For example, the hub and authority centrality assigns a score to nodes in the network in a way that is proportional to the importance of its neighbours. For a given weighted network graph, this involves solving the following problem

$$(G'G) h_v = \lambda_h h_v, \quad (GG') a_v = \lambda_a a_v, \quad (\text{A2})$$

where h_v and a_v are the hub score and authority score vectors and λ_h and λ_a are the largest eigenvalues of $G'G$ and GG' respectively.

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