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with Heterogeneous Consumers.
A Conjectural Approach to Mass Litigation**

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Accuracy and Costs of Dispute Resolution with Heterogeneous Consumers A Conjectural Approach to Mass Litigation

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Abstract

This study focuses on the tradeoff between accuracy and costs in consumer mass dispute resolution. The model investigates how a defendant-firm formulates its settlement offer given the fact that faces a large population of heterogeneously risk-averse claimants. Although firms can learn something about consumers' reaction to a given settlement offer thanks to repeated litigation, equilibrium settlement can be far –i.e. lower– from the correct claim value. From the societal perspective, this brings up the issue related to the desirability of out-of-the-court cheaper resolution and the risk of underpricing the value at stake. The paper also deals with the effects of aggregate litigation as an additional means to resolve consumer mass litigation. Results show that the overall cost of dispute resolution (aggregated costs of in-court-litigation plus costs of inaccuracy) is generally decreasing in the unit litigation cost. This suggests that litigation costs may be used as a leverage to favour not only more, but also more accurate, settlements. Finally, aggregate litigation might push the business defendant to negotiate a more generous settlement.

JEL classifications: D81, D83, K13, K41.

Keywords: Bayesian learning, Conjectural Equilibria, Consumer litigation, Risk Aversion, Settlement.

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1. Introduction

Law and economics literature has extensively shed light on several trade-offs existing between in-court mechanisms (trials) and various out-of-the court alternative dispute resolution systems. While the former – when accessible – are considered *costly* (both for the parties and for society) but legally *accurate* in the sense that can serve correct law enforcement and fair redress of meritorious complaints, the latter are desirable since can settle disputes rapidly, with lower costs, though producing less accurate outcomes.¹ Brunet, who extensively compares out-of-the court dispute resolution means (ADRs in his article) and conventional litigation, especially focusing on the accuracy of dispute resolution outcome, underlines that “quality justice contains a substantive component. [...] The relationship of ADR to substantive law is uneasy. [...] The ADR passion is to terminate disputes quickly and at low costs to disputants. While these characteristics possess genuine procedural value, the ADR process may give short shrift to substantive law”.² More informal, straightforward, quick and direct the procedures are, more accuracy may be jeopardized.³ On the other hand, literature has underlined the importance of accurate dispute resolution outcomes in the sense that disputes should be decided for an amount that is considered the proper legal sanction on primary behavior.⁴

This contrast between in-court and out-of-the-court dispute resolution systems becomes even more relevant when considering *consumer mass disputes*.⁵ Violations of the consumer law and product liability involve many victims and, even when individual claims are small, the resulting aggregate damage is significant. In such a case, costs, formalities, complexity of legal procedures may curb many consumers from proceeding in court. In turn, consumer protection is diluted and firms’ misbehavior under-deterred. This can be exacerbated by risk aversion of consumers who suffer from the uncertainty of a trial thus preferring inaction (Faure 2013). Small

¹ We opt for terms like “out-of-the-court” and “in-court” dispute resolution systems because we do not focus on a specific alternative dispute resolution scheme (ADR). In the case of out-of-the-court dispute resolution, we will generically refer to *settlement*. It must be noted that, except when the focus is on a specific *ADR*, the distinction between a generic settlement and ADRs tends to vanish, especially if ADRs are considered as less formal, cheaper, and less referred to substantive law than trial. When this is the case, many considerations on the trade-off between settlement and trial apply (see Daughety and Reinganum, 2012). One of the first seminal contributions specifically on ADR in law and economics literature is Shavell (1995). Besides consideration on costs and accuracy involved by the two families of dispute resolution means, part of the literature underlines that judicial decisions can be considered as a public good since able to determine legal precedent. The latter is (more or less) binding for future litigation and help reducing uncertainty of both the law (Dari-Mattiacci and Deffains, 2007 and Dari-Mattiacci et al., 2011) and future judicial decisions (Fon and Parisi, 2006 and Rampa and Saraceno, 2016).

² Brunet (1987, page 54).

³ See Brunet (1987). Stipanovich (2004) provides an extensive overview of empirical studies on the use and the quality of ADR in the United States; the author concludes that “there will continue to be tensions between the desire for more perfect adjudication on the one hand, and less expensive, more efficient, or more flexible choices on the other” (page 912). On the other hand, but limited to federal government litigation, Bingham et al. (2008) conclude that judicial decisions and ADR outcomes are comparable in terms of quality. On the accuracy of settlement and trial see Dari-Mattiacci and Saraceno (2017).

⁴ On the value of accuracy in adjudication, see Cooter and Rubinfeld (1989), Kaplow, (1994); Kaplow and Shavell (1994) and (1996), Bebchuk and Chang (1996). On the differences between settlement and trial from a deterrence perspective, see Polinsky and Rubinfeld (1988).

⁵ Focusing on a set-up where legal effort by the plaintiff improves the accuracy of the legal process, Friedman and Wickelgren (2008) show that settlement can lower social welfare because it reduces the accuracy of legal outcomes. They provide medical malpractice claims as an example of a context where this trade-off can often emerge.

claims⁶, dispersed losses and rational apathy of consumers well justify the debate on dispute resolution means that are alternative to the ordinary judicial proceeding (Faure and Weber, 2015). Generally, out-of-the-court disputes resolution means are discussed as tools of strengthening consumer law enforcement since they enable consumers to complain by reducing individual procedural costs. On the other hand, out-of-the-court settlement often lack the quality that should be guaranteed by judicial decisions (or at least by judge-scrutinized settlements) with consequences on fairness of redress and/or optimal deterrence. For consumer mass disputes, commentators observe that disputes are often settled for amounts that are significantly lower than those typically obtained in trial.⁷

Consumer damage remedies range from *the most frequently used* procedures⁸ including very informal and cheap *direct negotiation* between claimant and business to proceedings in small-claims-courts that are like the ordinary courts (although more flexible and accessible).⁹ In between there is a continuum of various direct and indirect (third-party-managed) procedures including Ombudsmen, online dispute resolution schemes¹⁰, several forms of mediation, formal, legal-intensive, and often expensive arbitration. In addition to these individual procedures, in some countries (and typically for a limited number of matters) representative and/or aggregate procedures are viable, including class action¹¹, consumer associations' actions¹², test procedures¹³, and actions of public bodies for injunctive reliefs¹⁴ that are associated with collective settlement schemes.¹⁵

Although we disregard peculiarities characterizing a specific legal system, it is evident that means of consumer redress move from informal and direct mechanisms aimed at settling the case rapidly, efficiently and

⁶ Wagner (2014) provides an interesting analysis of the role that ADRs can potentially play in consumer law enforcement, shedding light also on limitations of that mechanism.

⁷ See Stipanovich (2004) in section II on the relation between ADR and conflict-management in commercial sectors. Weber (2015), in particular signals problems related to risks of capture that can reduce fairness of out-of-the-court decisions provided by insufficiently independent bodies. The so-called *coupon settlement* can involve problems of under-compensation and under-deterrence (see Borenstein, 1996).

⁸ Stuyck et al. (2007) provides a very comprehensive description and evaluation of the means of consumer redress viable by consumers alternatively to ordinary proceedings in European countries. See also Stipanovich (2004).

⁹ Weber (2015, FN 12) defines small-claims procedures as procedures somewhere in between ADR and traditional civil litigation.

¹⁰ Simply by navigating on line, it is evident that out-of-the-court dispute resolution is becoming more and more viable through ODR platforms, both at institutional level (see for instance the European Commission ODR platform, (<https://webgate.ec.europa.eu/odr/main/index.cfm?event=main.home.chooseLanguage>) and at business level (see for instance Modria platform, <http://modria.com/about-us/>).

¹¹ On the economics of class action see –among others – the seminal work by Bernstein (1978). In many countries, especially in European countries, representative class action is not viable.

¹² On the economic analysis of group litigation and actions by associations according to the European models, see Schaefer (2000). In many European countries, only consumer association can proceed against firms on behalf of damaged consumer. Often individual damage redress can be obtained only through an additional individual action.

¹³ See, for instance, the German Kapitalanlagegeschäftsführungsgesetz that is viable only for securities cases.

¹⁴ In the U.S. the Attorney General's offices typically establish divisions aimed at enforcing consumer protection and mediating consumers' complaints; if mediation is not successful, consumers must pursue remedies on their own. Faure et al. (2008) provides a comparative overview of schemes involving public authorities for public law enforcement.

¹⁵ A quick recognition of some recent cases in the U.S. allows us to verify that refunding or coupon/ discount settlements are common solutions that are collectively negotiated in various sectors including food&beverage, mobile service providers, credit cards, etc. See for instance the RED BULL CASE (2015) <http://archive.jsonline.com/blogs/news/278564831.html>, the STARKIST CASE (California, 2015) <https://www.tunalawsuit.com/Home.aspx>, the SPRINT CASE <https://www.sprintrefundpsms.com/Home.aspx>, the LCD case (Washington, 2016, <http://www.lcdsettlement.atg.wa.gov>).

maintaining both consumer trust and company's reputation to more and more formal, legal-intensive, and costly procedures aimed at fairly ascertain legal rights and obligations of litigants.

The first kind of means is beneficial for both parties and society because disputes are resolved at low costs (monetary and non-monetary). On the other hand, resulting redress may imply very inaccurate (usually undervalued) *pricing* of company's misbehavior (Faure 2013). When consumer disputes, especially those involving many victims, are resolved for low amounts, even though claimants are satisfied, there are relevant consequence in terms of insufficient deterrence and incentive to primary behavior (Faure 2009). Conversely, individual trials could provide more accurate legal evaluation of the violations at the cost of higher burdens for both parties and society.

The instant paper focuses on the tradeoff between accuracy and costs in consumer mass dispute resolution by working on a dichotomous model of means of consumer redress. As the extremes of the range of means to resolve consumer disputes, two polarized schemes can be assumed. On the one hand, we can imagine the company that makes a take-or-leave-it settlement offer to a population of damaged claimants to avoid any further action (we opted for a one-directional non-negotiable settlement offer from business to consumer in order to grasp consumers' apathy)¹⁶. On the other hand, we can imagine that consumers who are unsatisfied by the settlement offer can opt for an individual legal action. For the present, we assume that consumers can proceed in court only individually.¹⁷ At a later stage (Section 4.1), we briefly discuss the possibility to proceed in court through an aggregate procedure.

The model allows us to investigate how the defendant-firm formulates its settlement offer given the fact that it faces a large population of heterogeneous claimants who suffered identical losses. Heterogeneity relates to personal features of the consumers and, specifically, to their risk aversion towards an uncertain in-court dispute resolution. In our setup, the firm does not know all the counterparties in details, hence it maintains a conjecture on their aggregate reaction to settlement offers; this conjecture is updated according to Bayesian rules. The defendant firm whose problem is minimizing the expected outflow related to the consumer mass dispute deals with the tradeoff between offering a relatively high settlement amount to satisfy the largest share of potential plaintiffs and risking many trials. If decided in court, individual disputes are assumed to be decided "accurately", on average. From the societal perspective, this framework brings up the issue related to the suitability of out-of-the-court cheaper resolution and the risk of underpricing the value at stake.

In addition, and this is the reason why we opted for a conjectural approach (see Hahn, 1977 and Dekel et al., 2004), our Bayesian-learning model deals with the fact that companies typically face repeated consumer mass disputes over time. Consequently, firms can learn (something about) how consumers react to a given settlement offer. Understanding how settlement offers evolve over time till reaching conjectural equilibria (no further

¹⁶ This kind of settlement offers is very common in the business sector as the first attempt to settle a mass of claims. Offers typically include return and refund, coupons, discounts for future purchases, etc.

¹⁷ In many legal systems, especially in European countries, class action procedures are not viable. Even in the U.S., in the case of consumer disputes, class action is often impeded by arbitration clauses accompanied by class-action bans or waivers.

changes in future offers) and then analyzing equilibrium features provide important insights. The model allows us to compare the equilibrium settlement offer and the fair and correct value of the claim. The possibility for the firm to learn from consumer behavior is compared to the fact that equilibrium settlement can be far from the *accurate* pricing of defendant misbehavior. Finally, the trade-off between accuracy and costs of dispute resolution is considered at aggregated level. We discuss the role played by risk aversion, litigation costs and the amount at stake in the determination of the equilibrium settlement amount and then on the double policy aim of minimizing costs of dispute resolution while guarantying accurate outcomes (and then the “right” level of damage compensation and deterrence)¹⁸.

To the best of our knowledge, no contributions to the law and economics literature deals with the dynamics of out-of-the-court consumer dispute resolution (here a direct settlement offer) and in-court litigation by accounting for repeated mass disputes, allowing the defendant to learn from previous settlements. Settlement is usually considered as a variously designed bargaining under asymmetric information; differences in the models mainly focus on the informative set characterizing each party (who has the informative advantage and which piece of information is asymmetric between parties) and on who makes the offer to the counterparty (for an overview, see Daughety and Reinganum, 2012). Some contributions consider settlement negotiations between asymmetrically skilled parties (Chopard, 2010). As in the largest part of the literature on settlement, we opt for a take-or-leave-it offer, and the latter is proposed by the defendant. However, three main innovations—all suitable since we are dealing with consumer mass disputes—characterize the present model. First, the defendant faces a number of heterogeneous counterparties. Second, asymmetric information concerns the nature of the multiple counterparty (consumers’ attitude towards risk) and not the nature of the case (either the probability of success in trial or the amount at stake of the lawsuit) as in the standard settlement models. This results in the fact that the defendant defines its offer like a monopolist who does not know the aggregated consumer demand for settlement. Third, while in previous contributions the uninformed party is typically assumed to ignore one (or more) parameter(s) with known support(s), here the defendant completely ignores even the functional family to which the demand for settlement belongs to.

The present contribution purports to add to the literature on litigation and settlement from a dynamic Bayesian perspective. With respect to another contribution that introduces Bayesian learning in the litigation dynamics (Rampa and Saraceno, 2016) and focuses on the effect of legal precedent on in-court-litigation over time, the present paper changes the perspective. It considers the impact of defendant’s experience in settling cases on settlement rate, settlement amount and overall costs and accuracy of dispute resolution outcomes. Heterogeneity of plaintiffs may generate different reactions to the settlement offer and then determine a different *settlement vs trial path*. Thanks to learning, progressive recognition of consumer behavior on the part of business may lead to more accurate settlements but it may also happen that settlement stabilizes to values that are very far from – i.e. far beneath – the *just* value of the dispute.¹⁹ Finally, this paper extends the use of

¹⁸ See Shavell (1997).

¹⁹ As we will clarify in the next sections, this may happen even if we would assume null litigation costs and a not very risk averse population of potential plaintiffs.

heterogeneous risk averse agent models of diffusion of new products (Bogliacino and Rampa, 2010) in order to frame the evolution of settlement offers by business to consumers.

Results show that, on the one hand, consumer risk aversion and variability in judicial decisions play a significant role in preventing litigation while inducing consumers to settle even when the case is meritorious and they will be not charged with any litigation cost, while the defendant offer is not particularly appealing. On the other hand, the impossibility for the defendant to know the actual demand for settlement by the consumers forces the former to define its offers on the base of conjectures. Although optimal offers are nonnegative for meritorious claims, proposed amounts are always smaller than what the defendant expects to pay in the case of in-court-litigation. Furthermore, although the experience of previous settlements (and related rates of acceptance/rejection) can help the defendant to formulate more accurate offers, the dynamics between consumers and business can crystallize in a such a way that large part of disputes are settled for amounts far beneath the fair value of the case.

Concerning the trade-off between costs and accuracy of dispute resolution, the model shows that the overall cost of dispute resolution (aggregated costs of litigation plus costs of inaccuracy) is decreasing in the unit litigation cost (provided that the conjectural equilibrium settlement amount is not very low). This suggests that litigation costs may be used as a leverage to favour not only more, but also more accurate, settlements.

This paper is organized as follows. Section 2 introduces a Bayesian model of repeated consumer mass disputes involving a single defendant (firm) and multiple plaintiffs (consumers); these latter are heterogeneous with respect to their risk aversion. The defendant formulates a settlement offer on the base of some conjectures and learns from the reaction of consumers to its settlement offers. Section 3 explains and derives the conjectural equilibrium (CE) concept to find both the CE share of consumers who settle the case and the CE settlement amount. In Section 4 we analyze the social welfare trade-off emerging between the goal of minimizing social costs of litigation and the aim of maximizing dispute resolution accuracy. We also briefly discuss a model extension where victims can proceed in court not only individually but also through an aggregate procedure. In the Conclusions, we comment on the implications of our model. Proofs and analytical steps are presented in detail in the Appendix.

2. The Model.

2.1. Setup and Hypotheses

Consider a firm D supplying a large population of consumers. At each date t , the firm faces analogous consumer mass disputes. Disputes are comparable because involve both the same (kind of) *mass of consumers*—the victims—and the same *legal objective merit*. The hypotheses below characterize the model: *Assumptions* 1-2 relate to the individual dispute between the firm and an individual consumer i , *Assumptions* 3-4 describe the population of victims, *Assumptions* 5-7 concern the defendant-firm.

Assumption 1 (Litigants; timing; remedies). There is a *continuum* of individuals indexed by $i \in (0,1)$, repeatedly suffering *similar* damages at every date $t \geq 0$; time is discrete. The defendant and the mass of victims stay the same at every date.²⁰ After suffering a damage at the beginning of date t , each victim can try to recover her/his loss at the same date t . Each victim i can choose among three routes at every date, depending on which of them gives her/him the highest utility:

- Filing an individual lawsuit to the court: in this case i 's reparation, *net of litigation costs*, is $r_{i,t} \in \mathbf{R}$. $r_{i,t}$ is a random variable due to the uncertainty of judicial decision. Assumption 2 elaborates on this.
- Accepting a take-it-or-leave-it settlement amount $s_t \geq 0$ possibly offered by the defendant: s_t is the same for all individuals willing to accept it; it is paid for certain, and cannot be negotiated. Settlement implies a null litigation cost for both the defendant and the victims.
- Staying apathetic and declining both the above routes.

Assumption 2 (Net reparation and cost allocation).

(a) $r_{i,t} \sim N(\rho, \sigma^2)$, $\forall i, t$; $r_{i,t}$ is i.i.d. across individuals and in time; ρ corresponds to the *true* value at stake corresponding to the *legal merit* of the case; $\rho > 0$.

(b) Total litigation costs of both parties $c > 0$ are shifted to the loser (English rule). We finally assume that $c < \rho$: total litigation costs are lower than the expected damage at stake.

Assumption 2a captures the fact that:

(i) There is a sort of average ‘consensus’ on the part of judges about the damage at stake; randomness depends on some idiosyncratic attitude —or ‘tremble’— of judges, and does not depend on time, nor on the identity of the victim. It is reasonable assuming the same distribution for each case since disputes emerging among consumers are analogous and origin from the same kind of cause of action.²¹

(ii) judges decide *correctly*, on average.

(iii) the assumption that $\rho > 0$ means that the case is *meritorious* in expected terms.²²

Assumption 2b is related to the fact that in the majority of the judicial systems, fee shifting in favour of the prevailing party applies, at least to some extent (Reimann, 2012). Moreover, even in the United States, where the general rule is that each party pays for its own attorney, there are many exceptions: several statutes at both the federal and state levels allow the winner to recover reasonable attorney's fees. This is particularly true in the case of consumer protection laws (Cohen, 2008).²³ A further reason why we opted for this setup is that, according to this assumption, litigation costs neither represent an obstacle to the commencement of a lawsuit by the victim, nor are an intrinsic incentive for the victim to settle: this allows us to focus on the effects of risk aversion as a force able to hinder in-court-litigation. Finally, note that introducing certain litigation costs that are allocated according to the American rule, only implies a smaller net value at stake for the victim. Our results would change only qualitatively.

²⁰We can also imagine that individuals change but the mass of victims who the firm faces has always the same features that are embodied by Assumptions 3 and 4.

²¹ The normality assumption is only a convenient approximation: while $r_{i,t}$ can well be negative, especially if a judge decides against the plaintiff and charges him for litigation costs, one does not expect that $r_{i,t}$ diverges to $-\infty/\infty$. However, if the mean ρ is large enough with respect to the variance, σ^2 , this is an acceptable approximation.

²²About the various definitions of meritorious lawsuits vs frivolous lawsuits, see Bone (1997). Often the concept of positive expected value lawsuit overlaps with the idea of a meritorious case, conversely negative expected value lawsuit corresponds to the idea of a frivolous case.

²³ See also (among other state law for consumer protection) the Californian Consumers Legal Remedies Act and the Maryland Consumer Protection Act.

In order to capture heterogeneity of the mass of victims, we assume that each victim is characterized by his/her own specific utility function. Heterogeneity concerns individual risk aversion that is measured relatively to the maximum degree of risk aversion present in the victims' population.

Assumption 3 (Victims' utility). Given any non-negative monetary sum m , the constant absolute risk aversion utility function²⁴ of victim i is $U_i = A - e^{-\gamma_i \delta m}$, with $A > 0$, $0 \leq \gamma_i \leq 1$ as the individual scale factor of risk aversion, and $\delta > 0$ as the maximum degree of risk aversion in the population. For simplicity, A and δ are assumed to be equal across individuals.

Now, define $q(\gamma)$ as the share of the victim population formed by individuals whose scale factors γ_i are lower than or equal to γ , $0 < \gamma \leq 1$.

Assumption 4 (Risk-aversion distribution across victims). $q(\gamma)$, obeys the *Kumaraswamy cumulative distribution function* (CDF)²⁵ with parameters g and h both greater than 1: $q(\gamma) = F(\gamma; g, h) = 1 - (1 - \gamma^g)^h$.

Despite its specificity, the Kumaraswamy distribution is versatile and easy to treat: it is very closely related to the Beta distribution (already used in previous analogous law and economics contributions²⁶). Like the Beta distribution, it is defined on the closed interval $[0, 1]$ and can take many different shapes depending on the parameter values. However, its CDF has a much simpler form than the Beta CDF. If both g and h are greater than one, the PFD is unimodal and the CDF is S-shaped. The location of the mode is increasing in g and decreasing in h .

Let us now focus on the defendant firm. At each date, it might be asked to bear outlays due to both damage compensation plus litigation costs for those cases that are filed in court (and then decided in favour of the plaintiff) and settlement (if any individual does accept it). Defendant's objective is to choose that settlement amount that minimises the expected total outlays.

As we did for the victims, we also characterize the defendant firm with respect to its information set and risk attitude. In particular,

Assumption 5 (Defendant knowledge and risk attitude)

(a) The defendant D knows ρ , c and the probability distribution of the net reparation.

²⁴ Constant absolute risk aversion (CARA) is a very general class of utility functions that are usually used in practice because of their mathematical tractability. The constant absolute risk aversion is measured by $\gamma_i \delta$. Spier and Prescott (2016) use CARA utility functions to model settlement and litigation under risk aversion.

²⁵ See Kumaraswamy (1980). The probability density function (PDF) of the Kumaraswamy distribution is $f(\gamma; g, h) = gh\gamma^{g-1}(1 - \gamma^g)^{h-1}$.

²⁶See Dari-Mattiacci (2007) and Rampa and Saraceno (2016).

(b) The defendant knows that victims are variously risk averse, but does not know individual victims' attitude towards risk. Specifically, it does not know victim's individual utility function (Assumption 3) nor risk-aversion distribution across victims (Assumption 4).

(c) The defendant is risk-neutral.

Assumptions 5a along with the fact that disputes are assumed to be meritorious in expected terms and cost are allocated according to the English rule (Assumption 2) implies that defendant know that the *gross* reparation that is paid to pay to each victim i in case of in-court-litigation is $R_{i,t} = r_{i,t} + c$ where $R_{i,t} \sim N(r_D, \sigma^2)$, where $r_D = \rho + c$. Note that, by Assumption 2b, $\rho < r_D < 2\rho$.

It is sound to assume 5b, since, although consumers are usually risk averse, a firm supplying a large mass of consumers cannot know each counterparty who is involved in a mass dispute. Even knowing the precise risk-aversion distribution across victims might be very difficult. The sense of this is clear: the defendant of a mass dispute involving many victims typically acts with much less than complete information.²⁷ It is worth noticing that, in our setup, the defendant's knowledge limitations about victims do not imply any strategic bargaining under asymmetric information because settlement is not negotiable (as it actually happens in other models with take-or-leave-it offers). Moreover, the usual common-prior assumption used in standard models is here absent.²⁸ On the other hand, as shown in the next section, knowledge constraints will lead to a certain level of litigation because the defendant might be unable to set its settlement offer to a level that satisfies the most "demanding" victim. Here, the focus is on the fact that the (professional) defendant can learn from experiencing settlements and trials in repeated mass disputes. We elaborate on this in Section 3.

Risk neutrality (5c) is a very common assumption; furthermore, it is well accepted for business agents, especially when dealing with many counterparties.²⁹

Given its limited knowledge of victims, in order to define its own optimal behaviour, the defendant needs to make some conjecture about how plaintiffs will react to any given settlement offer. Once the offer is bidden, though the defendant still ignores risk aversion of its victims, it can observe the aggregate victims' reaction (acceptance/rejection); then it can update its conjecture by considering the share of victims who actually accepted the settlement offer at t . In each subsequent time ($t+1$), the defendant defines its optimal offer based on the conjecture that has been updated accounting for what it observed at the previous time t (De Groot, 1970).³⁰ The two following assumptions describe the process in a quite intuitive way.

²⁷ Behavioral economics and Macroeconomics are devoting attention to models that are characterized by radical incomplete information, as opposed to Harsanyi's (1967-8) incomplete information. See e.g. Angeletos and Lian (2016).

²⁸ Law and economics literature has extensively dealt with the strategic implications of asymmetric (or private) information on settlement bargaining. Depending on the bargaining protocol (take-it-or-leave-it, simultaneous bids to a mediator, etc.), the nature of private information (probability of succeeding, amount at stake, investment in legal advising, etc.), and the side of information asymmetry (one-side, two-side asymmetries), implications change, especially in terms of dispute selection for trial; on the other hand, the general insight is that asymmetric/private information is the main cause of settlement failure. Readers interested in comprehensive reviews of settlement bargaining according to a traditional law and economics approach are referred to Daughety and Reinganum (2012) and Wickelgren (2013).

²⁹ The number of theoretical contributions on litigation and settlement providing models where litigants are assumed to be risk-neutral is boundless.

³⁰ This approach extends Rampa and Saraceno (2016) to the process of conjecture-updating on the part of the defendant in relation to the observation of victims' behavior in settlement.

Assumption 6 (Defendant's conjecture). Given Assumption 5a, the defendant guesses anyway that the number of victims who accept a settlement (file a lawsuit) is increasing (decreasing) in the settlement amount offered, s . More precisely, it guesses the following:

- 6.1 Settlement acceptance.** The agent believes that, for given *positive* values of b and s , and for given value of a , the conditional distribution of the share q_S of individuals who accept the settlement is approximated by a normal variable with mean $q_S = a + bs$ and known precision, 1 for simplicity.
- 6.2 Lawsuit share.** The defendant believes that the share of individuals who file a lawsuit is $q_L = 1 - q_S$.
- 6.3 Minimal coherence.** Besides $b > 0$, the defendant believes $a + br_D > 0$ and $br_D > a$.
- 6.4 Independence.** q_L and q_S are independent of $R_{i,t}$, for any i and t .

The linearity and normality assumption of 6.1, namely resorting to OLS, corresponds to a common practice of applied scholars and observers when facing new data, at least as a first step.³¹ Assumption 6.2 is reasonable; as clarified in Section 3, since the settlement offer is nonnegative, then the defendant expects that a plaintiff who rejects its offer does that because expecting to obtain more utility from trial. Assumption 6.3, can be easily understood by remembering that r_D is the expected outlay for any single lawsuit (and therefore the limit-amount for settlement): the part $a + br_D > 0$, while being obvious for $a > 0$ due to the positivity of b , means that a high settlement amount ($r_D > \rho$) is expected to be able to induce at least some individuals to accept the offer. The part concerning $br_D > a$ (obvious when $a < 0$ given the positivity of b) means that the *additional* effect of offering a high settlement amount ($r_D > \rho$) with respect to setting $s = 0$ is greater than the number of individuals who are induced to settle by a very low amount $s \approx 0$. In other words, we are assuming that there is enough room for settlement. Finally, Assumption 6.4 simply relates to the fact that gross reparations depend on judges' trembles on the case, not on victims' and defendant's characteristics.

Assumption 7 (Defendant's prior and learning).

(a) The defendant does not know a , b . Its prior distribution on the couple (a, b) is a normal bivariate, and the mean and precision hyper-parameters of this distribution at date t are the following vector \mathbf{z} and symmetric “precision” matrix \mathbf{H} ³²:

$$\mathbf{z}_t = \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix}, \quad \mathbf{H}_t = \begin{bmatrix} \eta_{1,t} & \eta_{12,t} \\ \eta_{12,t} & \eta_{2,t} \end{bmatrix},$$

with α_t and β_t as the defendant's conjecture on a and b , respectively, at t ; β_t , $\eta_{1,t}$, and $\eta_{2,t}$ are positive, while $\eta_{21,t}$ is non-negative. We assume also $\eta_{12,0} = 0$. Finally, we define $\eta_{i,0} = \eta_i$ for simplicity.

(b) After the announcement of the settlement amount at date t , s_t , the defendant can observe the *actual* share $q_S(s_t)$ of individuals who accept the settlement. Hence, it can update its prior along Bayesian lines. We call “prior” the conjecture at date t , and call “posterior” the updated conjecture after the victims' behaviour at date t has been observed. The posterior at date t is to be interpreted as the prior conjecture at date $t + 1$.

³¹ The OLS or *linear least squares* is a standard method for estimating the unknown parameters of a linear equation modeling the relationship between a dependent variable (here the share of victims who accept settlement) and one independent variables (here the settlement amount).

³² *Precision (or robustness) of the subjective prior* is the term typically used in Bayesian statistics. Precision is somehow inversely measured by the variance.

Assumption 7a depends on Assumption 6.1 that the variance of q_s is unitary and implies that the form of the precision matrix \mathbf{H}_t is a special one (see De Groot, 1970, chapter 11); this, however, is immaterial for our results, as we shall see, due to the agent's risk neutrality. The assumption $\eta_{12,0} = 0$ is quite reasonable, since the agent has no reason to hypothesise any specific initial value for co-variances.

2.2. Parties' choices

Given the Assumptions 1-7, we are now able to characterize parties' behavior. At each time t , both the victims and the defendant are interested in maximizing their individual expected utility.

Victims' choices

Let us start with victims' behaviour. If victim i gives up, i.e. he/she neither files a lawsuit nor accepts a settlement, her utility is $U_i = A - 1$. On the other hand, the utility drawn from accepting the settlement offer is certain: hence, if i accepts settlement offer $s_t \geq 0$, the resulting utility is $U_i = A - e^{-\gamma_i \delta s_t}$. Finally, the utility of filing a lawsuit is uncertain: hence, an individual computes the *expected* utility of filing a lawsuit at date $t \geq 1$, and this is done on the basis of Assumptions 1 and 2. Therefore, the individual expected utility of filing a lawsuit at time t is $E(A - e^{-\gamma_i \delta r_{i,t}}) = A - e^{-\gamma_i \delta \rho + \frac{\gamma_i^2 \delta^2}{2} \sigma^2}$ (Details in Appendix A.1.). The latter expression illuminates the role of risk aversion. Given the mean ρ and the risk-aversion parameter $\gamma_i \delta$, an increase in variance depresses the expected utility. Furthermore, a higher risk-aversion parameter depresses the expected utility if $\sigma^2 > \frac{\rho}{\gamma_i \delta}$, that is if the variance is relatively important. See Figure 1.

Figure 1

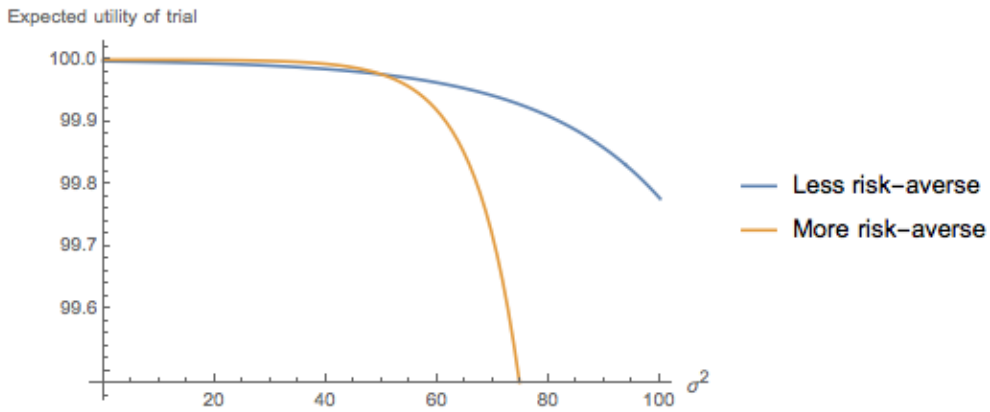


Figure is for $A=100$, $\rho=20$, $\gamma_i \delta=0.3$ (blue line), 0.5 (yellow line).

This given, one easily checks that victim i will decide as follows at each date $t \geq 1$:

1. If $\rho - (\gamma_i \delta \sigma^2 / 2) \leq 0$ and $s_t = 0$, then he/she neither files a lawsuit nor accepts the settlement.
Otherwise:
2. If $s_t > \rho - (\gamma_i \delta \sigma^2 / 2)$, he/she accepts the settlement;
3. If $s_t \leq \rho - (\gamma_i \delta \sigma^2 / 2)$, he/she files a lawsuit.

It is worth noticing that it is *not* the case that all individuals choose the same option, since individual decisions depend on individual risk-aversion parameters. On the one hand, if $s_t \geq \rho$, nobody will file a lawsuit and

everybody accepts the settlement. On the other hand, if $s_t < \rho$, it might be that a lower-than-one share of individuals accept the settlement. It can well happen also that the share of individuals accepting the settlement remains empty even if the settlement sum is positive. According to case (3), this happens when, for the most risk-averse individual ($\gamma_i = 1$), $\rho - (\delta\sigma^2/2) \geq s_t > 0$. Finally, the set of those filing a lawsuit might be non-empty even if the settlement sum is zero. Supposing indeed that $s_t = 0$ and that there are victims such that $\rho - (\gamma_i\delta\sigma^2/2) > 0$, those individuals who are characterized by $\gamma_i < \frac{2\rho}{\delta\sigma^2}$ will file a lawsuit. Given Assumption 4, the share of victims who, thus, go in court is $q_L(s_t = 0) = F\left(\frac{2\rho}{\delta\sigma^2}; g, h\right)$. In this case, since settlement is null, $1 - q_L(0)$ is the share of victims who give up (case 1). However, as soon as s_t becomes positive, all the victims who do not go in court suddenly accept the settlement offer. Therefore, for $s_t > 0$, the share of victims who accept settlement is $1 - q_L(s_t) = q_S(s_t)$.

Now, focus on the settlement condition (2): it can be written as $\gamma_i \geq \frac{2(\rho-s_t)}{\delta\sigma^2}$. Given Assumption 4, the share of victims who accept the settlement offer in t is $q_S(s_t) = 1 - F\left(\frac{2(\rho-s_t)}{\delta\sigma^2}; g, h\right) = \left(1 - \left(\frac{2(\rho-s_t)}{\delta\sigma^2}\right)^g\right)^h$. Note that one requires $\frac{2(\rho-s_t)}{\delta\sigma^2} \leq 1$, i.e. $s_t \geq \rho - \frac{\delta\sigma^2}{2}$, otherwise we set $q_S(s_t) = 0$. These implications are summarized by the following fact:

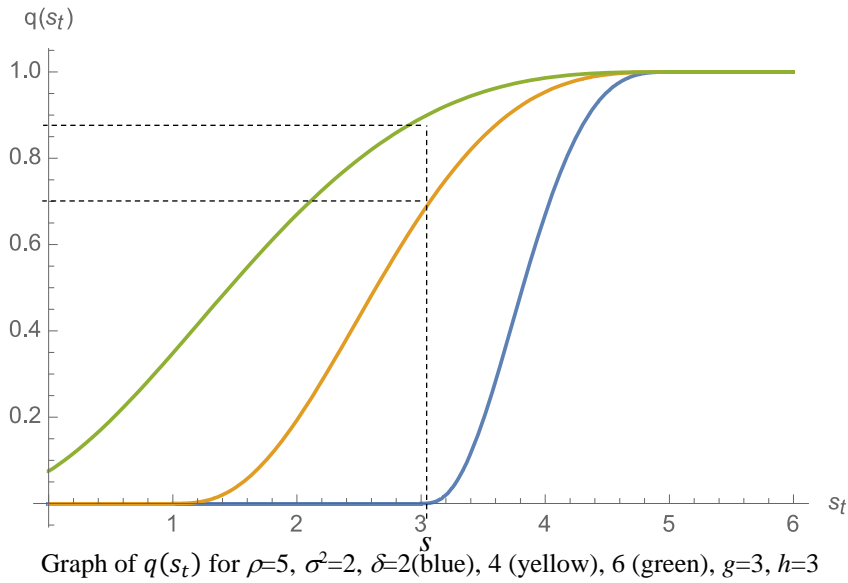
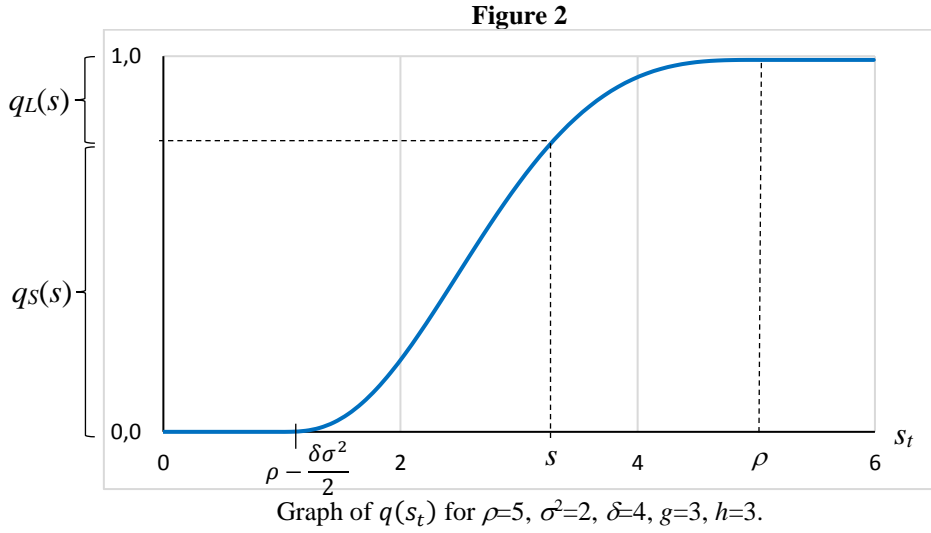
FACT 1 (a) *Given Assumption 4, the share of victims who accept the settlement offer in t is:*

$$q_S(s_t) = \begin{cases} 0 & s_t < \rho - \frac{\delta\sigma^2}{2} \\ \left(1 - \left(\frac{2(\rho-s_t)}{\delta\sigma^2}\right)^g\right)^h & \rho - \frac{\delta\sigma^2}{2} \leq s_t \leq \rho \\ 1 & s_t > \rho \end{cases} \quad (1)$$

(b) $q_S(s_t)$ is strictly increasing for $\rho - \frac{\delta\sigma^2}{2} \leq s_t \leq \rho$. Furthermore, for a given s , $q_S(s)$ is increasing in both the maximum degree of risk aversion (δ) and the variance of the trial outcome (σ^2), conversely is decreasing in the expected merit of the case (ρ). A positive intercept (for $\rho - \frac{\delta\sigma^2}{2} < 0$) can be interpreted as the share of victims who give up when the settlement offer is null.

Figure 2 (upper panel) depicts a typical graph of $q_S(s_t)$ for given values of the parameters: in correspondence to a generic settlement offer s , the share of victims who accept the settlement offer is determined; $q_L(s) = 1 - q_S(s)$ is the share of individuals who file a lawsuit. Figure 2 (lower panel) depicts $q_S(s_t)$ for given values of the parameters, varying δ .

We conclude that the higher the maximum degree of risk aversion in the population, the higher the share of population who accept a given settlement offer. Clearly, variability in trial outcomes (σ^2) has a similar impact. Finally, ceteris paribus, consumers are more prone to accept a given settlement offer for small claims (low ρ).



The defendant's choice

Now, consider the defendant's problem. Given Assumptions 5-7, the defendant's expected total outlays $O_t(s)$ resulting from a settlement amount s that it offers at date t are:

$$E_{D,t}[O_t(s)] = (\alpha_{t-1} + \beta_{t-1}s)s + (1 - \alpha_{t-1} - \beta_{t-1}s) \cdot r_D$$

The expectation is taken with respect to the prior formed at date $t-1$; specifically, the defendant's conjecture at $t-1$ about the share of victims who will accept the settlement offer at t is $(\alpha_{t-1} + \beta_{t-1}s)$, while the complement to 1 is the defendant's conjecture about the share of victims who will go in court. The second addend of the expected total outlays derives from the independence assumption 7.4.³³

Given its risk-neutrality, the defendant chooses the settlement amount that minimises the expected total outlays. Hence (as proved in Appendix A.3):

³³The expectation of the product is the product of the expectations.

FACT 2 (a) At date t the defendant chooses:

$$s_t = \frac{\beta_{t-1}r_D - \alpha_{t-1}}{2\beta_{t-1}} \quad (2)$$

(b) $0 < s_t < r_D$: For the defendant, it is optimal to offer less than the expected outlay for any single lawsuit; meanwhile, it is optimal to offer a positive settlement amount. For $\alpha_t > 0$ one has $s_t < \frac{r_D}{2}$, while for $\alpha_t < 0$ one has $s_t > \frac{r_D}{2}$.

(c) Recalling Assumption 5b and the optimal behaviour of risk averse consumers, we must conclude that $s_t \leq \rho$.

(d) The optimal settlement offer in (2) corresponds to a forecast about the share of individuals who will accept the settlement at t :

$$q_{S,t}^e = \frac{\alpha_{t-1}}{2} + \frac{\beta_{t-1}r_D}{2} \quad (3)$$

Recall that, by Assumption 7.2, the defendant's forecast about the share of individuals who file a lawsuit is $q_{L,t}^e = 1 - q_{S,t}^e$.³⁴

The defendant's learning

As we explained above, once the settlement amount s_t is offered at date t , all victims decide (accept/reject) as described above, and then the defendant can observe the *actual* share $q_S(s_t)$ of individuals who accept its offer. Although the defendant cannot yet perfectly infer victims' risk aversion, it updates its prior along Bayesian lines (Assumption 7). Posterior is interpreted as the prior of date t , that the defendant uses for its choice at date $t+1$.

Define the vector $\mathbf{x}'_t = [1 \quad s_t]$, i.e. the vector of the “regressors” of the equation $q_{S,t} = \alpha_t + \beta_t s_t$ that the defendant recursively estimates. Under our assumption, the updated hyper-parameters (De Groot, 1970, chapter 11) are as follows:

$$\mathbf{z}_t = [\mathbf{H}_{t-1} + \mathbf{x}_t \mathbf{x}'_t]^{-1} [\mathbf{H}_{t-1} \mathbf{z}_{t-1} + \mathbf{x}_t q_S(s_t)] \quad \text{and} \quad \mathbf{H}_t = \mathbf{H}_{t-1} + \mathbf{x}_t \mathbf{x}'_t \quad (4)$$

By inspecting (4) — after few passages — we conclude the following:

FACT 3 (a) $\mathbf{z}_t = \mathbf{z}_{t-1} + [\mathbf{H}_{t-1} + \mathbf{x}_t \mathbf{x}'_t]^{-1} [\mathbf{x}_t (q_S(s_t) - \mathbf{x}'_t \mathbf{z}_{t-1})]$: that is, the updated parameters are equal to the previous ones, plus the term $[\mathbf{x}_t (q_S(s_t) - \mathbf{x}'_t \mathbf{z}_{t-1})]$, containing a forecast error $(q_S(s_t) - \mathbf{x}'_t \mathbf{z}_{t-1})$, ‘deflated’ by the inverse of the (updated) precision matrix.

(b) The precision matrix \mathbf{H}_t ‘grows’ in time (in the sense of the positive-definite-matrix ordering). Hence, the updating process slows down over time.

In other terms, on the one hand, if and only if the defendant observes $q_S(s_t) = \mathbf{x}'_t \mathbf{z}_{t-1} = \alpha_{t-1} + \beta_{t-1} s_t$, then $\mathbf{z}_t = \mathbf{z}_{t-1}$. On the other hand, since the precisions grow, the updating process gets slower and slower. Finally,

³⁴ Note that, since the case is meritorious, the defendant's forecast is consistent with the effective share of people who file a lawsuit.

the defendant does *not* learn asymptotically any ‘true’ parameter, as we shall presently see.

2.3. Preliminary implications of the model

The model above seems to effectively gather most of the dynamics of consumer mass disputes. On the one hand, risk aversion and uncertainty of judicial decisions favour settlement; the latter seems to be particularly appealing for consumers when involved in small expected redress claims. On the other hand, though the defendant has incentives to offer a positive amount to settle the emerging disputes (since cases are meritorious in expected terms), the optimal offer is not only underneath the expected outlay for a single lawsuit but even smaller than the fair value of the case. This is an advantage for the defendant, but may imply adverse consequences of settlement in terms of optimal deterrence and fair redress, depending on the “distance” between the legal merit of the case and the actual settlement amount. Finally, although the defendant never *perfectly* learns how the heterogeneous population of its consumers will react to a given settlement offer (even asymptotically – see Fact 3–), settlement experience leads the defendant to adjust offers over time as long as the learning process provides further elements such as to change the offer. In the next section, we characterize those *equilibrium* situations when, for a given population of damaged consumers and a given type of disputes, settlement offers crystallize in their stationary state (namely, when no further learning takes place).

3. Conjectural Equilibria

We study how the defendant’s settlement offers, the shares of victims who either settle the case or litigate in court, litigation costs and accuracy of legal outcomes interrelate, resulting in aggregated costs of dispute resolution. To this end, we focus on those situations when the acceptance rate stops being informative for the defendant and the settlement offers crystallize in their *stationary state*.

On the one hand, the defendant’s priors determine its behaviour resulting in a given settlement offer. On the other hand, the interaction between parties’ choices described in Section 2.2. determines the share of victims who accept to settle. In turn, the latter can induce the defendant to adjust future offers according to its posterior; posteriors correspond to the defendant’s priors that are updated depending on the experienced settlements. We term this dynamical system *learning dynamical system* and focus on its *stationary states*, i.e. those configurations such that, if the system is set in one of these positions, stays there *forever*. In some well-defined sense, we can interpret these positions as *conjectural equilibria* (*CE*, hereafter): in fact, no force displaces the system from any of these positions.

3.1. CE characterisation

Although in Appendix A.4 we provide a *formal* definition of *CE*, let us introduce the concept intuitively. When working with a dynamical system characterized by a set of state-variables, the stationary states of this dynamical system are those configurations of the systems’ variables such that the system stays there forever. As we saw at the end of Section 2.2, the choice of the defendant depends uniquely on the mean hyper-parameters of its conjecture. Hence, if these parameters remain unchanged from date t to date $t+1$, the

defendant's choice (the settlement offer) stays unchanged as well (recall that parameters are lagged by one period with respect to the choice). On the other hand, if the settlement offer stays unchanged, also the victims' choice (and consequently the acceptance rate) remains unchanged.

We name *CE* each stationary state of our dynamical system; clearly, a *CE* is such that both hyper-parameters and also choices stay constant *forever*.

Now, we are ready to write down precisely the conditions for a conjectural equilibrium. Since the *CE* definition implies that equilibrium variables remain constant forever, we omit the time subscripts. As transparent from expression (4) and from Fact 3a, these conditions boil down to requiring that the actual observation of $q_S(s)$ on the part of the defendant confirms its expectations. Hence, the *CE* condition is represented by the following single equation:

$$q_S^e(s) = q_S(s).$$

Since we derived $q_S^e(s)$ in (3) and $q_S(s)$ in (1), we can write the condition above as $\frac{\alpha}{2} + \frac{\beta r_D}{2} = \left(1 - \left(\frac{2(\rho-s)}{\delta\sigma^2}\right)^g\right)^h$, provided that $\rho - \frac{\delta\sigma^2}{2} \leq s \leq \rho$. By substituting the actual offer s as derived in (2), the resulting *CE* condition is:

$$\frac{\alpha}{2} + \frac{\beta r_D}{2} = \left(1 - \left(\frac{2(\rho - [\frac{r_D}{2} - \frac{\alpha}{2\beta}])}{\delta\sigma^2}\right)^g\right)^h \quad (5)$$

Now, some characterisations of *CE* can be proved. Proofs are provided in Appendix (A.4).

Proposition 1.

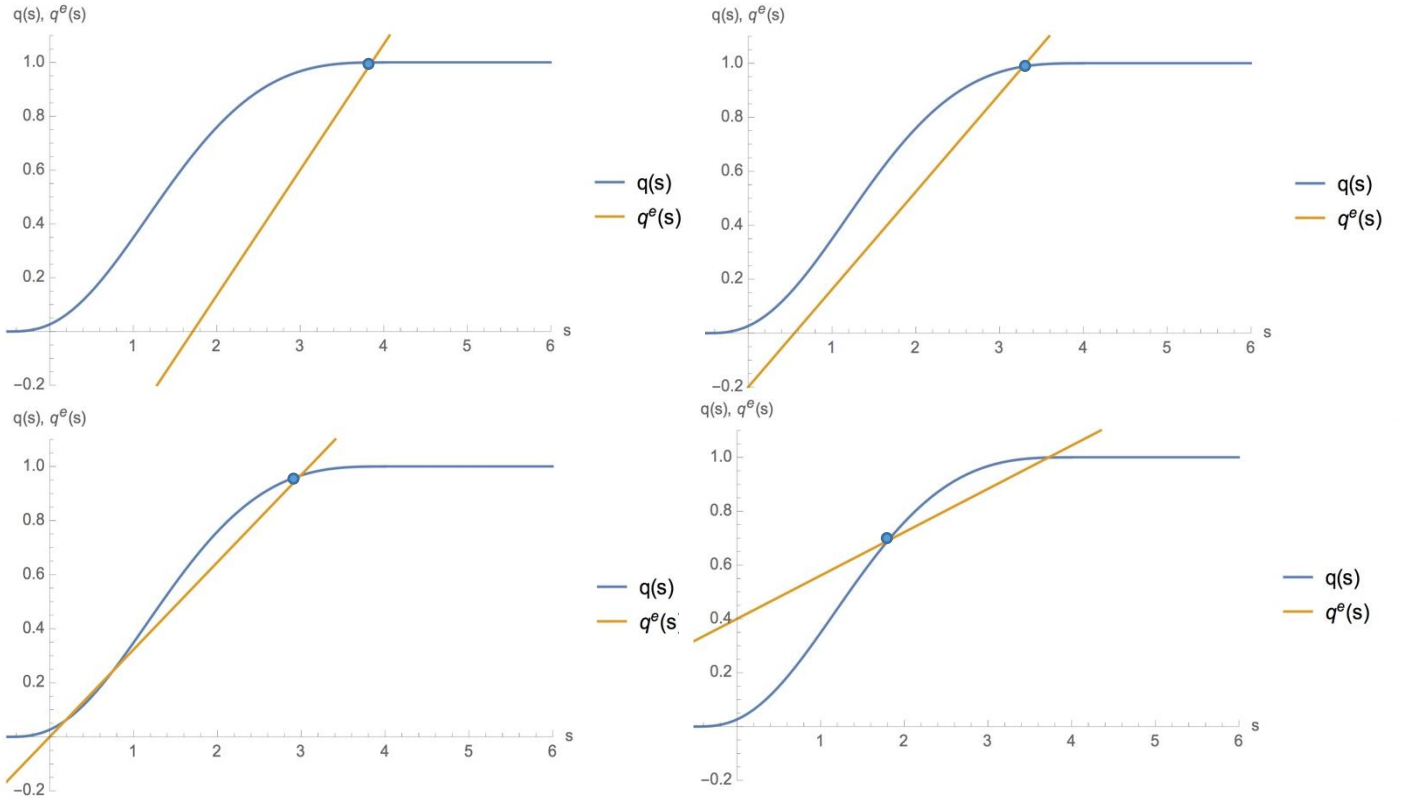
- a. *The set of CEs —the various combinations of state-variables solving condition (5)— forms a one-dimensional continuum set, the “CE set”, in the space of the defendant's hyper-parameters (α, β) .*³⁵
- b. *In this set, the equilibrium settlement amount s is always increasing in β .*
- c. *In this set, the parameters α and β always move in opposite directions: when α increases (decreases) β decreases (increases).*
- d. *The only CE such that $s = \rho$, and hence $q_S(s) = 1$, requires $\alpha < 0$.*

The term “conjectural equilibrium” is now clearer: *depending on defendant's conjectures*, we have different possible *CEs*. There is *not* a unique *CE* (nor a *finite* set of *CE*'s) of the system. Since we interpret conjectures as primitives (like preferences), there does not exist a unique equilibrium conjecture: the only requirement to have equilibrium conjectures is that these latter are confirmed by actual outcomes; this is why not all conjectures can be *CE*'s: however, a continuous subset of them are.

³⁵ For those readers who are not used to deal with very technical implications: Part a of Proposition 1 simply means that there exists an infinite number of pairs of parameters α and β satisfying (5). For “technical readers”, the *CE* set is a manifold.

As shown in Figure 3, to each different *CE* there corresponds a different equilibrium share $q_S(s)$ of individuals accepting the equilibrium settlement s (and thus of individuals filing a lawsuit). This result does *not* depend on the assumption that the defendant's conjectures are linear: *any* functional family characterized by *at least two parameters* that we might use to model conjectures will give the same general result. Finally, *which* of the infinite *CE*'s will actually establish depends on conjectures as of date zero (which can be seen as *initial conditions*) and on the learning dynamics. Figure 3 helps us to exemplify different possible *CE* situations. In each panel, the straight line represents a possible conjecture of the defendant defined as $q_S^e(s) = \alpha + \beta s$. The sigmoidal line represents $q_S(s)$ as already done in Figure 2. By definition, a *CE* requires the intersection between the two lines.

Figure 3. A sequence of possible *CE* situations



Graph of *CE*s for $\rho=4$, $c=3$, $\sigma^2=\delta=g=h=3$. From panel A to panel D *CE* parameters changes: α gradually increases and β decreases ($\alpha = -0.8, -0.2, 0, 0.4$; $\beta = 0.467, 0.362, 0.322, 0.161$).

Moving from panel A to panel D, passing through panel B and C, α gradually increases.³⁶ As made clear by Proposition 1, as α increases β decreases, and the equilibrium values of s and $q_S(s)$ decrease, as well. One might rightly observe that, for some values of α and β , there might be more than one intersection between the two lines. However, one soon realizes that in parts C and D of Figure 3 only one of them can be a *CE*. Recalling that, for a given pair of parameters α and β , the choice of the damaging agent is uniquely defined as $s = \frac{r_D}{2} -$

³⁶ For given parameters, *CE* parameters α and β can be easily calculated. Simulations are available upon request.

$\frac{\alpha}{2\beta}$, we observe that in panel C α has been assumed null; this implies that only the intersection corresponding to $s = \frac{r_D}{2}$ is a *CE*. Finally, since both the equilibrium s decreases smoothly as α increases and s must be positive and lower than ρ , the only *CE* of Part D is the one indicated in the graph.

4. A Policy Discussion of Model's Results

4.1. Policy Trade-off Analysis

Facts 1-3 besides Proposition 1 provide an interesting framework concerning the dynamics that characterizes repeated consumer mass disputes between a population of victim-consumers and a given defendant-business.

On the one hand, consumer risk aversion and variability in judicial decisions play a significant role in preventing litigation while inducing consumers to settle, even when the case is meritorious but the defendant offer is not particularly appealing (FACT 1). On the other hand, the impossibility for the defendant to know the actual demand for settlement by the consumers forces the former to define its offers on the base of conjectures. Although optimal offers are nonnegative for meritorious claims, proposed amounts are not only smaller than what the defendant expect to pay in the case of in-court-litigation but even smaller than the *true* value at stake (FACT 2). Furthermore, although the experience of previous settlements (and related rates of acceptance/rejection) can help the defendant to formulate more accurate offers (FACT 3), the dynamics between consumers and business can crystallize in a such a way where large part of disputes are settled for amounts far beneath the fair value of the case (PROPOSITION 1).

Given this picture, it becomes particularly interesting to analyse the trade-off that we introduced in Section 1, when parties' behaviour establishes in a *CE*. In particular, we are interested in analysing the trade-off between costs and accuracy of dispute resolution, by focusing on the *overall* cost of dispute resolution K . This can be measured as the sum of the aggregated costs of litigation – the first term of (6) – plus the costs of inaccuracy – the second term of (6) –.

$$K \equiv (1 - q_S)c + q_S(\rho - s) \quad (6)$$

Since we assumed that courts decide correctly on average (Assumption 2), the main leverage that can be used by the policy maker to affect the relevant policy trade-off which is described in (6) remains c .

As well known in the long-established literature on settlement, higher (individual) litigation costs favour settlement.³⁷ As proved in Appendix A.5., this is also confirmed in the instant paper by the following fact:

FACT 4 *In CE, the settlement rate q_S is increasing in litigation costs c since $\left. \frac{dq_S}{dc} \right|_{CE} > 0$.*

³⁷ The literature agreeing on that is boundless; here we only mention a recent literature review by Wickelgren (2013).

Fact 4 implies that higher litigation costs tend to curb the overall cost of dispute resolution by favouring settlement. Moreover, since settlement amount is increasing in c^{38} —this is immediately evident recalling that $s = \frac{1}{2} \left(\rho + c - \left(\frac{\alpha}{\beta} \right) \right)$ —also the distance between the fair value of the case and the settlement amount that is accepted to resolve the case (inaccuracy of the individual resolution) decreases. On the other hand, at aggregated level, when many cases are settled, inaccuracy increases because many cases are resolved for a value that is lower than their fair value. This is especially true when the *CE* settlement amount is very far from the fair value of the case.

To disentangle the trade-off implied by K , we focus on its relationship with c (details in Appendix A.6):

$$\begin{aligned} \left. \frac{dK}{dc} \right|_{CE} &= \left. \frac{dK}{d(r_D - \rho)} \right|_{CE} = \left(\frac{\partial K}{\partial (r_D - \rho)} + \frac{\partial K}{\partial q_S} \frac{dq_S}{d(r_D - \rho)} \right) \Big|_{CE} + \frac{\partial K}{\partial \frac{\alpha}{\beta}} \frac{d\frac{\alpha}{\beta}}{d(r_D - \rho)} \Big|_{CE} = \\ &= \left(\left(1 - \frac{3}{2} q_S \right) + \frac{F' \frac{2}{\delta \sigma^2} \beta}{F' \frac{2}{\delta \sigma^2} + \beta} \cdot (\rho - r_D + \rho - s) + \frac{1}{2} q_S \cdot \frac{F' \frac{2}{\delta \sigma^2} - \beta}{F' \frac{2}{\delta \sigma^2} + \beta} \right) \end{aligned} \quad (7)$$

Now, the relationship between the *overall* cost of dispute resolution and litigation costs in *CE* can be characterized:

Proposition 2

- a. In *CE*, when the settlement amount is large enough, the overall cost of dispute resolution is decreasing in the litigation costs, i.e. $\left. \frac{dK}{dc} \right|_{CE} < 0$.
- b. In *CE*, when litigation costs and the settlement amount are small enough, the overall cost of dispute resolution can be increasing in the litigation costs, i.e. $\left. \frac{dK}{dc} \right|_{CE} > 0$.

Since the settlement amount is increasing in litigation costs we can add a final consideration:

FACT 5 *When litigation costs c (that are charged on the defendant because of the English rule and the fact that the case is meritorious) increase enough to induce a sufficiently large *CE* settlement amount, overall costs of dispute resolution decrease.*

Proposition 2 underlines the enormous role of settlement in decreasing the overall cost of dispute resolution, provided that the settlement amount that is bargained by parties is large enough to avoid a systematic undervaluation of the fair value of the cases. On the other hand, if the adverse effects of inaccuracy prevail on the savings due to settlement, the latter is detrimental from a social welfare perspective.³⁹

³⁸ The unambiguous positive relation between the settlement amount and litigation costs is strongly related to the fact that we assume that fees are shifted according to the losing party and cases are meritorious.

³⁹ On this, see also Friedman and Wickelgren (2008).

Despite the assumptions of the present model, intuitions behind this result are quite general. Proposition 2 may also support the debate about the suitability of a court approval of settlement in the case of consumer mass disputes.

The implications of Fact 5 are more circumscribed since the possibility to induce a larger settlement by charging higher litigation costs rests on the fact that these costs are upon the defendant. However, this result sheds light on the fact that litigation costs have a double nature. On the one hand, they tend to distort optimal incentives (especially when courts are able to decide very accurately their cases, as it is assumed here).⁴⁰ On the other hand, higher costs charged upon the unmeritorious party (here the defendant, by assumption) may be used to contrast dilution effects due to cheap settlements.⁴¹

4.2. Aggregate procedures for Consumer Mass Litigation

In this section, we briefly discuss the possible effects induced by the introduction of aggregated procedures besides the usual individual litigation. Without claiming to be exhaustive, the instant extension analyses the advantages that are often associated to aggregate procedures without referring to any specific procedure (i.e. the U.S. class action). Aggregated procedures which apply in the case of consumer mass litigation are usually aimed at overcoming consumers' apathy; from this perspective, it must be noted that these actions are usually *solicited* by lawyers (as for class action) or by consumer associations (or analogous promoters). Victims who opt for an aggregate procedure typically cannot also file an individual lawsuit (independently on the fact that the procedure is an opt-out-like-class-action-procedure or an opt-in procedure). Individual litigation remains a feasible option for those who neither accept settlement nor participate to the aggregate procedure. The latter is appealing only if perceived as *easier* than individual litigation. For the sake of simplicity, and because we are mainly interested in the impact of risk aversion, we do not characterize aggregate procedures differently than individual litigation with respect to litigation costs. Since victims are somehow aggregated in a sole lawsuit, the case is discussed once and decided according to a single judicial decision.

Let us consider the point of view of the business-defendant. It guesses that the more appealing is the offer of the lawyer who solicits the aggregated procedure, the higher is the number of victims it will have to face in court, considering both individual trials and the aggregate procedure.

Given these points, in order to include the option for an aggregate procedure in the model, we assume the following:

Assumption 8 (Aggregate procedure).

8.1 Timing. Once consumers suffered the damage, a solicitor proposes an aggregate procedure characterized by parameter θ (discussed below); then the defendant offers s to settle the case. Finally, victims choose among aggregate procedure, settlement, and individual lawsuit.

⁴⁰ On this, see Polinsky and Rubinfeld (1988).

⁴¹ On this, see Dari-Mattiacci and Saraceno (2017).

- 8.2 Judicial decision on the aggregate procedure.** The aggregate procedure (identified by subscript AP) has the *per participant* random result which is defined as for the individual lawsuit, $r_{AP,t} \sim N(\rho, \sigma^2), \forall i, t$.
- 8.3 Litigation costs.** As in the case of individual litigation, per participant litigation costs are still $0 < c < \rho$, shifted to the loser.
- 8.4 Aggregate procedure agreement.** The solicitor publicly offers a sure amount equal to $\theta\rho$ to each victim who participates in the aggregate procedure. Given his/her costs, θ cannot exceed $\bar{\theta} < 1$. The *per participant* random revenue for the solicitor is $r_{AP,t}$.
- 8.5 Solicitor's risk attitude.** The solicitor is risk neutral.
- 8.6 Defendant's conjecture about the aggregate procedure.** The defendant knows θ . The defendant understands that the higher is θ , the higher is the number of victims it will have to face in lawsuit, either individually or mediated by the $AP(\theta)$. In particular, the defendant believes that, given any conjecture $(\alpha_{t-1}, \beta_{t-1})$, the share of those who will *accept* the settlement is reduced by the proportion $(1 - \theta)$ with respect to what would happen when the aggregate procedure is not available.⁴²

The victims' choice.

Given Assumption 8.1-8.5, the solicitor is confident he/she will get ρ on average for each victim who opts for the aggregate procedure; hence, he/she expects a gain $\rho - \theta\rho > 0$ for each participant, provided that $\theta \leq \bar{\theta}$. Given victims' utility functions (Assumption 3), victim i gets the (sure) utility $U_{i,AP} = A - e^{-\gamma_i \theta \rho}$ from the aggregate procedure $AP(\theta)$. Recalling that victim i gets $U_{i,S} = A - e^{-\gamma_i \delta s}$ if accepting the settlement while getting $EU_{i,L} = A - e^{-\gamma_i \delta \rho + \frac{\gamma_i^2 \delta^2}{2} \sigma^2}$ if filing an individual lawsuit, we conclude the following:

1. If $\theta\rho \geq s$, a share $q_{AP}(\theta\rho)$ of the victims choose $AP(\theta)$, and no one chooses to settle; the complementary share $1 - q_{AP}(\theta\rho)$ files an individual lawsuit. The shares $q_{AP}(\theta\rho)$ and $q_L(\theta\rho) = 1 - q_{AP}(\theta\rho)$ can be computed as already done in Section 2: the condition for accepting $AP(\theta)$ is $\gamma_i \geq \frac{2\rho(1-\theta)}{\delta\sigma^2}$, and $q_{AP}(\theta\rho) = 1 - F\left(\frac{2\rho(1-\theta)}{\delta\sigma^2}; g, h\right) = \left(1 - \left(\frac{2\rho(1-\theta)}{\delta\sigma^2}\right)^g\right)^h$.
2. If $\theta\rho < s$, the share $q_S(s_t) = 1 - F\left(\frac{2(\rho-s_t)}{\delta\sigma^2}; g, h\right) = \left(1 - \left(\frac{2(\rho-s_t)}{\delta\sigma^2}\right)^g\right)^h$ accepts the settlement and the complementary share files an individual lawsuit. These shares correspond to those already computed in Section 2.

⁴² We could assume any proportion that is inversely related to θ ; $(1 - \theta)$ is quite reasonable: indeed, for $\theta = 1$ the share of those *accepting* the settlement does actually go to zero. Recall that it is not worthwhile for the defendant to set $s > \rho$.

The defendant's choice

Given Assumption 8.6, the defendant understands that if $s \leq \theta\rho$ it will have to pay on average ρ to *all* victims, plus the cost c . Hence, if it can induce at least some victims to accept the settlement, it will pay less. Therefore, the expected outlay of the defendant is:

$$E_{D,t}[O_t(s)|\theta] = \begin{cases} (\alpha_{t-1} + \beta_{t-1}s)(1 - \theta)s + [1 - (\alpha_{t-1} + \beta_{t-1}s)(1 - \theta)] \cdot r_D & \text{for } s > \theta\rho \\ r_D & \text{for } s \leq \theta\rho \end{cases}$$

In order to minimize the expected outlay, the defendant offers the following settlement amount (details are provided in Appendix):

$$s_t = \begin{cases} \frac{r_D}{2} - \frac{\alpha_{t-1}}{2\beta_{t-1}} & \text{for } s > \theta\rho \\ \theta\rho + \varepsilon < \rho, \text{ (small } \varepsilon) & \text{for } s \leq \theta\rho \end{cases} \quad (8)$$

Although our stylized aggregate procedure is simple and disregards several specific procedural features, (8) provides an interesting picture. When the defendant is relatively pessimistic about the possibility of inducing victims to settle, the proposed amount is as if no *AP* exists ($s_t > \theta\rho$ means that the defendant deems that a “high” s is needed). In fact, *pessimistic* conjectures by themselves represent a driver able to induce a relatively high settlement amount. Conversely, when the defendant is relatively optimistic and might be induced to offer a relatively low amount to settle, the threat of an *AP* goads it to offer a settlement amount higher than what it would do in the absence of the *AP* alternative. The higher is the *competitiveness* of the *AP* (higher θ), the more effective is the threat of *AP* in inducing a relatively higher s .

Although limited, the present extension sheds light on the capacity of procedures that are able to reduce litigation-related risks (from the perspective of the victims) to force higher settlement amounts.

5. Conclusions

The main aim of this paper is contributing to the literature on repeated settlement-litigation dynamics between a business-defendant and a mass of heterogeneously risk-averse consumers. From the societal perspective, we focus on the trade-off between cheap but inaccurate settlement and expensive but accurate in-court dispute resolutions.

Our Bayesian-learning model shows that settlement offers evolve over time till reaching some conjectural equilibrium. Although the possibility for the firm to learn from consumer behavior can allow a better pricing of the value at stake, equilibrium settlement can be far from the *accurate* pricing of defendant misbehavior. This is due to the very existence of a multiplicity of conjectural equilibria. In addition, risk aversion of consumers plays a significant role: on the one hand, it is able to discourage litigation by inducing settlement; on the other hand, it induces victims to settle for relatively small amounts, even when the case is meritorious.

Concerning the trade-off between costs and accuracy of dispute resolution, the model shows that the aggregated costs of litigation plus costs of inaccuracy are decreasing in the unit litigation cost (provided that

the latter is upon the defendant and the conjectural equilibrium settlement amount is not too inaccurate with respect to the merit of the case). Therefore, we conclude that litigation costs may be used not only to favour settlement—as well known in the literature—but also to promote more accurate settlements of meritorious cases.

Thanks to a briefly extension, we discuss the impact of an aggregate procedure which represents an alternative means to resolve consumer disputes through the mediation of a solicitor who firsthand takes the risk of in-court litigation. According to the model, aggregate litigation is an effective threat able to push the business-defendant to negotiate a more generous settlement.

These results contribute to the debate about the concerns that the frequent use of settlement in consumer mass disputes might generate a systematic under-pricing of business misbehaviour. On the other hand, in the case of meritorious disputes, both higher litigation costs charged to the defendant and procedures (like many aggregate procedure) implying a reduction of litigation-related risks can contribute to promote more accurate settlements.

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Appendix

A.1 . From Assumption 1 it follows that, for given $\gamma_i \delta$, $\gamma_i \delta r_{i,t} \sim N(\gamma_i \delta \rho, \gamma_i^2 \delta^2 \sigma^2)$. On the other hand, it is well-known that, if r is a normal variable with mean ρ and variance σ^2 , then e^{-r} is *log-normal* with mean $e^{-\rho + \sigma^2/2}$. Therefore, we conclude that i 's expected utility of filing a lawsuit at time t is $E(A - e^{-\gamma_i \delta r_{i,t}}) = A - e^{-\gamma_i \delta \rho + \frac{\gamma_i^2 \delta^2}{2} \sigma^2}$. \square

A.2 (FACT 1)

$q_s(s)$ is studied over the interval $\rho - \frac{\delta \sigma^2}{2} \leq s_t \leq \rho$:

$$\begin{aligned} \frac{dq_s(s)}{ds} &= \frac{2^g gh \left(1 - \left(\frac{2(\rho - s)}{\delta \sigma^2}\right)^g\right)^{h-1} \left(\frac{\rho - s}{\delta \sigma^2}\right)^{g-1}}{\delta \sigma^2} > 0 \\ \frac{dq_s(s)}{d\rho} &= -\frac{dq_s(s)}{ds} < 0 \\ \frac{dq_s(s)}{d\sigma^2} &= (\rho - s) \frac{dq_s(s)}{ds} \geq 0 \end{aligned}$$

A.3 (FACT 2)

(a)

$$\min_{s_t} E_{A,t}[O_t(s)]$$

First order condition: $\beta_{t-1}s + (\alpha_{t-1} + \beta_{t-1}s) - \beta_{t-1}r_D = 0 \rightarrow s = \frac{\beta_{t-1}r_D - \alpha_{t-1}}{2\beta_{t-1}}$

Second order condition ($2\beta_{t-1} > 0$) is verified.

(b)

$s_t < r_D$: to verify that, remember that Assumption 7.3 about $a + br_D > 0$ implies $-\frac{\alpha}{\beta} < r_D$.

$s_t > 0$: to verify that, remember the part $br_D > a$ of Assumption 7.3 that implies $\frac{\alpha}{\beta} < r_D$.

(d) By replacing the optimal settlement offer provided in equation (2) into the defendant's forecast about the share of individuals who will accept the settlement at t we obtain $q_{S,t}^e = \alpha_{t-1} + \beta_{t-1}s_t = \frac{\alpha_{t-1}}{2} + \frac{\beta_{t-1}r_D}{2}$.

A.4 (PROPOSITION 1)

A learning dynamical system can be described through the notation $\mathbf{v}_t = L(\mathbf{v}_{t-1})$ whose state-variables are described by a vector \mathbf{v} . The stationary states of this dynamical system will be characterized by configurations of its variables such that the system stays there forever. The *relevant state-variables* of our learning dynamical system are $q_S(s_t)$ plus the defendant's hyper-parameters: these variables are described by the row-vector $\mathbf{v}_t = [\alpha_t \quad \beta_t \quad q_S(s_t) \quad \eta_{1,t} \quad \eta_{2,t} \quad \eta_{12,t}]$. All this given, we posit the following,

Definition 1. A *Conjectural Equilibrium (CE)* at date $t-1$ is a vector \mathbf{v}_{t-1} such that $\mathbf{v}_t = \mathbf{v}_{t-1}$ under the operation of the learning dynamical system. In other terms, it is a fixed point of the learning dynamical system $\mathbf{v}_t = L(\mathbf{v}_{t-1})$.

Clearly, a *CE* is a stationary state of our dynamical system, and it is such that, not only hyper-parameters, but also choices stay constant *forever*.

Part a

We have a single equation in two variables, α and β : hence, we have a one-dimensional set (or, technically, *manifold*) of couples (α, β) solving condition (5). \square

Part b

Taking advantage of some results obtained until now, write condition (5) as $H(\alpha, \beta) = \frac{\alpha}{2} + \frac{\beta r_D}{2} - G\left(\frac{2\rho - r_D + \frac{\alpha}{\beta}}{\delta\sigma^2}\right) = 0$, where $G(\cdot) = 1 - F(\cdot)$. This implicit equation defines the *CE* set, and the relationship between α and β in the set derives from the Implicit Function Theorem.

The partial derivatives of H with respect to α and β are $\frac{\partial H}{\partial \alpha} = \frac{1}{2} + F' \frac{1}{\beta\delta\sigma^2}$, and $\frac{\partial H}{\partial \beta} = \frac{r_D}{2} - F' \cdot \frac{\alpha}{\beta^2\delta\sigma^2}$. Hence, in

the set, we have $\frac{d\alpha}{d\beta|_{CE}} = -\frac{\partial H/\partial \beta}{\partial H/\partial \alpha} = \frac{F' \cdot \frac{\alpha}{\beta^2\delta\sigma^2} \cdot \frac{r_D}{2}}{F' \cdot \frac{1}{\beta\delta\sigma^2} + \frac{1}{2}}$. Recalling that in a *CE* $s = \frac{r_D}{2} - \frac{1}{2} \frac{\alpha}{\beta|_{CE}}$, to understand the

behaviour of the equilibrium settlement amount s in the set as a function of β , we compute $\frac{ds}{d\beta|_{CE}} = \frac{\partial s}{\partial \beta} + \frac{\partial s}{\partial \alpha} \cdot$

$\frac{d\alpha}{d\beta|_{CE}} = \frac{\alpha}{2\beta^2} - \frac{1}{2\beta} \cdot \frac{F' \cdot \frac{\alpha}{\beta^2\delta\sigma^2} \cdot \frac{r_D}{2}}{F' \cdot \frac{1}{\beta\delta\sigma^2} + \frac{1}{2}} = \frac{1}{2\beta} \left(\frac{\alpha}{\beta} - \frac{\frac{\alpha}{\beta} F' \cdot \frac{1}{\beta\delta\sigma^2} \cdot \frac{r_D}{2}}{F' \cdot \frac{1}{\beta\delta\sigma^2} + \frac{1}{2}} \right)$. After some algebra, we observe that $\frac{ds}{d\beta|_{CE}} > 0$ if $\frac{\alpha}{\beta} +$

$r_D > 0$. Given the Assumption 7.3 we conclude that $\frac{ds}{d\beta|_{CE}} > 0$. \square

Part c

To prove this part, consider $\frac{d\alpha}{d\beta|_{CE}} = \frac{F' \cdot \frac{\alpha}{\beta^2\delta\sigma^2} \cdot \frac{r_D}{2}}{F' \cdot \frac{1}{\beta\delta\sigma^2} + \frac{1}{2}}$ as derived above. The denominator is always positive, so we

concentrate on the numerator: its sign is clearly equal to the sign of $\frac{\alpha}{\beta} \frac{1}{\beta} F' \cdot \frac{2}{\delta\sigma^2} - r_D$. As long as α is negative,

we have $\frac{d\alpha}{d\beta|_{CE}} < 0$. Now, observe that $F' \cdot \frac{2}{\delta\sigma^2}$ is the derivative of the *actual* relation between settlement offer

and acceptance share $q_s(s) = 1 - F\left(\frac{2(\rho-s)}{\delta\sigma^2}; g, h\right)$ (given that $\rho - \frac{\delta\sigma^2}{2} \leq s \leq \rho$), while β is the derivative of the *conjectured* relation $q_s^c = \alpha + \beta s$.

Therefore, for positive α , as long as the conjectured relation is steeper than the actual one at a conjectural equilibrium, we still have $\frac{d\alpha}{d\beta|_{CE}} < 0$, since Assumption 7.3 guarantees that $\frac{\alpha}{\beta} < r_D$. It follows that as α increases β keeps decreasing in the *CE* set, and the equilibrium s decreases as well, as we proved in

part (b) of this Proposition. This implies that the equilibrium point keeps shifting to South-West in the (s, q_s) as showed in Figure 3.

Suppose now that, following this path, a certain point is reached such that $F' \cdot \frac{2}{\delta\sigma^2}$ becomes large enough to turn $\frac{d\alpha}{d\beta|_{CE}}$ positive. Now, as α keeps increasing, β should start increasing, and the *CE* point should move to

North-East in the (s, q_S) space. Therefore, if this were the case, one would observe a CE point already observed in the first phase (when β was decreasing), however characterized by a higher α and a higher β . Call (α_1, β_1) the parameter pair of the first observation and (α_2, β_2) that of the second observation of the *same* CE point, with $\alpha_2 > \alpha_1$. Since the CE point is the same in the two cases, one must have that $q_S^e(\alpha_1, \beta_1) = q_S^e(\alpha_2, \beta_2)$ and $s(\alpha_1, \beta_1) = s(\alpha_2, \beta_2)$. Given (3) and (2), these latter equalities correspond to $\frac{\alpha_1}{2} + \frac{\beta_1 r_D}{2} = \frac{\alpha_2}{2} + \frac{\beta_2 r_D}{2}$ and $\frac{r_A}{2} - \frac{\alpha_1}{2\beta_1} = \frac{r_A}{2} - \frac{\alpha_2}{2\beta_2}$, respectively. The second equality implies that $\alpha_2 = \alpha_1 \frac{\beta_2}{\beta_1}$; since we assumed $\alpha_2 > \alpha_1 > 0$, this would indeed imply that $\beta_2 > \beta_1$. However, the first equality, coupled with the second one and after some passages, implies $\alpha_1 \left(1 - \frac{\beta_2}{\beta_1}\right) = r_D(\beta_2 - \beta_1)$: but this *impossible*. Therefore, as α keeps increasing, $\frac{d\alpha}{d\beta|_{CE}}$ must be always negative. \square

Part d

This particular equilibrium requires $q_S^e(\alpha, \beta) = 1$ and $s(\alpha, \beta) = \rho$. Given (3) and (2), these latter equalities correspond to $\frac{\alpha}{2} + \frac{\beta r_D}{2} = 1$ and $\frac{r_D}{2} - \frac{\alpha}{2\beta} = \rho$. After some algebra, these equalities imply $\alpha = \frac{r_D - 2\rho}{r_D - \rho}$, which is negative for $\frac{1}{2}r_D \leq \rho \leq r_D$, as guaranteed by Assumptions 5a and 5b (see Page 8, top). \square

A.5 (FACT 4)

We are interested in analysing q_S with respect to c , provided that we focus on CE.

$$\frac{dq_S}{dc}\bigg|_{CE} = \frac{dq_S}{d(r_D - \rho)}\bigg|_{CE} = \left(\frac{\partial q_S}{\partial(r_D - \rho)} + \frac{\partial q_S}{\partial\left(\frac{\alpha}{\beta}\right)} \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right)\bigg|_{CE}$$

Since $q_S = 1 - F = \left(1 - \left(\frac{\rho - (r_D - \rho) + \frac{\alpha}{\beta}}{\delta\sigma^2}\right)^g\right)^h$, then $\frac{\partial q_S}{\partial(r_D - \rho)} = F' \frac{1}{\delta\sigma^2}$ and $\frac{\partial q_S}{\partial\left(\frac{\alpha}{\beta}\right)} = -F' \frac{1}{\delta\sigma^2}$.

Now, focus on $\frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)}\bigg|_{CE}$. We rewrite the CE condition presented in (5) as following:

$$\begin{aligned} \frac{\beta}{2} \left(r_D + \frac{\alpha}{\beta} + \rho - \rho \right) &= \left(1 - \left(\frac{\left(2\rho - \left[r_D - \frac{\alpha}{\beta} \right] \right)}{\delta\sigma^2} \right)^g \right)^h \\ \frac{\beta}{2} \left((r_D - \rho) + \frac{\alpha}{\beta} + \rho \right) &= \left(1 - \left(\frac{\left(\rho - (r_D - \rho) + \frac{\alpha}{\beta} \right)}{\delta\sigma^2} \right)^g \right)^h \end{aligned}$$

As we did in Part b of A.4., in order to exploit the Implicit Function Theorem, the previous condition is rewritten as:

$$H\left(\frac{\alpha}{\beta}, (r_D - \rho)\right) \equiv \frac{\beta}{2} \left(\frac{\alpha}{\beta} + (r_D - \rho) + \rho \right) - \left(1 - \left(\frac{\rho - (r_D - \rho) + \frac{\alpha}{\beta}}{\delta\sigma^2} \right)^g \right)^h$$

In order to be in the CE set (along the CE manifold) where condition (5) is verified, the following must hold:

$$H\left(\frac{\alpha}{\beta}, (r_D - \rho)\right) = \frac{\beta}{2} \left(\frac{\alpha}{\beta} + (r_D - \rho) + \rho \right) - 1 + F\left(\frac{\rho - (r_D - \rho) + \frac{\alpha}{\beta}}{\delta\sigma^2}\right) = 0.$$

According to the Implicit Function Theorem, we compute $\left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = -\frac{\frac{\partial H}{\partial(r_D - \rho)}}{\frac{\partial H}{\partial\left(\frac{\alpha}{\beta}\right)}}$. Since $\frac{\partial H}{\partial\left(\frac{\alpha}{\beta}\right)} = \frac{\beta}{2} + F' \frac{1}{\delta\sigma^2}$

and $\frac{\partial H}{\partial(r_D - \rho)} = \frac{\beta}{2} - F' \frac{1}{\delta\sigma^2}$, we conclude that:

$$\left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = \left(\frac{F' \frac{1}{\delta\sigma^2} - \frac{\beta}{2}}{F' \frac{1}{\delta\sigma^2} + \frac{\beta}{2}} \right)$$

Therefore,

$$\left. \frac{dq_S}{dc} \right|_{CE} = F' \frac{1}{\delta\sigma^2} - F' \frac{1}{\delta\sigma^2} \frac{F' \frac{1}{\delta\sigma^2} - \frac{\beta}{2}}{F' \frac{1}{\delta\sigma^2} + \frac{\beta}{2}} = F' \frac{1}{\delta\sigma^2} \cdot \left[1 - \frac{F' \frac{1}{\delta\sigma^2} - \frac{\beta}{2}}{F' \frac{1}{\delta\sigma^2} + \frac{\beta}{2}} \right] = \frac{F' \frac{2}{\delta\sigma^2} \beta}{F' \frac{2}{\delta\sigma^2} + \beta} > 0$$

□

A.6 (PROPOSITION 2 and FACT 5)

We are interested in analysing K , provided that we focus on CE .

$$\begin{aligned} K &\equiv (1 - q_S)(r_D - \rho) + q_S(\rho - s) = \\ &= (1 - q_S)(r_D - \rho) + q_S \left(\rho - \frac{r_D}{2} + \frac{\alpha}{2\beta} \right) \\ &= (1 - q_S)(r_D - \rho) + \frac{1}{2} q_S \left(\rho - (r_D - \rho) + \frac{\alpha}{\beta} \right) \end{aligned}$$

In particular, we analyse its relationship with c .

$$\left. \frac{dK}{dc} \right|_{CE} = \left. \frac{dK}{d(r_D - \rho)} \right|_{CE} = \frac{\partial K}{\partial(r_D - \rho)} + \left. \frac{\partial K}{\partial q_S} \frac{dq_S}{d(r_D - \rho)} \right|_{CE} + \left. \frac{\partial K}{\partial \frac{\alpha}{\beta}} \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE}$$

$$\begin{aligned} \left. \frac{dK}{dc} \right|_{CE} &= \left. \frac{dK}{d(r_D - \rho)} \right|_{CE} = \left(1 - q_S - \frac{1}{2} q_S \right) + \left. \frac{dq_S}{d(r_D - \rho)} \right|_{CE} \frac{1}{2} \left(\rho - 3(r_D - \rho) + \frac{\alpha}{\beta} \right) + \frac{1}{2} q_S \left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = \\ &= \left(1 - \frac{3}{2} q_S \right) + \left. \frac{dq_S}{d(r_D - \rho)} \right|_{CE} \frac{1}{2} \left(2\rho - 2(r_D - \rho) - r_D + \frac{\alpha}{\beta} \right) + \frac{1}{2} q_S \left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = \\ &= \left(1 - \frac{3}{2} q_S \right) + \left. \frac{dq_S}{d(r_D - \rho)} \right|_{CE} \left(\rho - (r_D - \rho) - \frac{1}{2} \left(r_D - \frac{\alpha}{\beta} \right) \right) + \frac{1}{2} q_S \left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = \end{aligned}$$

Recalling that that $s = \frac{1}{2} \left(r_A - \frac{\alpha}{\beta} \right)$, we obtain:

$$\left. \frac{dK}{dc} \right|_{CE} = \left(1 - \frac{3}{2} q_S \right) + \left. \frac{dq_S}{d(r_D - \rho)} \right|_{CE} (\rho - r_D + \rho - s) + \frac{1}{2} q_S \left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE}$$

From A.5. we already know that $\left. \frac{dq_S}{dc} \right|_{CE} = \frac{F' \frac{2}{\delta\sigma^2} \beta}{F' \frac{2}{\delta\sigma^2} + \beta}$ and $\left. \frac{d\left(\frac{\alpha}{\beta}\right)}{d(r_D - \rho)} \right|_{CE} = \frac{F' \frac{1}{\delta\sigma^2} - \frac{\beta}{2}}{F' \frac{1}{\delta\sigma^2} + \frac{\beta}{2}}$.

Then, we can conclude that:

$$\left. \frac{dK}{dc} \right|_{CE} = \left(1 - \frac{3}{2} q_S \right) + \frac{F' \frac{2}{\delta\sigma^2} \beta}{F' \frac{2}{\delta\sigma^2} + \beta} (\rho - r_D + \rho - s) + \frac{1}{2} q_S \frac{F' \frac{1}{\delta\sigma^2} - \frac{\beta}{2}}{F' \frac{1}{\delta\sigma^2} + \frac{\beta}{2}}$$

By inspecting $\left. \frac{dK}{dc} \right|_{CE}$, we observe that

- i. the first addend is negative for $q_S > 2/3$;
- ii. the sign of the second addend depends on its second term since the first term is always positive. The second term is negative when $s_{|CE}$ is not too much smaller than ρ ; increasing litigation costs $c = (r_D - \rho)$ guarantee that the term is even more negative;
- iii. the third addend is negative when the slope of the sigmoid function describing the probability of settlement acceptance ($F' \frac{1}{\delta \sigma^2}$) is smaller than the slope of the function describing the conjecture of the defendant about the probability that its offer will be accepted ($\frac{\beta}{2}$). This happens more and more easily when $s_{|CE}$ increases and approaches ρ from the left.

Given this analysis, and recalling that in CE a large s implies that both q_S and $\frac{\beta}{2}$ are relatively large (Proposition 1), we conclude that when s is sufficiently large then the overall cost of dispute resolution K is decreasing in s . Furthermore, given the observation ii. and the fact that s is increasing in c we also conclude that when s is sufficiently large, then K is decreasing in c .

A.6 (Aggregate procedure)

$$E_{D,t}[O_t(s)|\theta] = \begin{cases} (\alpha_{t-1} + \beta_{t-1}s)(1-\theta)s + [1 - (\alpha_{t-1} + \beta_{t-1}s)(1-\theta)] \cdot r_D & \text{for } s > \theta\rho \\ r_D & \text{for } s \leq \theta\rho \end{cases}$$

$$\min_s E_{D,t}[O_t(s)|\theta]$$

If $s > \theta\rho$: the derivative of $(\alpha_{t-1} + \beta_{t-1}s)(1-\theta)s + [1 - (\alpha_{t-1} + \beta_{t-1}s)(1-\theta)] \cdot r_D$ with respect to s is $\alpha_{t-1}(1-\theta) + 2\beta_{t-1}(1-\theta)s - \beta_{t-1}(1-\theta) \cdot r_D$. Equating this derivative to zero yields to $s_t = \frac{r_D}{2} - \frac{\alpha_{t-1}}{2\beta_{t-1}}$ for $s > \theta\rho$.

If $s \leq \theta\rho$, then the defendant should fix $\tilde{s}_t = \theta\rho + \varepsilon < \rho$ (small ε): this allows him to spend less than fixing $s_t \leq \theta\rho$.

Therefore, we conclude that the optimal settlement amount is:

$$s_t = \begin{cases} \frac{r_D}{2} - \frac{\alpha_{t-1}}{2\beta_{t-1}} & \text{for } s > \theta\rho \\ \theta\rho + \varepsilon < \rho, & (small \ \varepsilon) \text{ for } s \leq \theta\rho \end{cases}$$