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Banks' leverage behaviour in a two-agent New Keynesian model

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Abstract

In a NK model with two types of rational agents, savers and capitalists, and non-maximizing banks, financial shocks do affect the macroeconomic dynamics depending on banks' behaviour as for their leverage ratio. We first show that the level of banks' leverage - which may be imposed by banks regulation - affects the steady state level of output, employment and consumption, as might be expected in a non-Modigliani-Miller world. Different banks' behaviour after a shock has widely different effects on the macroeconomic dynamics: passive leverage results to be shock absorbing and capable of neutralizing an initial financial shock, whilst procyclical behaviour implies higher and more persistent instability and distributive effects than the constant leverage behaviour. Finally, we show that the interaction of procyclical leverage with hysteresis in output and employment stregthens the persistence of financial shocks.

JEL: E32; E44; E70; G01.

Keywords: Leverage, Procyclicality; Two-agent model; Non-maximising banks.

1 Introduction

In the aftermath of the 2007 financial crisis, the high level of leverage of financial intermediaries has commonly been identified as the main weakness of ailing advanced economies and, consequently, as one of the major causes of the crisis (Financial Stability Forum, 2009). Many observers pointed at leverage procyclicality – i.e. the increase (decrease) of leverage following an increase (decrease) of total assets value - as an amplification mechanism of business cycles upturns and downturns (Adrian and Shin, 2010), despite the presence of mildly stabilizing monetary policy. The procyclicality of leverage may indeed fuel a supply side financial accelerator complementing (or substituting for) the demand

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side financial accelerator pioneered by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) in explaining business cycle's booms and recessions. During upturns asset prices rise and - for a given value of debt - banks' leverage goes down. However, if banks target their leverage, the purchase of new assets will be financed by issuing new debt (deposits) in order to restore the targeted leverage. Such a mechanism may also work in the reverse, whenever there is a negative shock to asset prices. The financial-induced propagation mechanism of exogenous shocks may even become self-enforcing if banks let their leverage be procyclical, which happens (for instance) if banks target their capital to a fixed proportion of their Value at Risk (VaR)[1].

Despite the relevance for policy-making, most workhorse general equilibrium models routinely employed in research and policy institutions lack a suitable interaction between banks' leverage behaviour and the rest of the economy. Some work has been done to address the dynamic impact of different macroprudential policies regarding capital and/or leverage requirements (La Croce and Rossi 2014). In the present paper we explicitly model different banks' reactions to shocks hitting the real price of their assets in an upswing or in a downswing of the financial cycle, in order to explore the ensuing macroeconomic dynamics. In other words banks are allowed to either strictly abide by the regulatory requirements (that is target the prescribed leverage ratio) or to be pro-cyclical as they actually were in the last two decades both in Europe and the US.

The paper thus contributes to the literature on the role of financial factors in business cycle fluctuations, by at once bringing together and grafting some "real world" behavioural features in a NK model. The NK literature has focused on the comparison between shocks originating in the banking sector and macroeconomic shocks (Gerali et al. (2010)), particularly stressing the substantial impact of an unexpected reduction in bank capital on the real economy. However, Gerali et al. (2010) abstracts from the role played by bank's leverage behaviour. Furthermore, as most of the papers comprising a banking sector[2]. Gerali et al. (2010) differentiates between patient households and impatient ones[3], and it does not analyze the interaction between two main characters which turn around the banking world: savers and capitalists. This paper moves a step forward, by considering a

[1] The empirical evidence on banks procyclical behaviour is large. See, among many others, Adrian T., Shin H.S. (2010); Baglioni A., Beccalli E., Boitani A., Monticini A. (2013) ; Beccalli E., Boitani A., Di Giuliantonio S. (2015).

[2] Bernanke, Gertler and Gilchrist (1999) and Iacoviello (2005) pioneered the introduction of credit and collateral requirements into macroeonomic models in order to study how shocks are transmitted or amplified in the presence of financial variables. These model assume that credit transactions take place through the market and do not assign any role to financial intermediaries such as banks.

[3] This setup has become increasingly popular in the recent literature. Mankiw (2000) analyses the classic savers-spenders model of fiscal policy in which "myopic" household, who merely consume their income, co-exist with standard, intertemporally optimising households. The classic savers-spenders model has been extended by, among others, Galì et al. (2007) and Bilbiie (2008) to include nominal rigidities and other frictions to study questions ranging from the effects of government spending to monetary policy analysis and equilibrium determinacy. See for instance Eggertsson and Krugman (2012) and Monacelli and Perotti (2012). These models are variants of the RBC-type borrower-saver framework proposed in for example, Kiyotaki and Moore (1997) and extended to a New Keynesian environment by Iacoviello (2005) and Monacelli (2010); for an early analysis see Becker (1980) and Becker and Foias (1987). See also Bilbiie, Monacelli and Perotti (2013).

New-Keynesian Dynamic Stochastic General Equilibrium model - henceforth, NK-DSGE model - characterized (i) by two types of agents (savers and capitalists) who interact in the credit market, (ii) non neutral monetary policy by assumption (Ravenna and Walsh, 2006) and (iii) the possibility of unemployment, due to labour market frictions such as the existence of hiring costs (Blanchard and Galí, 2010; Abbritti, Boitani and Damiani, 2012).

Savers and capitalists differ in their risk aversion, and both agents are intertemporal maximizers. However, in our model savers do not hold shares but supply savings to banks (i.e. savings take the form of deposits), while capitalists hold both bank and firm shares. Banks' assets are loans to firms and bonds issued by firms (owned by capitalists). Banks are modeled as representative non-maximising agents who want to keep a fixed proportion between loans and bonds in the asset side of their balance sheet. Three possible behavioural rules in response to an exogenous decrease in the real value of bonds are examined. All these rules fulfill the steady state constant proportionality of banks' assets. The rules are as follows: (i) the bank accepts whatever leverage ratio is determined by the interplay of an initial shock and the aggregate dynamics (passive leverage); (ii) the bank follows a target leverage ratio (in our model the bank is assumed to target the macroprudential leverage level); (iii) the bank follows a procyclical behaviour according to which assets vary proportionally in response to a change in bank's profit. In case (i) and (iii) the bank returns to the regulated leverage level only in the long run.

The main results of the paper can be summarized as follows. As may be expected, the non-Modigliani-Miller world assumed in our model delivers a steady state equilibrium of the economy which is influenced by the level of banks' leverage. We find that higher leverage implies higher steady-state values of output and employment. As the leverage level is a macro-prudential policy instrument, our model tells us that macro-prudential policy is non neutral in a steady state. Another key finding emerges as we compare the above-mentioned different bank's leverage behaviours. If banks are passive with respect to leverage, the financial shock is entirely absorbed by a change in banks' equity and is not transmitted to real variables such as employment and output. The model confirms the intuition that the bank's procyclical behaviour after a shock implies a higher and more persistent instability effect than the constant leverage behaviour. The procyclicality of banks' behaviour tends to amplify the effects of the shock on real variables. The two agents framework allows us to also address distributive issues. A negative shock to the real value of bonds, despite an initial short-lived distributive effect from capitalists to savers, has a persistent and large distributive effect in the opposite direction - i.e. from savers to capitalists. Positive shocks have opposite and symmetric effects.

In order to explore the interaction between different "real world" features, we compare a model characterized by constant productivity with the same model under hysteresis. Empirical studies support the hysteresis hypothesis that recessions have permanent effects on the level of output[4]. We analyze the implications of hysteresis in a DSGE model in the presence of a shock to the real value of firms' bonds. Following Engler and Tervala (2016), we assume a simple learning-by-doing mechanism where demand-driven changes

[4]See Ball (2014); Blanchard et al. (2015), Fatas and Summers (2016)

in employment can permanently affect the level of productivity, leading to hysteresis in output. We show that, with hysteresis, the effect of the shock on real variables is more persistent. The proposed model allow us to capture two relevant features of contemporary economies: fluctuations in employment and unemployment and distributional effects ensuing from different patterns of banks' behaviour. The DSGE framework adopted does not allow one to mimic the sort of "financial instability hypothesis" advanced by Hyman Minsky (1992): the economy displays dynamic stability, i.e. it goes back to a steady state sometimes after a financial shock. However shock amplification due to leverage targeting and procyclical leverage is at work. The model suggests that even a constant leverage regulatory requirement is not sufficient to fully stabilise the economy and prevent distributive effects between savers and capitalists. Full stabilisation in our model-economy is only achieved when banks have a de facto anti-cyclical leverage such as the passive leverage behaviour simulated in Section 3.

The remainder of the paper is organized as follows. Section 2 spells out the model economy, while Section 3 analyses the effects of a shock to the real value of firms' bonds in our NK model with savers, capitalists and non-maximising banks. It compares the results obtained under two different bank's leverage behaviours, and then it compares results under banks' procyclical leverage in the absence and in the presence of hysteresis. Section 4 summarizes the main findings and concludes.

2 The model

This section outlines our economic environment, which consists of two types of households, savers and capitalists, both deriving utility from consumption goods and leisure. Furthermore, we consider a New-Keynesian model with imperfectly competitive goods markets and sticky prices. We follow Schmitt-Grohé and Uribe (2004) and consider a closed production economy populated by a continuum of monopolistically competitive producers. Each firm produces a differentiated good by using as an input the labour services supplied by households. We capture labour market frictions through hiring costs increasing in labour market tightness, defined as the ratio of hires to the unemployment pool (Blanchard and Galì, 2010; Abbritti, Boitani and Damiani, 2012).The prices of consumption goods are assumed to be sticky à la Rotemberg (1982). Following De Grauwe, Macchiarelli (2015), banks are implicitly included in this model, since the interest rate is the price of credit. However, in order to account for banks explicitly, we introduce non-maximising banks, which are assumed to collect deposits from savers and lend to capitalists (firms), besides holding bonds issued by firms.

2.1 Households

There is a continuum of households [0,1] indexed by j, all having the same utility function:

$$U(C_{j,t}, N_{j,t}) = \frac{C_{j,t}^{1-\sigma_j}}{1-\sigma_j} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi},$$

where $\varphi > 0$ is the inverse of the labour supply elasticity. The agents differ in their risk aversion coefficient σ_j. Specifically, we assume that there are two types of agents, $j = s, c$, $\sigma_s > \sigma_c$.

All households (regardless of their discount factor) consume an aggregate basket of individual goods $k \in [0, 1]$, with constant elasticity of substitution ε: $C_t = \left[\int_0^1 C_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$, $\varepsilon > 1$. Standard demand theory implies that total demand for each good is $C_t(k) = \left[\frac{P_t(k)}{P_t}\right]^{-\varepsilon} C_t$, where $C_t(k)$ is total demand of good k, $\frac{P_t(k)}{P_t}$ its relative price and C_t aggregate consumption.[5] The aggregate price index is $P_t^{1-\varepsilon} = \int_0^1 P_t(k)^{1-\varepsilon} dk$.

A share $(1 - \lambda)$ of households are less risk-averse. Consistent with the equilibrium outcome (discussed below) that less risk-averse agents are actually capitalists, we impose that less risk-averse agents also hold all the shares of firms and banks. They could also be labeled as "capitalists" (indexed by c).

Each capitalist chooses consumption, hours worked and shareholdings (of banks and firms), solving the standard intertemporal problem:

$$\max E_t \left[\sum_{i=0}^{\infty} \beta U(C_{c,t+i}, N_{c,t+i})\right],$$

subject to the sequence of constraints:

$$C_{c,t} + \Omega_{c,t}^f V_t^f + \Omega_{c,t}^b V_t^b \leq \Omega_{c,t-1}^f \left(V_t^f + \Gamma_t^f\right) + \Omega_{c,t-1}^b \left(V_t^b + \Gamma_t^b\right) + w_t N_{c,t} \tag{1}$$

where E_t is the expectations operator, $C_{c,t}, N_{c,t}$ are consumption and hours worked by risk-taker agent, w_t is the real wage. V_t^f is the real market value at time t of shares of firms, Γ_t^f are real dividend payoffs of these share and $\Omega_{c,t}^f$ are firms' shareholdings. V_t^b is the real market value at time t of shares in banks, Γ_t^b are real dividend payoffs of these shares and $\Omega_{c,t}^b$ are banks' shareholdings.

The Euler equations - for firms shareholdings and bank shareholdings - and the intratemporal optimality condition are as follows:

$$V_t^f = \beta E_t \left[\left(\frac{C_{c,t}}{C_{c,t+1}}\right)^{\sigma_c} \left(V_{t+1}^f + \Gamma_{t+1}^f\right)\right] \tag{2}$$

$$V_t^b = \beta E_t \left[\left(\frac{C_{c,t}}{C_{c,t+1}}\right)^{\sigma_c} \left(V_{t+1}^b + \Gamma_{t+1}^b\right)\right],$$

$$N_{c,t}^{\varphi} C_{c,t} = W_t. \tag{3}$$

The rest of the household on the $[0, \lambda]$ interval are more risk-averse (and will save in equilibrium, hence we index them by s for savers). They face the intertemporal constraint:

$$C_{s,t} + D_{s,t} \leq \frac{1 + i_{t-1}}{\pi_t} D_{s,t-1} + w_t N_{s,t}. \tag{4}$$

[5] This equation holds in aggregate because the same static problem is solved by both types of households.

The Euler equation for deposits, $D_{s,t}$, is as follows:

$$C_{s,t}^{-\sigma_s} = \beta \left(\frac{1+i_t}{\pi_{t+1}} \right) C_{s,t+1}^{-\sigma_s} \tag{5}$$

and the deviation of savers' labour supply, $N_{s,t}$, from its steady state value, N_s, depends on the deviation of the labour demand, N_t, from its steady state value, N. Savers labour supply matches demand. Hence:

$$N_{s,t} - N_s = N_t - N. \tag{6}$$

2.2 Firms

There are infinitely many firms indexed by k on the unit interval $[0, 1]$, and each of them produces a differentiated variety of goods with a constant return to scale technology:

$$Y_t(k) = z_t N_t(k) \tag{7}$$

where $N_t(k)$ denotes the quantity of labour hired by firm k in period t and z_t represents the state of technology which is assumed to be common across firms. Employment in firm k evolves according to:

$$N_t(k) = (1-\delta) N_{t-1}(k) + H_t(k), \tag{8}$$

where $\delta \in (0, 1)$ is an exogenous separation rate, and $H_t(k)$ represents the measure of workers hired by firm k in period t. Note that new hires start working in the period they are hired. At the beginning of period t there is a pool of jobless available for hire, and whose size we denote by U_t. We refer to the latter variable as beginning-of-period unemployment (or just unemployment, for short). We make assumptions below that guarantee full participation, i.e., at all times all individuals are either employed or willing to work, given the prevailing labour market conditions. Accordingly, we have:

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1-\delta) N_{t-1}, \tag{9}$$

where $N_t \equiv \int_0^1 N_t(k)\, dk$ denotes aggregate employment. We introduce an index of labour market tightness, t_t,which we define as the ratio of aggregate hires to unemployment:

$$t_t \equiv \frac{H_t}{U_t}. \tag{10}$$

This tightness index t_t will play a central role in what follows. It is assumed to lie within the interval $[0, 1]$. Only agents in the unemployment pool at the beginning of the period can be hired ($H_t \leq U_t$). In addition and given positive hires in the steady state, shocks are assumed to be small enough to guarantee that desired hires remain positive at all times. Note that, from the viewpoint of the unemployed, the index t_t has an alternative interpretation. It is the probability of being hired in period t, or, in other words, the job finding rate. Below we use the terms labour market tightness and job finding rate interchangeably. Hiring labour is costly. The cost of hiring for an individual firm is given by $g_t H_t(k)$, expressed in terms of the CES bundle of goods. g_t represents the marginal cost per hire, which is independent of $H_t(k)$ and taken as given by each individual firm.

While g_t is taken as given by each firm, it is an increasing function of labour market tightness. Formally, we assume:

$$g_t = z_t K t_t, \tag{11}$$

where K is a positive constant. The relevance of g_t in our model economy is strictly related to the extensive margin hypothesis: each firm may adjust its optimal amount of labour by recruiting additional workers and thus paying the hiring cost; the relevance of hiring costs emerges even in more general models, where extensive margin adjustments are accompanied by intensive margin adjustments, provided the first kind of adjustment does not play a trivial role. Firms may bear advertising, screening, and training costs and may incur in firing costs when protection legislation imposes legal restrictions. Vacancies are assumed to be filled immediately by paying the hiring cost, which is a function of labour market tightness. For future reference, let us define an alternative measure of unemployment, denoted by u_t, and given by the fraction of the population who are left without a job after hiring takes place in period t. Formally, and given our assumption of full participation, we have:

$$u_t = U_t - H_t = 1 - N_t. \tag{12}$$

Following Rotemberg (1982), we assume that firms face quadratic adjustment costs:

$$\frac{\gamma}{2}\left(\frac{P_t(k)}{P_{t-1}(k)} - 1\right)^2$$

expressed in the units of the consumption good and $\gamma \geq 0$.The benchmark of flexible prices can easily be recovered by setting the parameter $\gamma = 0$. The present value of current and future profits reads as:

$$E_t\left\{\sum_{i=0}^{\infty} Q_{t,t+i}\left[\begin{array}{c} P_{t+i}(k)\,Y_{t+i}(k) + P_{t+i}L_t^d(k) + P_{t+i}B_t^s(k) - W_{t+i}N_{t+i}(k) \\ -\frac{1+\rho_{t-1}}{\pi_t}L_{t-1}^d(k) - \frac{1+i_{t-1}}{\pi_t}B_{t-1}^s(k) - P_{t+i}\frac{\gamma}{2}\left(\frac{P_{t+i}(k)}{P_{t+i-1}(k)} - 1\right) \end{array}\right]\right\},$$

where $Q_{t,t+i}$ is the discount factor in period t for nominal profits i periods ahead. Assuming that firms discount at the same rate as capitalists β ($Q_{t,t+i} = \beta\left(\frac{C_{c,t}}{C_{c,t+1}}\right)^{\sigma_c}$), which is assumed to be strictly lower than β_s, implies that entrepreneurs are, in equilibrium, net borrowers. Each firm faces the following demand function:

$$Y_t(k) = \left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} Y_t^d,$$

where Y_t^d is aggregate demand and it is taken as given by any firm k. Firms choose processes prices $P_t(k)$, the desired amount of labour input $N_t(k)$, loans demand L_t^d and bond supply $B_{t\geq 0}^s$ so as to maximise nominal profits subject to the production function and the demand function, while taking as given aggregate prices and quantities $\left\{P_t, W_t, Y_t^d\right\}_{t\geq 0}$. Let the real marginal cost be denoted by:

$$mc_t = \frac{(1+\rho_t)(w_t + g_t)}{z_t} \tag{13}$$

Notice that, as in Ravenna and Walsh (2006), real marginal costs depend directly on the nominal interest rate. This introduces the so called *cost channel* of monetary transmission into the model. If firms' costs for external funds rise with the short-run nominal interest rate, then monetary policy cannot be neutral, even in the presence of flexible prices and flexible interest rates. We assume that the amount of bond supply that banks are willing to buy from entrepreneurs is constrained by the value of their collateral, exogenously fixed. The bond supply constraint is thus:

$$B_t^s \leq \bar{B}^s \tag{14}$$

The assumption on the discount factor β and of "small uncertainty" allows us to solve the model by imposing an always binding bond supply constraint for the entrepreneurs. Then, at a symmetric equilibrium where $P_t(k) = P_t$ and $N_t(k) = N_t$ for all $k \in [0,1]$, profit maximisation and the definition of the discount factor imply that:

$$\pi_t(\pi_t - 1) = \beta E_t \left[\left(\frac{C_{c,t}}{C_{c,t+1}} \right)^{\sigma_c} \pi_{t+1}^2 (\pi_{t+1} - 1) \right] + \frac{\epsilon z_t N_t}{\gamma} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) \tag{15}$$

which is the standard Phillips curve according to which current inflation depends positively on expected future inflation and current marginal cost.

The aggregate f.o.c across firms with respect to the supply of bonds reads as:

$$1 + i_t = \frac{1}{\beta} E_t \left[\pi_{t+1} \left(\frac{C_{c,t+1}}{C_{c,t}} \right)^{\sigma_c} \right] [1 + \phi_t] \tag{16}$$

This condition, together with (1) and (18), implicitly defines a supply of bonds function such as:

$$B^s = f(i_t, C_{c,t+1}, C_{c,t}, \phi_t, \pi_t)$$

where $f'_{i_t} > 0$; i.e. the supply of bonds is positively related to the policy rate of interest, all else being equal. In addition, the constraint on bond supply ϕ_t depends positively on the policy rate and takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the bond supply constraint will always bind in steady state. The aggregate f.o.c. across firms with respect to the demand for loans is:

$$1 + \rho_t = \frac{1}{\beta} E_t \left[\pi_{t+1} \left(\frac{C_{c,t+1}}{C_{c,t}} \right)^{\sigma_c} \right] \tag{17}$$

which, once again together with (1) and the (18) implicitly defines a loan demand function such as:

$$L^D = b(\rho_t, C_{c,t+1}, C_{c,t}, \pi_t)$$

where $b'_{\rho_t} < 0$; i.e. the demand for loans is a decreasing function of the interest rate on loans, all else being equal.

The firm profit function in real terms is given by:

$$\Gamma_t^f = Y_t + L_t^d + B_t^s - w_t N_t - g_t H_t - \frac{1 + \rho_{t-1}}{\pi_t} L_{t-1}^d - \frac{1 + i_{t-1}}{\pi_t} B_{t-1}^s - \frac{\gamma}{2} (\pi_t - 1)^2 \tag{18}$$

2.3 Banks and monetary authority

Banks play a central role in our model since they intermediate all financial transactions between agents in the model. The only saving instrument available to risk-averse households is bank deposits. Entrepreneurs may borrow either by applying for a bank loan or by selling its bonds to a bank.

We shall assume non-maximising identical banks in order to explore different behavioural patterns[6]. The first key ingredient in our simple banks modeling strategy is that they obey a balance sheet identity:

$$B_t^d + L_t^s = D_t + E_t^b \tag{19}$$

stating that banks can finance their bond holdings B_t^d and their loans supply L_t^s using either deposits D_t or bank equity E_t^b. For a commercial bank, the leverage is thus the ratio between its assets and equity.

$$\frac{1}{\kappa} = \frac{L_t^s + B_t^d}{B_t^d + L_t^s - D_t} \tag{20}$$

When a firm borrows money from a bank, it must pay an interest which normally exceeds the interest rates that savers receive for deposits. Hence, the cost of a loan from banks, ρ, is usually equal to the rate savers receive (here equal to the *risk-free* rate set by the central bank, i) plus a spread, x.

$$\rho_t = i_t + x_t. \tag{21}$$

The next step in the analysis is to determine what the spread between the borrowing and the deposit rate is. The aggregate real profits of banks are:

$$\Gamma_t^b = D_t - L_t^s - B_t^d + \frac{1+\rho_{t-1}}{\pi_t} L_{t-1}^s + \frac{1+i_{t-1}}{\pi_t} B_{t-1}^d - \frac{1+i_{t-1}}{\pi_t} D_{t-1}. \tag{22}$$

As for the monetary authority we shall assume the Central Bank sticks to a pure inflation targeting rule, i.e. it sets the nominal interest rate in response to fluctuations in inflation (we assume for simplicity that target inflation is one)[7]:

$$\log \frac{1+i_t}{1+i} = \phi_\pi \log \frac{\pi_t}{\pi} \tag{23}$$

2.4 Aggregation and market clearing

In an equilibrium of this economy, all agents take as given the evolution of exogenous processes. A rational expectations equilibrium is then as usually a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for

[6] At this stage of our analysis it is not relevant distinguishing between commercial and investment banks.

[7] The introduction of more complex Taylor rules, comprising some macroprudential targets, is on the research agenda.

all agents and all markets clear at any given time t. Specifically, labour market clearing requires that labour demand equals total labour supply:

$$N_t = \lambda N_{s,t} + (1-\lambda)\, N_{c,t}. \tag{24}$$

Equity market clearing implies that firms' share holdings of each capitalist are:

$$\Omega^f_{c,t} = \Omega^f_{c,t-1} = \frac{1}{1-\lambda}, \tag{25}$$

and banks share holdings of each capitalist are:

$$\Omega^b_{c,t} = \Omega^b_{c,t-1} = \frac{1}{1-\lambda}. \tag{26}$$

Finally, by Walras' Law the goods market also clears. The aggregate resource constraint specifies that produced output will be consumed or, if saved, will finance the cost of hiring (for a proof see the Appendix):

$$Y_t = C_t + g_t H_t + \frac{\gamma}{2}\,(\pi_t - 1)^2\,, \tag{27}$$

where

$$C_t = \lambda C_{s,t} + (1-\lambda)\, C_{c,t} \tag{28}$$

is aggregate consumption. All loans issued by the banks will be demanded by firms. Market clearing for loans implies:

$$L^d_t = L^s_t. \tag{29}$$

Finally, all bonds issued by the firms will be demanded by banks. Market clearing for bonds implies:

$$B^d_t = B^s_t. \tag{30}$$

2.5 Steady State

We focus on a deterministic steady state where inflation is one, i.e. at the Central Bank target. To simplify the analysis, we make the further assumption that savers supply a fixed amount of labour in steady state: $N_s = 1$. This assumption is consistent with the view of a constant employment for risk-averse households, as well as a consumption level proportional to the real wage (Galì, Lòpez-Salido and Vallés, 2004).

In the steady state the leverage ratio is assumed to be equal to $\frac{1}{\kappa}$. The values of the macroeconomic variables of interest are reported in the third and fourth column of Table (**1**) If $\kappa = 0.09$, which is consistent with the Basilea 3 requirement (Gerali et al., 2010), output and aggregate consumption are larger and the distribution (as measured by the ratio of capitalists consumption to savers consumption) is skewed towards savers. That is the level of bank's leverage influences the steady state equilibrium of the model economy and that higher leverage implies higher steady-state values of output and a lower rate of

Variable	Symbol	Value of κ	
		$\kappa = 0.03$	$\kappa = 0.09$
Invest. Cons.	C_c	2.7738	2.9795
Saver Cons.	C_s	0.7689	0.7450
Aggr. Cons.	C	0.9694	0.9684
Output	Y	0.9766	0.9756

Table 1: Steady State

inequality. This is not surprising: in a non-Modigliani-Miller world - i.e. a world with financial hierarchy - the steady state value of output depends on the spread between the interest rate on loans and the risk free interest rate. Such a spread is influenced by the level of banks' leverage. As the steady state leverage ratio is chosen by the regulator we find that banks regulators affect directly the steady state level of output and employment. However, higher leverage of banks also adds to the instability of the economy, i.e. when banks have a higher leverage a financial shock (to the real value of bonds) has a larger impact on real variables and for a longer time, as we shall show in the following section.

2.6 Bank's leverage rules

In "normal" times bonds' value is assumed to be constant and equal to B^s. The objective of the analysis below is to determine the consequences of a negative bonds' value shock[8]. Below we assume that such a shock follows the exogenous process:

$$B_t^s = \bar{B}^s + u_t^b,$$

where:

$$u_t^b = \theta_b u_{t-1}^b - \varepsilon_t^b,$$

and $\theta_b \in [0, 1)$ is a measure of the persistence of the shock.

We consider three different bank's leverage behavioural rules. We assume that all rules fulfill a common condition: the dynamics of loans supply follows the exogenous dynamics of bond supply, taking account of the equilibrium in the bonds market:

$$\frac{B_t^d}{L_t^s} = \frac{B^d}{L^s}. \tag{31}$$

We label this condition as constant assets proportion (CAP). The three behavioural rules are as follows:

1. The bank accommodates whatever leverage ratio is determined by the shock (provided the CAP condition (31) is fulfilled) and by the macroeconomic dynamic adjustment towards the steady state. Deposits are kept constant and the shock is absorbed by an increase in the bank's equity.

[8] A symmetric analysis can be carried out as for positive shocks.

2. The bank has a target leverage ratio and keeps constant the SS proportionality of assets. Thus, while the dynamic of loans supply continues to be explained by (31), the dynamic of deposits follows the constant leverage rule (hereafter, constant leverage*)*.

$$\frac{B_t^d + L_t^s}{E_t^b} = \frac{1}{\kappa}$$

   Under this behavioural rule banks continuously fulfill the regulatory requirements concerning leverage. That is banks stick to the steady state leverage ratio, adjusting loans and deposits to accommodate whatever shock to the real value of their bond holdings.

3. The bank follows a procyclical behaviour according to which future assets, A_t, vary in response to an increase of bank's profit. From equation (22) it is apparent that a change in a bank's profit is equivalent to a change in its equity, and a change in equity, given deposits and loans may only be due to a change in bonds. Therefore, a shock to the real value of bonds is equivalent to a change in profits. After a negative shock to the real price of bonds the value of total assets decreases (equity decreases) and the Value at Risk of the bank increases sharply and its insolvency becomes more likely (Adrian, Shin, 2010). If the bank targets its capital ratio to its VaR, a decline in VaR pushes the bank to deleveraging by selling assets (and reducing debts) until the desired VaR has been restored. This behaviour can be captured by the simple procyclical leverage rule (hereafter, procyclical leverage):

$$\Delta A_{t+1} = \Delta \Gamma_t^b.$$

   together with (31).

   Notice that in our model even a procyclical leverage is not allowed to be explosive (either permanently increasing or permanently decreasing). The model has a strong built-in stabilization mechanism such that banks' procyclical leverage will not be monotonic and will actually slowly converge back to its steady state regulatory level.

3 Model Dynamics

3.1 Calibration

Before we start showing our results, we briefly show the baseline calibration of the model's parameters. That calibration is summarized in the top panel of Table (2) .

3.2 The dynamic impact of a shock under alternative banks' behaviours

In this section we shall examine the different dynamic paths followed by the economy under the two types of bank behaviour listed above. We show the impulse response functions (IRFs) of our baseline model in the face of a negative firm's bond value shock. Figures (1) and (2) display selected IRFs to a 1% decrease in bonds' value. The dynamics can be

	Description	*Value*
φ	Curvature of labour disutility	5
β	discount factor	0.99
σ_c	capitalist's risk aversion coeff.	1
σ_s	saver's risk aversion coeff.	2
$\bar{B}^s$	Constraint on bond supply	0.5
ϕ_π	Taylor rule coefficient	1.5
ε	Elasticity of substitution (goods)	6
α	Index of price rigidities	0.75
κ	Inverse of leverage ratio	0.09
λ	Savers' share	0.9
δ	Separation rate	0.08
B	Level of hiring cost	0.12
μ	elasticity of productivity	0.02
ϕ_z	persistency of productivity	0.99

Table 2: Baseline Calibration

described as follows. In particular, Figure (1) shows that, if (31) holds, a decrease of bonds real value generates an equivalent decrease of loans real value. If banks follow a passive leverage behaviour, loans decrease on impact proportionally to the decrease in bonds' real value. Deposits are kept constant and the shock does not affect the policy rate and the spread. As a consequence, under passive leverage real variables are not affected: output and employment stay constant at their steady state level, whilst no distributive effect is at work. Thus passive leverage is stabilising.

If deposits can vary, and leverage is kept constant by banks, a decrease in asset real value will be absorbed both by a decrease in bank equity real value and by a decrease in deposits. The latter will be the cause of a policy rate decrease, and a consequent widening of the spread, reinforced by a shortage of loans. Notice that a shortage of loans alone wouldn't be enough to affect the spread substantially. Figure (2) underlines the real and redistributive effects of the shock when banks have either constant or procyclical leverage[9]. It can be seen that the spread increase has recessionary effects on economic activity. And if, on impact, there is a redistributive[10] effect from capitalists to savers - the loss of income for savers is smaller than that for capitalists - after few periods the redistributive effect is reversed, because an increase in firms' profits favours capitalists which increase their consumption, differently from savers who do not benefit from increased profits.

The pro-cyclical behaviour of banks definitely reinforces the impact of a shock to the real value of bonds and, as for macroeconomic real variables the impact is prolonged. On impact, leverage is free to move, and it favours an increase in deposits. But, as soon as the procyclical behaviour starts to bite, deposits decrease to guarantee an increase of

[9] Passive leverage, as already said, has no dynamic real effect.

[10] We approximate redistribution through the ratio between investors/capitalists consumption and savers consumption.

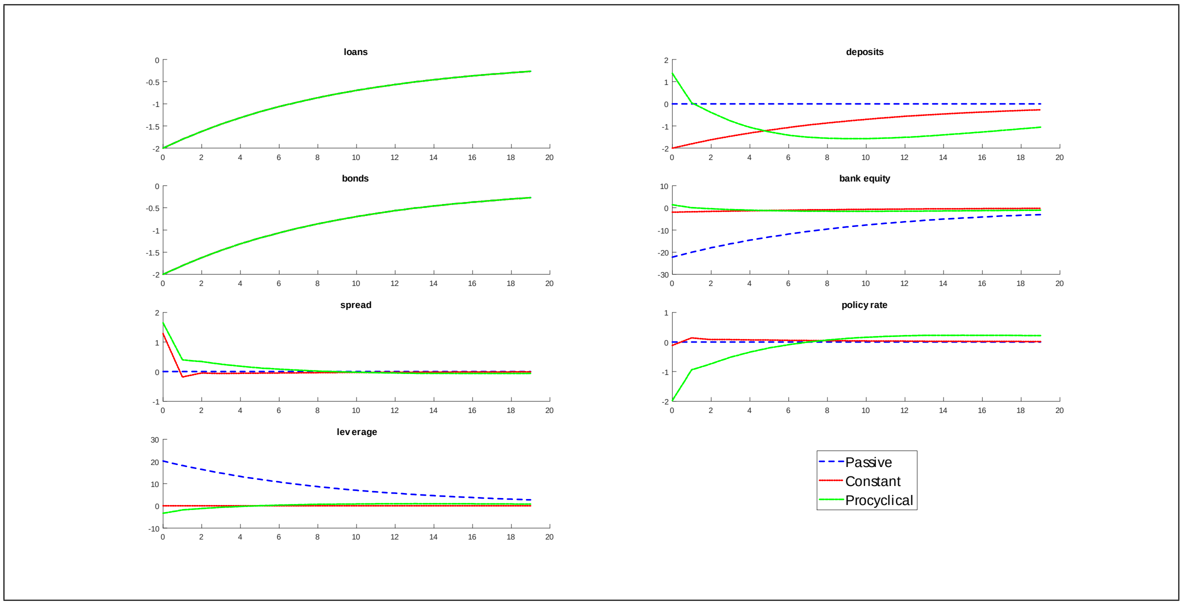

Figure 1: **Baseline Model: Banking Variables**

bank equity. This has a strong downward pressure on the policy rate and a persistent upward pressure on the spread. Hence, the bank procyclical behaviour strengthens both the recessionary and the redistributive effects of the shock that we have explained before, as the green line in Figure (2) shows.

3.3 The interaction of procyclical leverage with hysteresis

The presence of hysteresis is likely to be central to the response of banking variables, and above all real variables over the business cycle. In this section we want to explore the interactions between hysteresis - due to complementarity between past employment and resent TFP - and procyclical behaviour of banks as defined above. We compare our baseline model with the same model in the presence of hysteresis. Chang et al. (2002) assume a simple skill accumulation mechanism through learning by doing, in which the skill level accumulates over time depending on past employment and that the skill level raises the effective unit of labour supplied by the household. We follow Tervala (2013) and Engler, Tervala (2016) in assuming that the level of productivity accumulates over time according to past employment, as follows:

$$z_t = \phi_z z_{t-1} + \mu N_{t-1} \tag{32}$$

where $0 \leq \phi_z \leq 1$ and μ are parameters ($\phi_z = 0.99, \mu = 0.02$). Equation (32) highlights that a change in the current labour supply changes the level of productivity in the next period, with an elasticity of μ.

Next we compare the dynamics of our baseline model with the dynamics of the model with hysteresis, in response to a negative shock to the real value of bonds when the bank's

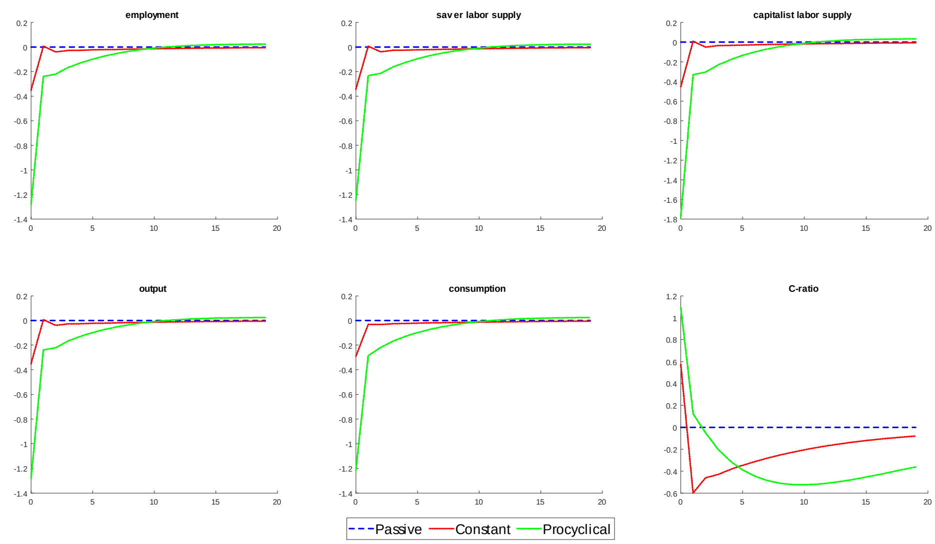

Figure 2: **Baseline Model: Real Variables**

leverage is procyclical.

Figures (3) and (4) display selected impulse responses (for banking and real variables respectively) to a twenty percent decrease in bonds' value. The presence of hysteresis does not change the pattern of reactions after the shock. However, as expected, the combination of hysteresis with bank's leverage procyclicality definitely increase the persistence of a financial shock on output, employment and aggregate consumption, whilst the distribution between capitalists and savers is "permanently" modified (it does not display convergence over 40 periods).

4 Conclusion

In the present paper we have constructed a NK model with two types of agents and non-maximising banks, in order to explore the macroeconomic dynamic effects of different attitudes towards leverage. The model explored in this paper allow us to capture two relevant features of contemporary economies: fluctuations in employment and unemployment and distributional effects ensuing from different patterns of banks' behaviour. The DSGE straightjacket does not allow us to go all the way to explain "booms" and "crises" by means of a formal model. We are not able to mimic the sort of "financial instability hypothesis" advanced by Hyman Minsky (1992). Our economy displays dynamic stability, i.e. it goes back to a steady state sometimes after a financial shock. However shock amplification and slower convergence is at work under leverage targeting and even more under procyclical leverage.

Indeed we are able to show how the behaviour of financial institutions and the macroprudential policy (i) affect the steady state of the economy and (ii) may amplify and

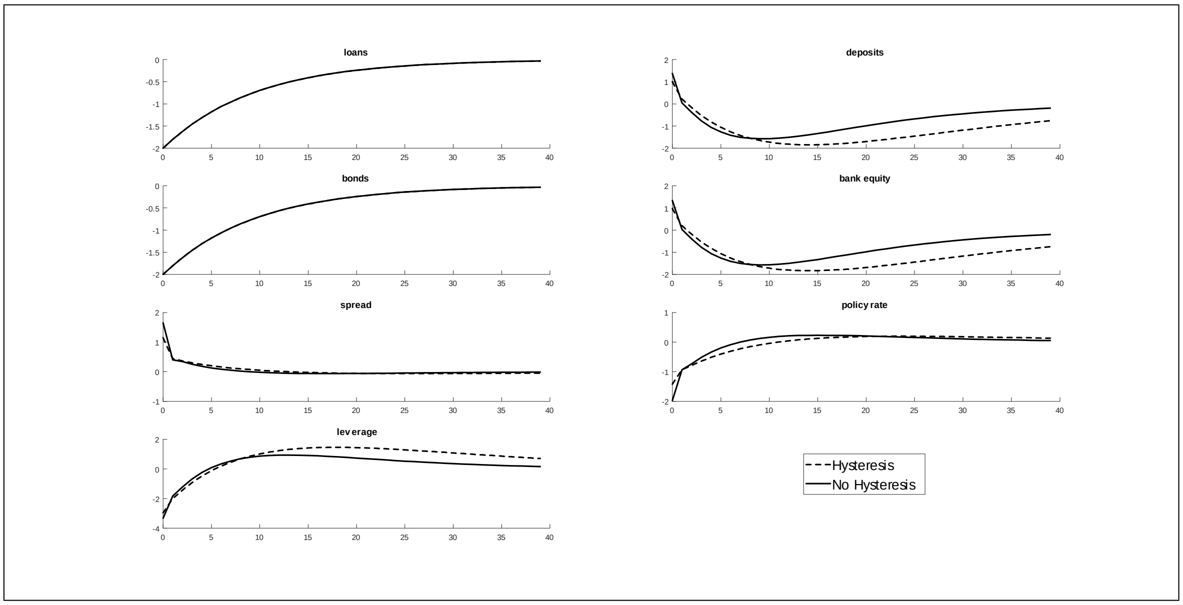

Figure 3: **Procyclical Leverage with Hysteresis: Banking Variables**

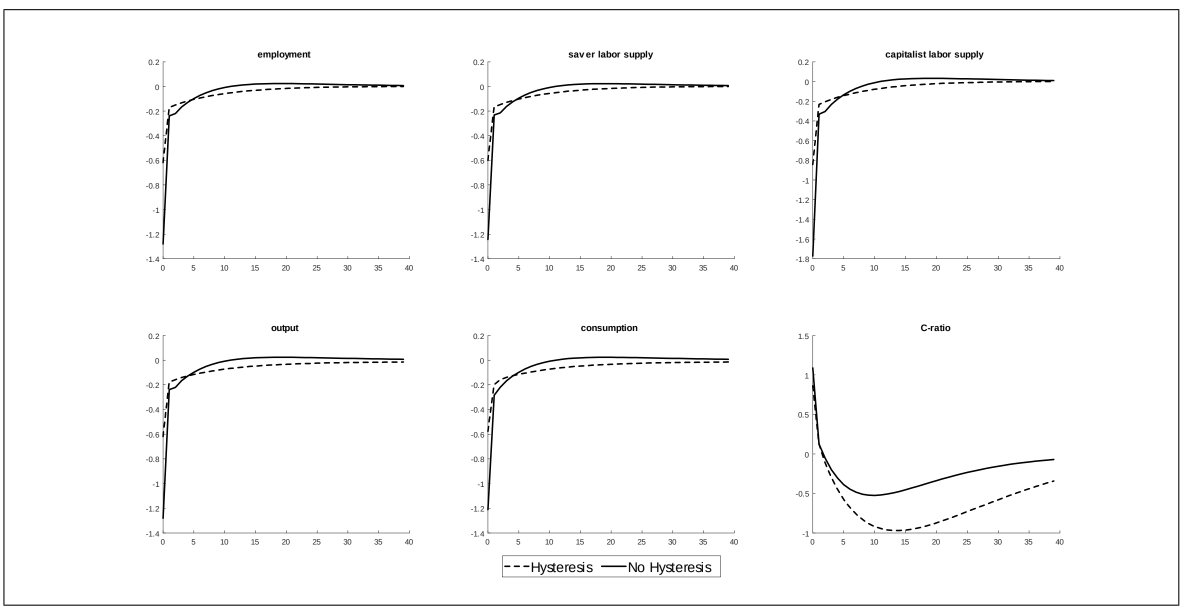

Figure 4: **Procyclical Leverage with Hysteresis: Real Variables**

prolong the impact of financial shocks. We first show that, by allowing a higher (lower) leverage, the macroprudential regulator allows the economy to have higher (lower) output and employment in a steady state. We then analyze the effects of a shock to the real value of firms' bonds under different banks' leverage behaviours. Remarkably, we find that banks' procyclical behaviour implies a higher and more persistent instability after a financial shock with respect to a constant leverage behaviour. On the other hand, a passive leverage behaviour is shock absorbing: output, employment and the distribution of consumption between capitalists and savers are not affected by the shock to the real value of banks' assets. The procyclicality of bank's behaviour tends to amplify the effects of the shock on real variables. A negative shock to the real value of bonds, despite an initial short-lived distributive effect from capitalists to savers, has a persistent and large distributive effect in the opposite direction - from savers to capitalists. Finally, we compare a model characterized by constant productivity with the same model under hysteresis. We show that, with hysteresis, the effect of the shock on real variables is more persistent. It must be noted that a constant leverage target such as it could be set by banks regulation is not sufficient to prevent shock amplification and distributive effects from taking place, although less pronounced than in a situation in which banks' leverage is allowed to be pro-cyclical. This result points to the need of an anti-cyclical regulation of banks leverage, somehow forcing a passive leverage behaviour, perhaps by means of a fine-tuned macro-prudential regulation.

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A Technical appendix

A.1 Capitalist's problem

$$\max \frac{C_{c,t}^{1-\sigma_c}}{1-\sigma_c} - \frac{N_{c,t}^{1+\varphi}}{1+\varphi},$$

subject to the sequence of constraints:

$$C_{c,t} + \Omega_{c,t}^f V_t^f + \Omega_{c,t}^b V_t^b \leq \Omega_{c,t-1}^f \left(V_t^f + \Gamma_t^f\right) + \Omega_{c,t-1}^b \left(V_t^b + \Gamma_t^b\right) + w_t N_{c,t}$$

1. **FOC wrt** $C_t^c : C_{c,t}^{-\sigma_c} + \lambda_t^* = 0 \Rightarrow \lambda_t^* = -C_{c,t}^{-\sigma_c}$

2. **FOC wrt** $N_{c,t} : -N_{c,t}^{\varphi} - \lambda_t^* w_t = 0 \Longrightarrow \lambda_t^* w_t = -N_{c,t}^{\varphi} \Longrightarrow N_{c,t}^{\varphi} C_{c,t} = w_t$

3. **FOC wrt** $\Omega_t^f : \lambda_t^* V_t^f - \lambda_{t+1}^* \left(V_{t+1}^f + \Gamma_{t+1}^f\right) = 0;$

   $\lambda_t^* V_t^f = \lambda_{t+1}^* \left(V_{t+1}^f + \Gamma_{t+1}^f\right);$

   $-C_{c,t}^{-\sigma_c} V_t^f = -C_{c,t+1}^{-\sigma_c} \left(V_{t+1}^f + \Gamma_{t+1}^f\right);$

   $V_t^f = \frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \left(V_{t+1}^f + \Gamma_{t+1}^f\right);$

4. **FOC wrt** $\Omega_t^b : \lambda_t^* V_t^b - \lambda_{t+1}^* \left(V_{t+1}^b + \Gamma_{t+1}^b\right) = 0;$

   $\lambda_t^* V_t^b = \lambda_{t+1}^* \left(V_{t+1}^b + \Gamma_{t+1}^b\right);$

   $-C_{c,t}^{-\sigma_c} V_t^b = -C_{c,t+1}^{-\sigma_c} \left(V_{t+1}^b + \Gamma_{t+1}^b\right);$

   $V_t^b = \frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \left(V_{t+1}^b + \Gamma_{t+1}^b\right);$

$$N_{c,t}^{\varphi} C_{c,t} = w_t$$

$$V_t^f = \beta E_t \left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \left(V_{t+1}^f + \Gamma_{t+1}^f \right) \right]$$

$$V_t^b = \beta E_t \left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \left(V_{t+1}^b + \Gamma_{t+1}^b \right) \right]$$

A.2 Saver's problem

$$\max \frac{C_{s,t}^{1-\sigma_s}}{1-\sigma_s} - \frac{N_{s,t}^{1+\varphi}}{1+\varphi},$$

$$C_{s,t} + D_{s,t} \leq \frac{1+i_{t-1}}{\pi_t} D_{s,t-1} + w_t N_{s,t}.$$

1. **FOC wrt** $C_{s,t} : C_{s,t}^{-\sigma_s} + \lambda_t^* = 0 \Rightarrow \lambda_t^* = -C_{s,t}^{-\sigma_s}$
2. **FOC wrt** $D_{s,t} : \lambda_t^* - \lambda_{t+1}^* \frac{1+i_t}{\pi_{t+1}} = 0 \Longrightarrow -C_{s,t}^{-\sigma_s} = -C_{s,t+1}^{-\sigma_s} \frac{1+i_t}{\pi_{t+1}}$

$$C_{s,t}^{-\sigma_s} = \beta \left(\frac{1+i_t}{\pi_{t+1}} \right) C_{s,t+1}^{-\sigma_s}$$

A.3 Firm's problem

$$\begin{aligned}
&P_t(k)Y_t(k) + P_t L_t^d(k) + P_t B_t^s(k) - W_t N_t(k) - \left(1+\rho_{t-1}\right) P_{t-1} L_{t-1}^d(k) - \left(1+i_{t-1}\right) P_{t-1} B_{t-1}^s(k) \\
&\quad -P_t \tfrac{\gamma}{2} \left(\tfrac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2 + \beta E_t \left[\tfrac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \left(\begin{array}{c} P_{t+1}(k)Y_{t+1}(k) + P_{t+1} L_{t+1}^d(k) \\ +P_{t+1} B_{t+1}^s(k) - W_{t+1} N_{t+1}(k) \\ -\left(1+\rho_t\right) P_t L_t^d(k) - \left(1+i_t\right) P_t B_t^s(k) \\ -P_{t+1} \tfrac{\gamma}{2} \left(\tfrac{P_{t+1}(k)}{P_t(k)} - 1 \right)^2 \end{array} \right) \right] = \\
&\quad = P_t(k) \left(\tfrac{P_t(k)}{P_t} \right)^{-\epsilon} Y_t^d + P_t L_t^d(k) + P_t B_t^s(k) - mc_t P_t x_t \tfrac{Y_t(k)}{x_t} - \left(1+\rho_{t-1}\right) P_{t-1} L_{t-1}^d(k) \\
&\quad - \left(1+i_{t-1}\right) P_{t-1} B_{t-1}^s(k) - P_t \tfrac{\gamma}{2} \left(\tfrac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2 + \\
&\quad + \beta E_t \left[\tfrac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}} \cdot \left(\begin{array}{c} P_{t+1}(z) Y_{t+1}(k) \\ +P_{t+1} L_{t+1}^d(k) \\ +P_{t+1} B_{t+1}^s(k) \\ -W_{t+1} N_{t+1}(k) \\ -\left(1+\rho_t\right) P_t L_t^d(k) \\ -\left(1+i_t\right) P_t B_t^s(k) \\ -P_{t+1} \tfrac{\gamma}{2} \left(\tfrac{P_{t+1}(k)}{P_t(k)} - 1 \right)^2 \end{array} \right) \right].
\end{aligned}$$

$$\max_{P_t(z)} P_t(k) \left(\tfrac{P_t(k)}{P_t} \right)^{-\epsilon} Y_t^d + P_t L_t^d(k) + P_t B_t^s(k) - mc_t P_t x_t \tfrac{Y_t(k)}{x_t} - \left(1+\rho_{t-1}\right) P_{t-1} L_{t-1}^d(k)$$

$$-\left(1+i_{t-1}\right)P_{t-1}B^s_{t-1}(k)-P_t\frac{\gamma}{2}\left(\frac{P_t(k)}{P_{t-1}(k)}-1\right)^2+$$

$$+\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.\begin{pmatrix} P_{t+1}(z)Y_{t+1}(k)\\ +P_{t+1}L^d_{t+1}(k)\\ +P_{t+1}B^s_{t+1}(k)\\ -W_{t+1}N_{t+1}(k)\\ -\left(1+\rho_t\right)P_tL^d_t(k)\\ -\left(1+i_t\right)P_tB^s_t(k)\\ -P_{t+1}\frac{\gamma}{2}\left(\frac{P_{t+1}(k)}{P_t(k)}-1\right)^2 \end{pmatrix}\right].$$

$$\max_{P_t(z)}\frac{P_t(z)^{1-\epsilon}}{P_t^{-\epsilon}}Y^d_t+P_tL^d_t(k)+P_tB^s_t(k)-mc_t\frac{P_t(k)^{-\epsilon}}{P_t^{-(\epsilon+1)}}Y^d_t-\left(1+\rho_{t-1}\right)P_{t-1}L^d_{t-1}(k)$$

$$-\left(1+i_{t-1}\right)P_{t-1}B^s_{t-1}(k)-P_t\frac{\gamma}{2}\left(\frac{P_t(k)^2}{P_{t-1}(k)^2}-2\frac{P_t(k)}{P_{t-1}(k)}+1\right)$$

$$+\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.\begin{pmatrix} P_{t+1}(k)Y_{t+1}(k)\\ +P_{t+1}L^d_{t+1}(k)\\ +P_{t+1}B^s_{t+1}(k)\\ -W_{t+1}N_{t+1}(k)\\ -\left(1+\rho_t\right)P_tL^d_t(k)\\ -\left(1+i_t\right)P_tB^s_t(k)\\ -P_{t+1}\frac{\gamma}{2}\left(\frac{P^2_{t+1}(k)}{P^2_t(k)}+1-2\frac{P_{t+1}(k)}{P_t(k)}\right) \end{pmatrix}\right].$$

$$(1-\epsilon)\underbrace{\left(\frac{P_t(k)}{P_t}\right)^{-\epsilon}Y^d_t}_{\text{Production}}+\epsilon mc_t\left(\frac{P_t(k)}{P_t}\right)^{-(\epsilon+1)}Y^d_t-P_t\gamma\frac{P_t(k)}{\left(P_{t-1}(k)\right)^2}$$

$$+\frac{P_t\gamma}{P_{t-1}(k)}+\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.P_{t+1}\gamma\left(\frac{P^2_{t+1}(k)}{P^3_t(k)}-\frac{P_{t+1}(k)}{P^2_t(k)}\right)\right]=0;$$

$$(1-\epsilon)x_tN_t+\epsilon mc_t\underbrace{\left(\frac{P_t}{P_t(k)}\right)}_{=1}\underbrace{\left(\frac{P_t(k)}{P_t}\right)^{-\epsilon}Y^d_t}_{\text{Production}}-P_t\gamma\frac{P_t(k)}{\left(P_{t-1}(k)\right)^2}+\frac{P_t\gamma}{P_{t-1}(k)}$$

$$+\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.P_{t+1}\gamma\left(\frac{P^2_{t+1}(k)}{P^3_t(k)}-\frac{P_{t+1}(k)}{P^2_t(k)}\right)\right]=0;$$

$$\frac{P_t\gamma}{P_{t-1}(k)}\left(\frac{P_t(k)}{P_{t-1}(k)}-1\right)=\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.P_{t+1}\gamma\frac{P_{t+1}(k)}{P^2_t(k)}\left(\frac{P_{t+1}(k)}{P_t(k)}-1\right)\right]+(1-\epsilon)x_tN_t+\epsilon mc_tx_tN_t$$

$$\underbrace{\frac{P_t}{P_{t-1}(k)}}_{\pi_t}\left(\underbrace{\frac{P_t(k)}{P_{t-1}(k)}}_{\pi_t}-1\right)=\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}.\left(\underbrace{\frac{P_{t+1}(k)}{P_t(k)}}_{\pi_{t+1}}\right)^2\left(\underbrace{\frac{P_{t+1}(k)}{P_t(k)}}_{\pi_{t+1}}-1\right)\right]+\frac{\epsilon x_tN_t}{\gamma}\left(mc_t-\frac{\epsilon-1}{\epsilon}\right)$$

$$\pi_t\left(\pi_t-1\right)=\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}\left[\pi^2_{t+1}.\left(\pi_{t+1}-1\right)\right]\right]+\frac{\epsilon x_tN_t}{\gamma}\left(mc_t-\frac{\epsilon-1}{\epsilon}\right)$$

$$\pi_t(\pi_t-1)=\beta E_t\left[\frac{C^{-\sigma_c}_{c,t+1}}{C^{-\sigma_c}_{c,t}}\pi^2_{t+1}(\pi_{t+1}-1)\right]+\frac{\epsilon x_tN_t}{\gamma}\left(mc_t-\frac{\epsilon-1}{\epsilon}\right)$$

$$\max_{L^d_t(k)}Y_t+L^d_t(k)+B^s_t(k)-w_tN_t(k)-\frac{(1+\rho_{t-1})}{\pi_t}L^d_{t-1}(k)-\frac{(1+i_{t-1})}{\pi_t}B^s_{t-1}(k)$$

$$
-\frac{\gamma}{2}\left(\frac{P_t(k)}{P_{t-1}(k)}-1\right)^2+\beta E_t\left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}}\cdot\left(\begin{array}{c} Y_{t+1}(k)+L_{t+1}^d(k) \\ +B_{t+1}^s(k)-w_{t+1}N_{t+1}(k) \\ -\frac{1+\rho_t}{\pi_{t+1}}L_t^d(k)-\frac{1+i_t}{\pi_{t+1}}B_t^s(k) \\ -\frac{\gamma}{2}\left(\frac{P_{t+1}(k)}{P_t(k)}-1\right)^2 \end{array}\right)\right]=
$$

$$
1-\beta E_t\left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}}\frac{1+\rho_t}{\pi_{t+1}}\right]=0;
$$

$$
1+\rho_t=\frac{1}{\beta}E_t\left[\frac{C_{c,t}^{-\sigma_c}\pi_{t+1}}{C_{c,t+1}^{-\sigma_c}}\right]
$$

$$
\max_{B_t^s(k)} Y_t+L_t^d(k)+B_t^s(k)-w_tN_t(k)-\frac{(1+\rho_{t-1})}{\pi_t}L_{t-1}^d(k)-\frac{(1+i_{t-1})}{\pi_t}B_{t-1}^s(k)
$$

$$
-\frac{\gamma}{2}\left(\frac{P_t(k)}{P_{t-1}(k)}-1\right)^2+\beta E_t\left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}}\cdot\left(\begin{array}{c} Y_{t+1}(k)+L_{t+1}^d(k) \\ +B_{t+1}^s(k)-w_{t+1}N_{t+1}(k) \\ -\frac{1+\rho_t}{\pi_{t+1}}L_t^d(k)-\frac{1+i_t}{\pi_{t+1}}B_t^s(k) \\ -\frac{\gamma}{2}\left(\frac{P_{t+1}(k)}{P_t(k)}-1\right)^2 \end{array}\right)\right]+\phi_t\left[B_t^s-\bar{B}^s\right].
$$

$$
1-\beta E_t\left[\frac{C_{c,t+1}^{-\sigma_c}}{C_{c,t}^{-\sigma_c}}\left(\frac{1+i_t}{\pi_{t+1}}\right)\right]+\phi_t=0;
$$

$$
1+i_t=\frac{1}{\beta}E_t\left(\frac{\pi_{t+1}C_{c,t}^{-\sigma_c}}{C_{c,t+1}^{-\sigma_c}}\right)[1+\phi_t]
$$

A.4 Equilibrium

$$
\lambda C_{s,t}+\lambda d_{s,t}^s-\lambda\frac{1+i_{t-1}}{\pi_t}d_{s,t-1}^s-\lambda w_tN_{s,t}+(1-\lambda)C_{c,t}+(1-\lambda)\Omega_{c,t}^fV_t^f+(1-\lambda)\Omega_{c,t}^bV_t^b
$$
$$
-(1-\lambda)\Omega_{c,t-1}^f\left(V_t^f+\Gamma_t^f\right)-(1-\lambda)\Omega_{c,t-1}^b\left(V_t^b+\Gamma_t^b\right)-(1-\lambda)w_tN_{c,t}
$$

If we consider:

$$
C_t=\lambda C_{s,t}+(1-\lambda)C_{c,t},
$$

labor market clearing condition:

$$
N_t=\lambda N_{s,t}+(1-\lambda)N_{c,t},
$$

and equity market clearing condition for firms:

$$
\Omega_{c,t}^f=\Omega_{c,t-1}^f=\frac{1}{1-\lambda}
$$

and banks:

$$
\Omega_{c,t}^b=\Omega_{c,t-1}^b=\frac{1}{1-\lambda}
$$

we obtain:

$$C_t + \lambda d^s_{s,t} - \lambda \frac{1+i_{t-1}}{\pi_t} d^s_{s,t-1} - \Gamma^f_t - \Gamma^b_t - w_t N_t = 0$$

If we substitute (18) and (22), we obtain:

$$\begin{aligned}
&C_t + \lambda d^s_{s,t} - \lambda \frac{1+i_{t-1}}{\pi_t} d^s_{s,t-1} \\
&-Y_t - L^d_t - B^s_t + w_t N_t + g_t H_t + \frac{1+\rho_{t-1}}{\pi_t} L^d_{t-1} + \frac{1+r_{t-1}}{\pi_t} B^s_{t-1} + \frac{\gamma}{2} (\pi_t - 1)^2 \\
&-D_t + L^s_t + B^d_t - \frac{1+\rho_{t-1}}{\pi_t} L^s_{t-1} - \frac{1+i_{t-1}}{\pi_t} B^d_{t-1} + \frac{1+i_{t-1}}{\pi_t} D^d_{t-1} - w_t N_t
\end{aligned}$$

Simplifying,

$$\begin{aligned}
&C_t + \lambda d^s_{s,t} - \lambda \frac{1+i_{t-1}}{\pi_t} d^s_{s,t-1} \\
&-Y_t - L^d_t - B^s_t + g_t H_t + \frac{1+\rho_{t-1}}{\pi_t} L^d_{t-1} + \frac{1+r_{t-1}}{\pi_t} B^s_{t-1} + \frac{\gamma}{2} (\pi_t - 1)^2 \\
&-D_t + L^s_t + B^d_t - \frac{1+\rho_{t-1}}{\pi_t} L^s_{t-1} - \frac{1+i_{t-1}}{\pi_t} B^d_{t-1} + \frac{1+i_{t-1}}{\pi_t} D^d_{t-1}
\end{aligned}$$

And considering that:

$$\lambda d^s_{s,t} = D_t$$

$$L^d_t = L^s_t$$

$$B^d_t = B^s_t$$

$$Y_t = C_t + g_t H_t + \frac{\gamma}{2} (\pi_t - 1)^2$$