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Productivity Shocks and Uncertainty Shocks in a Model with Endogenous Firms Exit and Inefficient Banks

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Abstract

We consider a NK-DSGE model with endogenous firms' creation and destruction together with monopolistic competitive banks, where defaulting firms do not repay loans to banks. This framework implies: i) an endogenous and countercyclical number of firms destruction; ii) an endogenous and countercyclical bank markup. We study the effects of a shock to both the level and the volatility of the aggregate productivity. In response to a level shock, the interaction between i) and ii) generates a stronger propagation mechanism with respect to a model with exogenous exit and to a model with efficient banks. Remarkably, our model shows that a shock to the volatility of productivity, referred as an uncertainty shock, is recessionary. It implies a decline in firms' creation, an increase in firms' destruction and an increase in banks' markup. Estimating a small BVAR we find that our theoretical results are well supported by the empirical responses to uncertainty shocks.

Keywords: firms' endogenous exit, countercyclical bank markup, productivity shocks, uncertainty shock, BVAR.

JEL codes: E32; E44; E52; E58

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1 Introduction

This paper contributes to the literature on firms dynamics. While this literature has concentrated on firms entry as a channel for understanding business cycle transmission of a productivity shock, it abstracts from the role played by firms exit. Furthermore, most of the papers with endogenous business creation consider a perfect financial market and do not analyze the interaction between firms dynamics and financial markets. This paper try to fill this gap, by considering a New-Keynesian Dynamic Stochastic General Equilibrium model - henceforth, NK-DSGE model - characterized by firms' endogenous entry and exit decisions, together with an imperfect competitive banking sector interacting with incumbent firms. In this respect, we assume that monopolistic competitive banks lend money to firms. Banks cannot insure against the risk of firms' default and thus they can incur in balance sheet losses every time a firm exit the market without repaying the loan. To prevent from these losses, banks increase their markup when the probability of firms default increases. As a result banks' markup is endogenous. Firms' exit is modeled using a modified version of the mechanism proposed by Melitz (2003) and Ghironi and Melitz (2005) for exporting firms. In particular, we assume that firms decide to produce as long as their specific productivity is above a cut-off level, which is determined by the level of productivity that makes the sum of current and discounted future profits equal to zero.

In this context, we study the dynamics of the model in response to two shocks: i) a shock to the level of the aggregate productivity, that is a standard productivity shock and ii) a shock to the volatility of the aggregate productivity, commonly referred as a shock to macroeconomic uncertainty. The main results of the paper can be summarized as follows.

First, in response to positive shocks to the level of productivity firms' profits opportunities increase and households invest in new firms. Thus, firms creation as well as their number increase. Total output increases and the economy enters into a boom. At the same time firms' exit probability decreases and consequently the number of firms failures decrease, thus being countercyclical. A direct consequence of this fact is that the propagation mechanism of productivity shocks, via the extensive margin of the good-market, becomes stronger than in a model with exogenous exit, as for example in Bilbiie, Ghironi and Melitz (2012) - henceforth, BGM (2012), where firms exit probability is constant, while the number of firms exit is mildly procyclical and therefore at odds with the data.[2] Besides this, we show that the endogeneity of firms' exit generates an additional shock transmission channel through the banking sector. Indeed, the indirect consequence of firms endogenous default is that, every time firms' exit probability increases, banks optimally increase their markup and thus their interest rate on loans. In other words, banks' markup is countercyclical, i.e. it increases in face of recessionary shocks, while it decreases in response to expansionary shocks. The countercyclicality of the banks' markup further amplifies the initial impact of the shock. To quantify the importance of the countercyclicality of the banks' markup, we compare the performance of our baseline model with an alternative model where exit is still endogenous, while the banking sector is efficient. In this case, the banks' markup is zero and banks' decisions are not affected by the probability of firms' default. The comparison between these two models clearly shows that the baseline model with inefficient banks generates a stronger propagation mechanism with respect to the model with efficient banks.

In the second part of the paper we consider the effects of an uncertainty shocks, i.e. of a shock to the volatility of the aggregate productivity. We show that this shock is recessionary. It is indeed followed by a decline in output, a decline in the number of new entrants and, by an increase in both firms destruction and banks' markup. Furthermore, we show that the interaction between the probability of firms' default and the bank markup generates a stronger and more prolonged recession in the medium run with respect to a model with efficient banks.

Finally, following the literature on uncertainty shocks, and in particular Leduc and Liu (2016) and Fernandez-Villaverde (2011), we estimate a small BVAR, using the CBOE Volatility Index (VIX) as a proxy for aggregate macroeconomic uncertainty. We compute the implied IRFs and we show that the theoretical IRFs are well supported by the empirical ones, at least qualitatively.

The procyclicality of firms entry and the countercyclicality of firms exit, both in terms of number and rates are well documented in the empirical literature. With respect to firms dynamics, Campbell (1998), using a sample

[2] The reason is the following. Suppose that a positive technology shock hits the economy. Since the exit probability is constant and firms' destruction is proportional to the total number of firms, exit increases during a boom instead of decreasing. A direct consequence of this fact is that the propagation mechanism of the shocks via the extensive margin of the good-market is weaker than what suggested by our baseline model.

of US manufacturing firms, found a positive correlation between entry rates and the growth rate of real GDP, he found instead an even stronger but negative correlation between the growth rate of real GDP and business' failures, implying that firm exit is countercyclical. Using a different dataset, Totzek (2009) and Vilmi (2011) found similar results. In their paper, BGM (2012) claim that the assumption of exogenous exit is adopted only for tractability. However, they also argue that some evidence, as for example that reported by Broda and Weinstein (2010), shows that product destruction is much less cyclical than product creation, at least at product level. Similar results, at plant level, are obtained by Lee and Mukoyama (2015), even thought they obtain these results using US Census annual data. To better motivate our paper, in the next section we show that the procyclicality of firms entry and the countercyclicality of firms exit is confirmed when considering their conditional responses to a productivity shock, which is the shock of interest in this paper. The empirical responses are obtained estimating a structural VAR, using two alternative measures of productivity: the utility adjusted TFP, computed by Basu et al (2006) and a measure of the labor productivity. As a proxy of firms entry and exit we use the BLS data on establishment births and deaths, which are the only updated sources available at quarterly level. The same VAR is estimated including real GDP, CPI inflation and a proxy of the bank markup. We show that, real GDP increases and inflation decreases in response to a positive shock to the level of productivity, whereas the bank markup decreases, thus being countercyclical as suggested by our baseline model.

The countercyclicality of the banks markup - often computed using as a proxy the banks' loan spread - is also found in several papers. Examples are Hannan and Berger (1991), Asea and Blomberg (1998) and more recently Lown and Morgan (2008), Nikitin and Smith (2009) and Kwan (2010). In particular, Kwan (2010) reported that the commercial and industrial loan rate spread has been of about 66 basis points higher (or 23% higher) than its long-term average in the aftermath of the recent financial crisis. Finally, Rousseas (1985) was the first to claim that banks desire to increase their markup to restore their profits, every time they fear a fall in the economic activity, followed by firms defaults and thus losses in their balance sheets. Dueker and Thornton (1997), Angelini and Cetorelli (2003), and more recently, Olivero (2010) and Aliaga-Diaz and Olivero (2012), all show that banks' markup is countercyclical.

The impact of firms' dynamics on business cycle has been studied in many papers. The seminal paper of BGM (2012) considers a model with endogenous firms entry and shows that the sluggish response of the number of producers (due to the sunk entry costs) generates a new and potentially important endogenous propagation mechanism for real business cycle models. In this respect, Etro and Colciago (2010) study a DSGE model with endogenous good market structure under Bertrand and Cournot competition and show that their model improves the ability of a flexible price model in matching impulse response functions and second moments for US data. Colciago and Rossi (2015) extend this model accounting for search and matching frictions in the labor market.[3] All these papers together with Lewis and Poilly (2012), Jaimovich and Floetotto (2008), also provide evidence that the number of producers varies over the business cycle and that firms dynamics may play an important role in explaining business cycle statistics. Bergin and Corsetti (2008) and Cavallari (2013) use a similar framework for analyzing an open economy. However, all these models consider a constant exit probability and are not able to disentangle the role of firms exit with respect to that of firms entry, thus missing an important characteristic of the business cycle.

To the best of our knowledge few papers try to model firms exit in a DSGE framework. Exceptions are Totzek (2009), Vilmii (2011), Cavallari (2015), Hamano and Zanetti (2015) and Cesares and Poutineau (2014). The closest to my paper are Totzek (2009) Cesares and Poutineau (2014) and Hamano and Zanetti (2015). All these papers consider a standard DSGE model without banking[4], with different timing and different exit condition.[5]

[3] They show that their model contributes to explain the volatility of the labor market variables and also stylized facts concerning the countercyclicality of price markups, the procyclicality of firms profits, the overshooting of the labor share of income and job creation by new firms.

[4] All these models consider an efficient financial market. Bergin at al (2014) study a model with endogenous firms entry and financial shocks. They show that entry contributes to the propagation of financial shocks. Using a different franmework, La Croce and Rossi (2014), find similar results. Both models however consider endogenous business creation but exogenous firms destruction and do not analyze the effects of uncertainty shocks.

[5] Totzek (2009) as well as Vilmi (2011) and Cesares and Poutineau (2014) assume that firms exit occurs at the end of the production period. In my model, exit occurs as soon as firms realize that their productivity is below the threshold and before starting producing. This implies that the average productivity changes along the business cycle and, as will be discussed in the paper, it also implies a stronger response of output. Importantly, Cesares and Poutineau (2014) assume that the stochastic discount factor is not affected dynamically by the endogenous firms exit probability. This also implies that the exit

Remarkably, none of these papers consider the effects of an uncertainty shocks. Furthermore, they consider The remainder of the paper is organized as follows. Section 2 provides an empirical motivation by reporting the dynamic responses of the US establishments births and deaths, as well as of a proxy of the US banks markup, to a shock to the level of the aggregate productivity. Section 3 spells out the model economy, while Section 4 contains the main results of the model. Section 5 estimates a small BVAR and shows the responses to an uncertainty shock. Technical details are left in the Technical Appendix.

2 Empirical Motivation

To further motivate our theoretical model, we now run Structural Vector Auto-Regressions (SVARs) to estimate the impulse response functions (IRFs) of the US Bank Markup and Establishments births and deaths to a temporary productivity shock. In order to verify the correct identification of a productivity shock, we also consider the IRFs of the US real GDP and that of the CPI inflation.[6] We consider two alternative and widely used measures of productivity: i) The TFP series based on growth accounting techniques, computed by Basu et al (2006), which is Utilization Adjusted TFP series,[7] labeled as $TFPu$;[8] ii) the series of measured labor productivity, labeled as $LabProd$, which is also widely used in the literature.[9] The other series con-

probability does not affect firms' decision on entering the market as well as firms pricing decisions. Furthermore, the authors consider a medium scale model with a large number of frictions that makes the model more suitable for policy analysis, however it makes the results and transmission channel of the exit margin more difficult to interpret. We take our model as simple as possible in order to better understand the role of the exit margin and its interaction with the banking sector.

[6] As suggested by the theory and by the wide empirical literature, if the shock is correctly identified the real GDP should increase on impact, while the inflation rate should decreases in response to a productivity shock.

[7] The importance of correcting for utilization effects in measured TFP has long been stressed in the literature (see, for example, Burnside et al. (1995) and references therein and Basu et al (2006)). Further, Chang and Hong (2006) argue in favour of using TFP growth instead of labour productivity as the latter is influenced by changes in the input mix.

[8] Chang and Hong (2006) argue in favour of using TFP growth instead of labour productivity as the latter is influenced by changes in the input mix.

[9] Labor productivity, defined as Real Output Per Hour of All Persons in the Non-farm Business Sector (OPHNFB) has been dowloaded from FRED. The utilization adjusted

sidered are: CPI inflation, real GDP, Establishment births and deaths and a proxy of the Banks Markup. We follow the literature starting from Rousseas (1985) and we compute the bank markup as the ratio between the US Bank Prime Loan rate and the Effective Fed Fund rate[10] We label these series respectively as: $INFL$, $RGDP_t$, $BIRTHS_t$, $DEATHS_t$, $BankMarkup_t$. All these series, except inflation and banks markup, are in logarithmic differences. Given the short time-span of the series of BIRTHS and DEATHS we estimate a SVAR(1) using the following sample: 1993Q3-2015Q1.

To identify productivity shocks we consider simple short-run restrictions. The short-run identification strategy is motivated by two reasons. First, by the very short sample available,[11] and second, by the type of the TFP shock considered in our theoretical model, which is persistent but temporary.[12] We use a short-run restriction based on the standard Cholesky decomposition where either $TFPu_t$ or the alternative measure of $LabProd_t$ are ordered first. This allows to consider the two alternative measures as the most exogenous ones, meaning that only productivity shocks affect these series on impact. The order of the other five variables is the following: $[TFPu_t\,(LabProd_t)\,, INFL, RGDP_t, BIRTHS_t, DEATHS_t, BankMarkup_t]$.[13] The resulting IRFs, jointly with 16% and 84% bootstrap confidence bands obtained from 10.000 draws, are plotted in Figure 1 and 2.

TFPu, is defined as the Business Sector TFP net of the Utilization of capital and Labor (dtfp_util). It can be downloaded, jointly with other related measures of productivity, from the websites of FED S.Francisco.

[10] Both series have been downloaded from FRED. RGDP, and CPI are also dowloaded from FRED. The series of RGDP, is the quarterly series of the US real Gross Domestic Product (GDPC1). The CPI series is the Consumer Price Index for All Urban Consumers (CPIAUCSL). The series of Inflation has been computed taking the quarter to quarter log-difference of the CPI. Finally the series of BIRTHS and DEATHS comes from the Bureau of Labor Statistics (BLS). They are all the US Private sector establishment births and deaths. Unfortunately, they are available at quarterly frequency starting only from 1993Q2.

[11] We also consider a SVAR with short-run sign restrictions. The resulting IRFs are qualitatively similar and results are available upon request. We take the short-run Cholesky identification strategy as the baseline one because it relies on weaker restrictions.

[12] Adding a non-stationary productivity dynamics would highly complicate the aggregation techniques of our model with heterogenoeous firms, without changing the qualitatively the results.

[13] Changing the order of the other five variables does not sensibly alters the resulting IRFs.

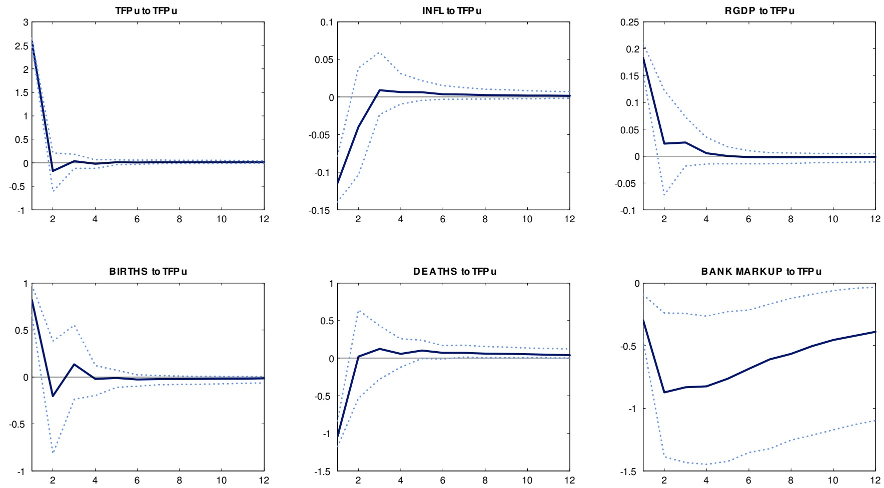

Figure 1. IRFs to a TFP shock identified using TFPu series.

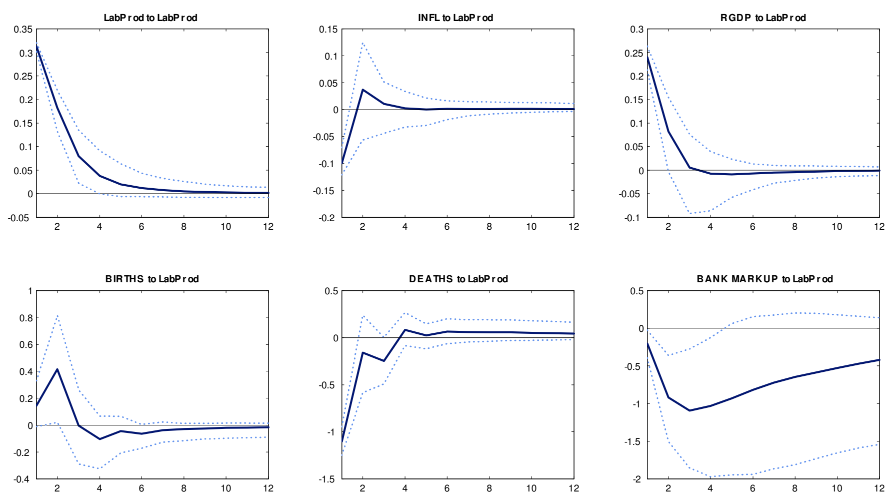

Figure 2: IRFs to a TFP shock identified using labor productivity series.

Notice that, the two SVAR models, with the two measures of productivity generate very similar IRFs. Indeed, in both models the inflation rate decreases on impact and remain below zero for several periods, while the real GDP increases in response to the shock. These patterns of RGDP and inflation suggest that such a shock is well identified. Importantly, notice

that in both models the series of establishments BIRTHS is procyclical and not particularly persistent. The series of DEATHS is instead countercyclical on impact and overshoots its long run value in the medium run. Similarly to the series of establishment DEATHS, the bank markup is countercyclical, even though it turns to be much more persistent and presents larger confidence bands.
The next Section presents a DSGE model able to qualitatively provide an explanation for these empirical findings.

3 The Model

The model considered is a closed economy composed by four agents: households, firms, banks and the monetary authority which is responsible for setting the policy interest rate.

3.1 Firms

The supply side of the economy is composed by: i) the intermediate good-producing firms equally distributed into a continuum of $k \in (0, 1)$ symmetric sectors. Each sector produces a continuum of differentiated goods $i \in N$ under monopolistic competition and flexible prices ii) The retail sector is composed by $j = k$ firms, competing under monopolistic competition. Each firm purchases all goods produced by the sector k, bundles it using a CES technology and set prices à la Rotemberg (1982).[14]

3.1.1 Firms: the Intermediate Sectors

Each sector k produces a continuum of differentiated intermediate goods $i \in N$, where N represents the mass of available goods produced by the sector. For the sake of simplicity, we assume one-to-one identification between a product and a firm. Firms in each sector k enjoy market power and set prices $P_{i,k,t}$ as a markup over their marginal costs. Since all sectors are identical we consider a representative intermediate sector and we remove the index k. In this context, the production function of firm i is,

$$y_{i,t} = A_t z_{i,t} l_{i,t} \tag{1}$$

[14] The retail sector is introduced only to separate the sticky price problem from that of firms dynamics.

where $l_{i,t}$ is the amount of labor hours employed by firm i, while $z_{i,t}$ is a firm specific productivity, which is assumed to be Pareto distributed across firms, as in Ghironi and Melitz (2005). The variable A_t is instead an aggregate AR(1) productivity shock.

The intermediate-goods producing firm i chooses the optimal price $P_{i,t}$ to produce $y_{i,t}$, maximizing its expected real profits, thus solving the following problem:

$$
\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} j_{i,t}, && (2) \\
& s.t. && \\
y_{i,t} \quad = \quad & A_t z_i l_{i,t}, && (3)
\end{aligned}
$$

where $j_{i,t}$ are firm i real profits, $\Lambda_{0,t}$ is the real stochastic discount factor, that will be defined below. The demand for the produced good $y_{i,t}$ comes from the retail sector and it is given by $y_{i,t} = \left(\frac{P_{i,t}}{P_t^I}\right)^{-\theta} y_t^R(k)$, where $y_t^R(k)$ is the aggregate demand of the retail firm k, with P_t^I being the Price Index of the intermediate sector and θ being the elasticity of substitution among intermediate goods of the same sector. Real profits, $j_{i,t}$ are given by:

$$j_{i,t} = \frac{P_{i,t}^I}{P_t} y_{i,t} - f^F + b_{i,t} - w_t l_{i,t} - \left(1 + r_t^b\right) b_{i,t}, \tag{4}$$

where $\frac{P_{i,t}^I}{P_t} y_{i,t}$ are total real revenues in term of the CPI index, $b_{i,t}$ is firms i real amount of borrowing from the banking sector at the beginning of time t. It is used by the firm to pay the fixed production cost $f_t^F = f^F$ for the period t to households[15]. Loans are paid back to the bank at the end of the same period at the net interest rate r_t^b. The variable w_t is the real wage and $l_{i,t}$ is firm i labor input. Using the retail sector demand and substituting for $w_t l_{i,t} = mc_{i,t} y_{i,t}$, the optimal problem can be rewritten as follows:

$$\max_{\{P_{i,t}\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[\frac{P_t^I}{P_t} \left(\frac{P_{i,t}}{P_t^I}\right)^{1-\theta} y_t^R(k) - mc_{it} \left(\frac{P_{i,t}}{P_t^I}\right)^{-\theta} y_t^R(k) - \left(1 + r_t^b\right) f^F \right]. \tag{5}$$

[15] Since we assume that households are the owner of firms and their plants, the fixed cost can be viewed as a constant cost that a firm pay to household in each period for using its plant. Alternatively, the fixed cost can be viewed as a constant lump-sum tax payed by firms to the Government. Considering the latter assumption would not affect the main results of the paper.

The FOC with respect to $P_{i,t}$ yields:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_{i,t}^{I}} &= (1-\theta)\frac{P_t^I}{P_t^I P_t}\left(\frac{P_{i,t}}{P_t^I}\right)^{-\theta} y_t^R(k) + \theta mc_{i,t}\left(\frac{P_{i,t}}{P_t^I}\right)^{-\theta-1}\frac{y_t^R(k)}{P_t^I} \\ &= 0. \end{aligned} \quad (6)$$

Multiplying by $\frac{P_t^I}{y_t^R(k)}$ and rearranging we get:

$$P_{i,t} = \frac{\theta}{\theta-1} mc_{i,t} P_t. \quad (7)$$

Equation (7) simply states that the optimal price of firm i is a markup over its nominal marginal costs, $mc_{i,t}^{Nom} = mc_{i,t}P_t$.

Then, defining $\rho_{i,t} = \frac{P_{i,t}}{P_t}$ we can rewrite the optimal price in relative terms,

$$\rho_{i,t} = \frac{\theta}{\theta-1} mc_{i,t} = \mu mc_{i,t}, \quad (8)$$

where $\mu = \frac{\theta}{\theta-1}$ is the gross markup.

Distribution of Productivity Draws According to Melitz (2003) and Ghironi and Melitz (2005), firm productivity draws are Pareto distributed. The cumulative distribution function (CDF) implied for productivity $z_{i,t}$ is $G(z_{i,t}) = 1 - \left(\frac{z_{\min}}{z_{i,t}}\right)^{\xi}$, while we denote by $g(z_{i,t}) = \xi \frac{z_{\min}^{\xi}}{z_{i,t}^{\xi+1}}$ the probability distribution function (PDF). The parameters $z_{\min}$ and $\xi > \theta - 1$ are scaling parameters of the Pareto distribution, representing respectively the lower bound and the shape parameter, which indexes the dispersion of productivity draws. As ξ increases dispersion decreases and firm productivity levels are increasingly concentrated towards their lower bound $z_{\min}$.

Endogenous Entry and Exit Prior to entry firms are identical and face a fixed sunk cost of entry $f^E > 0$. Entrants are forward looking, so that the entry condition will be

$$\tilde{v}_t = \widetilde{j}_t + \beta E_t \left(1 - \eta_{t+1}\right) \tilde{v}_{t+1} = f^E, \quad (9)$$

where $\tilde{v}_t$ is the average firms value, given by the sum of current average profits, $\widetilde{j}_t$, and the next period discounted average value of firms, i.e.

$\beta E_t \left(1-\eta_{t+1}\right)\tilde{v}_{t+1}$. Notice that $\tilde{v}_{t+1}$, is discounted not only by β but also by the probability of firms default in the next period η_{t+1}, which dynamically affects firms decision on entry, thus creating an important transmission channel between exit and entry decisions. Indeed, the higher the probability of firms' default, the lower is firms expected average value and thus the lower will be firms entry. Notice that with respect to Bilbiie at al (2012) the extra term $\widetilde{j}_t$ in the entry condition comes from the fact that we assume that there is no time to build for new entrants. Indeed, our timing assumptions are the following. Upon entrance new entrants borrow from the banks to pay the fixed production cost f^F. This cost is paid at the beginning of each production period by both new entrant and incumbent firms.[16] Immediately after, they both draw their firm specific productivity level from a Pareto distribution. Then, the aggregate shock arrives and firms immediately start producing, unless they decide to exit. Exiting firms do not repay loans to banks. Using this timing assumption, the decision of new entrants to exit the market is identical to the decision of incumbent firms. In particular, both new entrants and incumbent firms decide to produce as long as their specific productivity $z_{i,t}$ is above a cutoff level $\overline{z}_t$. The latter value is the level of productivity that makes the sum of current and discounted future profits (i.e. the firms value) equal to zero. Otherwise, firms will exit the market before producing. The cut off level of productivity, $\overline{z}_t$, is therefore determined by the following exit condition:

$$v_t\left(\bar{z}_t\right) = j_{\bar{z},t}\left(\bar{z}_t\right) + \beta E_t\left\{\left(1-\eta_{t+1}\right)v_{t+1}\left(\bar{z}_{t+1}\right)\right\} = 0, \tag{10}$$

with

$$j_t\left(\bar{z}_t\right) = y_t\left(\bar{z}_t\right) - w_t l_{\bar{z},t} - \left(1+r_t^b\right)f^F, \tag{11}$$

where $j_t\left(\bar{z}_t\right)$ are current profits of the firm with a productivity $z_{i,t} = \overline{z}_t$. In other words, before they start producing both new entrants and incumbents know exactly their time t profits. Consequently, if the sum of these profits and of all their expected future profits is non-positive they will exit the market before producing. The exit probability $\eta_{t+1} = 1 - \left(\frac{z_{\min}}{\bar{z}_{t+1}}\right)^{\xi}$ is thus endogenously determined. As in Ghironi and Melitz (2005), the lower bound productivity $z_{\min}$ is low enough relative to the production costs so that $\overline{z}_t$

[16] Notice that the entry cost and the production cost are two different cost. The first one is a sunk-cost payed only once and only by new entrants, before entering the market. While the second one is payed in every period by both firms types, i.e. incumbents and new entrants.

is above $z_{\min}$. In each period, this ensures the existence of an endogenously determined number of exiting firms: the number of firms with productivity levels between $z_{\min}$ and the cutoff level $\overline{z}_t$ are separated and exit the market without producing.

Notice that, under these assumptions the number of firms in the economy at period t will be:

$$N_t = (1 - \eta_t)\left(N_{t-1} + N_t^E\right). \tag{12}$$

3.2 Average and Aggregate Variables

From now on for any generic variable x we use $x_{i,t} = x_{i,t}(z_{i,t})$ to indicate a variable belonging to the firm with productivity equal to $z_{i,t}$. Analogously $x(\tilde{z}_t)$ indicates the value of the same variable belonging to the firm whose productivity is equal to the average productivity $\tilde{z}_t$. We define the average value of the variable x as $\tilde{x}$. We show that not always $\tilde{x} = x(\tilde{z}_t)$. Finally, we define aggregate variables using capital letters.

3.2.1 Firms Average Productivity

Following Ghironi and Melitz (2005), the average productivity of the intermediate good sector is:

$$\tilde{z}_t \equiv \left[\frac{1}{1 - G(\overline{z}_t)} \int_{\overline{z}_t}^{\infty} z_{i,t}^{1-\theta} dG(z_{i,t})\right]^{\frac{1}{\theta-1}}, \tag{13}$$

where $1 - G(\overline{z}_t) = \left(\frac{z_{\min}}{\overline{z}_t}\right)^{\xi}$ is the share of firms with a level of productivity $z_{i,t}$ above the cut off level $\overline{z}_t$. In other words, it is the firms' probability to remain in the market and produce at time t.

3.2.2 Aggregate Price Index and the Average Relative Price: the Intermediate Sector

The aggregate price level of the intermediate sector k is defined as

$$\begin{aligned} P_t^I(k) &= \left[\frac{1}{1 - G(\overline{z}_t)} \int_{\overline{z}_t}^{\infty} N_t (P_{i,t})^{1-\theta} g(z_i)\, dz_i\right]^{\frac{1}{1-\theta}} \\ &= N_t^{\frac{1}{1-\theta}} \left[\frac{1}{1 - G(\overline{z}_t)} \int_{\overline{z}_t}^{\infty} (P_{i,t})^{1-\theta} g(z_i)\, dz_i\right]^{\frac{1}{1-\theta}} \end{aligned} \tag{14}$$

since each intermediate sector k faces the demand of the retail sector k, solving the Dixit Stiglitz problem of the retail sector we find that the demand of good i is $y_{i,t}(z_{i,t}) = \left(\frac{P_{i,t}}{P_t^I(k)}\right)^{-\theta} Y_t^R(k)$, where $Y_t^R(k)$ is the aggregate demand of the retailer k. Solving for $P_{i,t}$

$$P_{i,t} = \left(\frac{y_{i,t}(z_{i,t})}{Y_t^R(k)}\right)^{-\frac{1}{\theta}} P_t^I(k) \tag{15}$$

and thus

$$P_t(\tilde{z}_t) = \left(\frac{y_t(\tilde{z}_t)}{Y_t^R(k)}\right)^{-\frac{1}{\theta}} P_t^I(k) \tag{16}$$

is the price of the firm with the average productivity $\tilde{z}_t$. Using (15) we can rewrite (14) as

$$P_t^I(k) = N_t^{\frac{1}{1-\theta}} \left[\frac{1}{1-G(\overline{z}_t)} \int_{\overline{z}_t}^{\infty} \left(\left(\frac{y_{i,t}(z_{i,t})}{Y_t}\right)^{-\frac{1}{\theta}} P_t^I(k)\right)^{1-\theta} g(z_i)\, dz_i\right]^{\frac{1}{1-\theta}}. \tag{17}$$

As shown in Melitz (2003) the relative output shares between two firms imply that $\frac{y_{i,t}(z_i)}{y_{k,t}(z_k)} = \left(\frac{z_{i,t}}{z_{k,t}}\right)^{\theta}$, and then $\frac{y_{i,t}(z_{i,t})}{y_t(\tilde{z}_t)} = \left(\frac{z_{i,t}}{\tilde{z}_t}\right)^{\theta}$. Using this result we can rewrite[17]

$$P_t^I(k) = N_t^{\frac{1}{1-\theta}} P_t^I(k) \left(\frac{y_t(\tilde{z}_t)}{Y_t}\right)^{-\frac{1}{\theta}}, \tag{18}$$

using equation (16) it implies that

$$P_t^I(k) = N_t^{\frac{1}{1-\theta}} P_t(\tilde{z}_t). \tag{19}$$

Due to symmetry across retail sector firms $P_t^I(k) = P_t^I$. Then, the aggregate price index of the intermediate sector is

$$P_t^I = N_t^{\frac{1}{1-\theta}} P_t(\tilde{z}_t). \tag{20}$$

Finally, since

$$P_t(\tilde{z}_t) = \left[\frac{1}{1-G(\overline{z}_t)} \int_{\overline{z}_t}^{\infty} (P_{i,t})^{1-\theta} g(z_i)\, dz_i\right]^{\frac{1}{1-\theta}}, \tag{21}$$

[17] See the Technical Appendix for details.

the average relative price is given by

$$\frac{P_t(\tilde{z}_t)}{P_t^I}\frac{P_t}{P_t} = N_t^{\frac{1}{\theta-1}} \tag{22}$$

and then

$$\rho(\tilde{z}_t) = N_t^{\frac{1}{\theta-1}}\rho_t^I \tag{23}$$

where we define $\rho(\tilde{z}_t) = \frac{P_t(\tilde{z}_t)}{P_t}$ and $\rho_t^I = \frac{P_t^I}{P_t}$.

Similarly, firms average profits are

$$\widetilde{j}_t = j(\widetilde{z}_t) = \rho^I N_t^{-1} Y_t - w_t N_t^{-1} L_t - \left(1 + r_t^b\right) f^F, \tag{24}$$

thus, they coincide with the profits of the firm that obtains the average productivity $\widetilde{z}_t$.[18]

3.2.3 Firms: Retailers

For the sake of simplicity we assume one-to-one relation between the number of retail sectors and the number of intermediate good-producing sectors. Each retailer $k \in (0,1)$ in the retail sector bundles the goods produced by the intermediate sector k under monopolistic competition, facing Rotemberg (1982) price adjustment costs. The new good of the retailer k is thus,

$$Y_t^{\varrho}(k) = \left[\int_{N_t} y_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}.$$

This good is sold to the household at the price $P_{k,t}^R$. Since all firms in the retail sector are identical, they all set the same price maximizing their real profits, $j_{k,t}^R$ given by:

$$j_{k,t}^R = \frac{P_{k,t}^R}{P_t} Y_t^R(k) - \frac{\int_{N_t} P_{i,t} y_{i,t}}{P_t} - pac_{k,t}, \tag{25}$$

$$s.t.: Y_t^R(k) = \left(\frac{P_{k,t}^R}{P_t}\right)^{-\theta} Y_t^d \tag{26}$$

[18] The derivation of average real profits and the proof for $\widetilde{j}_t = j(\widetilde{z}_t)$ is in the Technical Appendix.

where $Y_t^R(k) = \left(\frac{P_{k,t}^R}{P_t}\right)^{-\theta} Y_t^d$ is the household demand for the differentiated final good k, with P_t being the CPI index, while Y_t^d is the aggregate demand for output. The term $pac_{kt} = \frac{\tau}{2}\left(\frac{P_{k,t}}{P_{k,t-1}} - 1\right)^2 \frac{P_{k,t}}{P_t} Y_t^R(k)$ represents the Rotemberg (1982), with $\tau > 0$. After solving the Dixit Stiglitz problem, according to which $P_t^I(k) Y_t(k) = \int_{N_t} P(i) y_t(i) di$, profits of the retail firm k can be rewritten as:

$$J_{k,t}^R = \left(\frac{P_{k,t}^R}{P_t} - \frac{P_t^I}{P_t}\right) Y_t^R(k) - \frac{\tau}{2}\left(\frac{P_{k,t}^R}{P_{k,t-1}^R} - 1\right)^2 \left(\frac{P_{k,t}^R}{P_t}\right)^{1-\theta} Y_t^d, \qquad (27)$$

and we can write the profit maximization function as

$$\max_{\{P_{k,t}\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[\left(\frac{P_{k,t}^R}{P_t} - \frac{P_t^I}{P_t}\right) Y_t^R(k) - \frac{\tau}{2}\left(\frac{P_{k,t}^R}{P_{k,t-1}^R} - 1\right)^2 \left(\frac{P_{k,t}^R}{P_t}\right)^{1-\theta} Y_t^d\right]$$

$$s.t.$$

$$Y_t^R(k) = \left(\frac{P_{k,t}^R}{P_t}\right)^{-\theta} Y_t^d$$

Substituting the constraint and solving for $P_{k,t}^R$ and imposing the symmetric equilibrium, that is $P_{k,t}^R = P_t$ and $Y_t^R(k) = Y_t$ yields to:

$$(1-\theta) + \theta\rho_t^I - \tau(\pi_t - 1)\pi_t - (1-\theta)\frac{\tau}{2}(\pi_t - 1)^2 + E_t\left\{\Lambda_{t,t+1}\tau(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right\} = 0 \qquad (28)$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate and where the stochastic discount factor, $\Lambda_{t,t+1}$, is defined as:

$$E_t\Lambda_{t,t+1} = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-1}(1 - \eta_{t+1})\right\}. \qquad (29)$$

Notice that, since the exit probability changes along the business cycle, it now affects the dynamics of the stochastic discount factor.

3.3 Aggregate Output and Price

Aggregate output is given by the following CES technology:

$$Y_t = \left[\int_0^1 (Y_{k,t})^{\frac{\theta-1}{\theta}} dk\right]^{\frac{\theta}{\theta-1}}, \qquad (30)$$

the aggregate price index is:

$$P_t = \left[\int_0^1 P_{k,t}^{1-\theta} dk\right]^{\frac{1}{1-\theta}}.$$

The Technical Appendix shows that the aggregate price and output can be rewritten as,

$$P_t = N_t^{\frac{1}{1-\theta}} P_t(\tilde{z}_t)\left(\rho_t^I\right)^{-1}, \tag{31}$$

$$Y_t = N_t^{\frac{\theta}{\theta-1}} y_t(\tilde{z}_t) = \rho_t(\tilde{z}_t) A_t \tilde{z}_t L_t. \tag{32}$$

3.4 Households

Households maximize their expected utility, which depends on consumption and labor hours as follows,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \frac{L_t^{1+\phi}}{1+\phi}\right), \tag{33}$$

where $\beta \in (0,1)$ is the discount factor and the variable L_t represents hours worked, while C_t is the usual consumption index:

$$C_t = \left(\int_0^1 C_{k,t}{}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{34}$$

where $C_{k,t} = \left(\int_{i\in N} C_{i,t}{}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$ is the good bundled by the retail sector and $C_{i,t}$ the production of the intermediate good-producing firm i. The parameter θ (being $\theta > 1$) is the elasticity of substitution between the goods produced in each sector. Households consume and work. They also decide how much to invest in new firms and in the shares of incumbent firms and how much to lend to the banking sector.

Households enter the period t earning an income from the deposits owned in the previous period $\frac{r_{t-1}^d}{\pi_t} D_{t-1}$, they then invest in a mutual fund of firms given by the sum of the already existing firms N_{t-1} and the new entrants at time t, N_t^E, where γ_t is the share of the mutual fund of firms held by the household, and $\widetilde{v}_t$ is the price paid, i.e. the firm value at the beginning of the period t. As previously discussed, both new entrants and incumbents firms borrow from the banking sector to pay the fixed production cost, they draw their firms specific productivity and then, after observing the aggregate shock, they decide whether to produce or exit the market. Those firms that are not separated produce and distribute their dividends $j_t(\tilde{z})$ to the household at the end of time t. At the end of the same period, the average value of the same share γ_t of mutual fund of firms will be $\widetilde{v}_{t+1}$. In addition to the labor income $w_t L_t$, and to the fixed costs received by the intermediate producers $F^F = N_t f^F$, households use dividends $j_t(\tilde{z})$, the new value of the mutual fund $\widetilde{v}_{t+1}$ and profits from retailers, j_t^R, to consume C_t or to save in the form of new deposits D_t. Thus, the household budget constraint is:

$$w_t L_t + F^F + \frac{r_{t-1}^d}{\pi_t} D_{t-1} + \underbrace{N_t \gamma_t \left(\widetilde{v}_{t+1} + j_t(\tilde{z})\right)}_{\text{End of period } t} + j_t^R = C_t + \left(D_t - \frac{D_{t-1}}{\pi_t}\right) + \underbrace{\left(N_{t-1} + N_t^E\right) \widetilde{v}_t \gamma_t}_{\text{Beginning of period } t}, \tag{35}$$

with

$$N_t = (1-\eta_t)\left(N_{t-1} + N_t^E\right). \tag{36}$$

Taking the first order conditions with respect to γ_t, D_t, C_t, L_t, combining households FOCs and imposing that in equilibrium $\gamma_t = \gamma_{t+1} = 1$, yields:

$$w_t = C_t L_t^{\phi}, \tag{37}$$

$$E_t \beta \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-1} \right\} = \frac{\pi_{t+1}}{\left(1 + r_t^d\right)}, \tag{38}$$

$$\widetilde{v}_t = E_t \beta \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left(1 - \eta_{t+1}\right) \left[\widetilde{v}_{t+1} + \widetilde{j}_t\right] \right\}, \tag{39}$$

which are respectively the households' labor supply, the Euler equation for consumption and the Euler equation for share holding.

3.5 The Banking Sector

3.5.1 Loans and Deposits Branches

The structure of the banking sector is a simplified version of Gerali et al. (2010). We assume that the bank is composed by two branches: the loan branch and the deposit branch. Both are monopolistic competitive, so that deposits from households and loans to entrepreneurs are a composite CES basket of a continuum of slightly differentiated products $j \in (0,1)$, each supplied by a single bank with elasticities of substitution equal to ε^b and ε^d

respectively. As in the standard Dixit–Stiglitz (1977) framework, loans and deposits demands are:

$$b_{j,t} = \left(\frac{r^b_{j,t}}{r^b_t}\right)^{-\varepsilon^b_t} b_t \quad \text{and} \quad d_{j,t} = \left(\frac{r^d_{j,t}}{r^d_t}\right)^{-\varepsilon^d_t} d_t, \tag{40}$$

where $b_{j,t}$ is the aggregate demand for loans at bank j, that is $b_{j,t} = \int_0^1 b_{k,j,t}dk = \int_0^1 \left[\int_{i\in N} b_{i,j,t}di\right]$, where $b_{k,j,t}$ is the total amount of loans demanded to bank j by sector k and b_t is the overall volume of loans to firms. Similarly, $d_{j,t}$ is the households aggregate demand for deposits to bank j, while d_t is the households overall demand for deposits.

The amount of loans issued by the loan branch can be financed through the amount of deposits, D_t, collected from households from the deposit branch or through bank capital (net-worth), denoted by K^b_t, which is accumulated out of retained earnings. Thus, the bank sector obey a balance sheet constraint,

$$B_t = D_t + K^b_t, \tag{41}$$

with the low of motion of the aggregate banking capital given by:

$$\pi_t K^b_t = (1-\delta^b)K^b_{t-1} + j^b_t, \tag{42}$$

where δ^b represents resources used in managing bank capital, while j^b_t are overall profits made by the retail branches of the bank.

Loans Rates and Deposits Rates Banks play a key role in determining the conditions of credit supply. Assuming monopolistic competition, banks enjoy market power in setting the interest rates on deposits and loans. This leads to explicit monopolistic markups and markdowns on these rates.

Each bank j belonging to the loan branch can borrow from the deposit bank j at a rate R^b_{jt}. We assume that banks have access to unlimited finance at the policy rate r_t from a lending facility at the central bank: hence, by the non-arbitrage condition $R^b_{j,t} = r_t$. The loan branch differentiates the loans at no cost and resell them to the firms applying a markup over the policy rate.[19] As in Curdia and Woodford (2009) we assume that banks are unable to distinguish the borrowers who will default from those who will repay, and

[19] All banks essentially serve all firms, providing slightly differentiated deposit and loan contracts.

so must offer loans to both on the same terms. The problem of the loan bank j is therefore,

$$\max_{\{r^b_{j,t}\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[r^b_{j,t} b_{j,t} (1-\eta_t) - r_t B_{j,t} - b_{j,t}\eta_t \right], \tag{43}$$

$$s.t. \quad b_{j,t} = \left(\frac{r^b_{j,t}}{r^b_t} \right)^{-\varepsilon^b_t} b_t, \tag{44}$$

where $b_{j,t} = \left(\frac{r^b_{j,t}}{r^b_t} \right)^{-\varepsilon^b} b_t$ is the demand for loans of bank j, $r^b_{j,t} b_{j,t} (1-\eta_t)$ are bank j net revenues, while $r_t B_{j,t}$ is the net cost due to the interest rate paid on the deposit rates. The additional term $b_{j,t}\eta_t$ is the amount of the notional value of the loans that it is not repaid by firms. This is a death weight loss for the bank and represents an extra-cost. From the FOC, after imposing symmetry across banks, i.e. $r^b_{j,t} = r^b_t$, and thus $b_{j,t} = b_t$ and $B_{j,t} = B_t = N_t f^F$, we get the equation for the optimal interest rate:

$$r^b_t = \left(\frac{\varepsilon^b}{\left(\varepsilon^b_t - 1\right)(1-\eta_t)} \right) (r_t + \eta_t), \tag{45}$$

where $\mu^{Lb}_t = \frac{\varepsilon^b}{\left(\varepsilon^b - 1\right)(1-\eta_t)}$ is the bank markup and $r_t + \eta_t$ is its marginal cost.[20] The bank marginal cost is the sum of two components: i) r_t, i.e. the net interest rate that the bank has to pay to the deposit branch for each loan. This is the only effective cost per loan in the case the bank is able to have back the notional value of the loan from defaulting firms. ii) η_t represents instead the additional cost per loan faced by the bank due to firms defaulting and not repaying the loan.

Notice that $\frac{d\left(\mu^{Lb}_t\right)}{d\eta_t} = \frac{1}{\varepsilon - 1} \frac{\varepsilon + 1}{(\eta_t - 1)^2} > 0$, implying a positive relationship between firms' exit and the value of the bank markup. Indeed, as the expected probability of exit increases, retail banks increase their markup and set higher interest rate. The intuition is straightforward. An increase in the firms' exit probability imply that the probability that a firm do not repay the loan increases. As a consequence the bank that has issued that loan faces

[20] Indeed, in the symmetric equilibrium total costs are given by $CT^b_t = r_t b_t + b_t \eta_t$. Thus bank's marginal costs are $MC^b_t = \frac{dCT^b_t}{db_t} = r_t + \eta_t$.

lower expected profits. To restore its profits the bank is forced to increase the interest rate on loan.

The deposit branch collects deposits from households and gives them to the loans unit, which pays r_t. The problem for the deposit branch is then

$$\max_{\left\{r^d_{j,t}\right\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[r_t D_{j,t} - r^d_{j,t} d_{j,t} - \frac{\kappa_d}{2} \left(\frac{r^d_{j,t}}{r^d_{j,t-1}} - 1 \right)^2 r^d_t d_t \right], \quad (46)$$

$s.t.$

$$d_{j,t} = \left(\frac{r^d_{j,t}}{r^d_t} \right)^{-\varepsilon^d} d_t \text{ and } D_{j,t} = d_{j,t}, \quad (47)$$

where $d_{j,t} = \left(\frac{r^d_{j,t}}{r^d_t} \right)^{-\varepsilon^d} d_t$ is the demand for deposits of bank j. From the FOC, after imposing symmetry across banks, i.e. $r^d_{j,t} = r^d_t$, and thus $d_{j,t} = d_t$ and $D_{j,t} = D_t$, we get the optimal interest rate for deposits,

$$r^d_t = \frac{\varepsilon^d}{\varepsilon^d - 1} r_t \quad (48)$$

$\frac{d\left(\frac{\varepsilon}{\varepsilon-1}\right)}{d\varepsilon} = -\frac{1}{(\varepsilon-1)^2} < 0$, i.e. the interest rate on deposits is markdown over the policy rate r_t.

Aggregate bank profits are the sum of the profits of the branches of the bank. Thus, they are also affected by the firms' exit probability and given by:

$$j^b_t = r^b_t B_t (1 - \eta_t) - r^d_t D_t - B_t \eta_t. \quad (49)$$

where $B_t \eta_t$ is the total amount of the loans not repaid to the banks.

3.6 Monetary Policy

To close the model we specify an equation for the Central Bank behavior. We simply assume that the monetary authority set the nominal interest rate r_t following a standard Taylor-type rule given by

$$\ln \left(\frac{1 + r_t}{1 + r} \right) = \phi_R \ln \left(\frac{1 + r_{t-1}}{1 + r} \right) + (1 - \phi_R) \left[\phi_\pi \ln \left(\frac{\pi_t}{\pi} \right) + \phi_y \ln \left(\frac{Y_t}{Y} \right) \right], \quad (50)$$

where $\ln\left(\frac{\pi_t}{\pi}\right)$ and $\ln\left(\frac{Y_t}{Y}\right)$ are respectively the deviations of inflation and output from their steady state values, ϕ_π and ϕ_y being the elasticities of the nominal interest rate with respect to these deviations. Finally, ϕ_r is the interest rate smoothing parameter.

4 Business Cycle Dynamics

In what follows we study the impulse response functions (IRFs) to two types of productivity shocks: i) a standard productivity shock, i.e. a shock to the level of the aggregate productivity A_t. ii) An uncertainty shock, which is instead a shock to the volatility of the aggregate productivity. We model this shock by using the stochastic volatility approach as proposed by Fernandez-Villaverde et al. (2011), i.e. assuming time varying volatility of the innovation of the aggregate productivity, labeled $\sigma_{a,t}$.

More in details, we assume that the aggregate productivity follows a process of the form:

$$\ln(A_t/A) = \rho_a \ln(A_{t-1}/A) + \sigma_{a,t} u_t^a, \tag{51}$$

where A is the steady state value of A_t and where the innovation u_t^a is a standard normal process. The time-varying standard deviation of the innovations, $\sigma_{a,t}$, that is the uncertainty shock, follows this stationary process:

$$\ln(\sigma_{a,t}/\sigma_a) = \rho_a \ln(\sigma_{a,t-1}/\sigma_a) + \eta_\sigma u_t^\sigma, \tag{52}$$

where the innovation u_t^σ is a standard normal process and η_σ is the (constant) standard deviation of the uncertainty shock. In this Section we study the model dynamics in response to both shocks, by taking into account each shock at the time.

4.1 Calibration

Calibration is set on a quarterly basis. The discount factor, β, is set at 0.99. The inverse of Frisch elasticity of labor supply is $\phi = 2$. As in BGM (2012), we set the steady state value of the exit probability η to be 0.025, this needs that ξ is set equal to 7.76. A value of $\eta = 0.025$ matches the U.S. empirical evidence of 10% of firms destruction per year. The elasticity of substitution among intermediate goods, θ, is set equal to 3.8, a value which is in line with Ghironi and Melitz (2005) and BGM (2012). It also ensures that the condition for the shape parameter $\xi > \theta - 1$ is satisfied in the model with endogenous exit. The lower bound of productivity distribution, $z_{\min}$, is equal to 1. Further, as in BGM (2012), Etro and Colciago (2010) and Colciago and Rossi (2012), we set the entry cost $f^E = 1$. The fixed costs f^F is set such that in all the economies considered they correspond to 5% of total output produced. We translate the Rotemberg cost of adjusting prices, τ, into an equivalent Calvo probability that firms do not adjusted prices equal to 0.67, a value close to the ones obtained in the empirical literature (see for example Christiano et al 2005, among others).

We calibrate the banking parameters as in Gerali et al. (2010). For the deposit rate, we calibrate $\varepsilon^d = -1.46$. Similarly, for loan rates we calibrate $\varepsilon^b = 3.12$. The steady-state ratio of bank capital to total loans, i.e. the capital-to-asset ratio, is set at 0.09. As done for the computation of the correlation with real GDP. When we run the shock to the level of the productivity, we set the parameters as follows: the steady state of productivity A is equal to 1, its standard deviation is 0.0035, while its persistence is set to 0.94, as found by Smets and Wouters (2007) and very close to the value found in our VAR, for the labor productivity.

The parameter of the uncertainty shock follows Leduc and Liu (2016), so that the persistence of the shock is $\rho_\sigma = 0.76$, the steady state of the shock, σ^a, is set to 0.01, while $\eta_\sigma = 0.392$;

Finally, we consider a Taylor rule, with $\phi_R = 0.75$, $\phi_\pi = 2.15$ and $\phi_y = 0.1$. This rule guarantees the uniqueness of the equilibrium. Further, these parameters are in the range of the values estimated for the US economy.[21]

4.2 Productivity Shocks

We now show the IRFs to a positive shock to the level of productivity. To capture the importance of the endogenous exit mechanism, we compare the IRFs of our baseline model (labeled as *Endogenous Exit)* with those of a model where firms exit probability is exogenous and constant (label as *Exogenous Exit).* In both models the banking structure is characterized by monopolistic competition in the loans and in the deposits branch and by the assumption that firms exiting the market do not repay the loan.

Then, to better understand the interplay between the endogenous exit

[21] See for example Smets and Wouters (2007). The qualitative results and the comparison with the exogenous exit model and with the model with efficient banks are not qualitatively altered by the choice of the Taylor rule.

mechanism and the banking sector, we compare the performance of our baseline model with a version of the model characterized by an efficient banking sector. In this case, we label the first model as *Endogenous Exit MB,* while we label the second model as *Endogenous Exit EB.* In the second model banks are efficient since they compete under perfect competition. Further, banks can completely insure against the risk of not having the loans repaid. These two assumptions imply that the bank markup is zero and that both the loan rate and the deposit rate collapse to the policy rate. Further, since banks profits are zero the bank balance sheet constraint becomes $D_t = B_t$, i.e. the banks net worth is zero.

4.2.1 Impulse Response Functions: Endogenous versus Exogenous Exit

Figure 3 shows the IRFs to a positive shock to the level of productivity, A_t, in the two models considered. Notice that, in both models a positive productivity shock lowers real marginal costs and creates expectations of future profits which lead to the entry of new firms. The entry margin results in a strong and persistent increase in output. This is the standard propagation mechanism implied by the BGM (2012) model. With the introduction of the endogenous exit margin, the number of firms exiting the market becomes countercyclical and the propagation of the shock is much stronger. The reason is threefold. First, the increase in the productivity leads to higher profits and thus to a lower cut-off level of productivity, $\bar{z}_t$ in the model with endogenous exit. This implies a reduction in firms' exit probability and thus a decrease in the number of firms exiting the market, further amplifying the response of output. Second, since firms entry decisions negatively depend on firms exit probability, also the response of new entrants is stronger in the endogenous exit model. Third, a decrease in the exit probability implies an higher probability for firms to repay the loan, which in turn induces banks to reduce their markups. The countercyclical markup results in countercyclical spread between the loan rate and the policy rate.[22] Firms' cost for borrowing reduces, further reducing firms profits and thus giving an extra boost to output. Finally, the model with endogenous exit not only implies a stronger propagation mechanism than the standard BGM (2012) framework, but it also matches three important stylized facts:

[22] The spread between the loan rate and the deposit rate is also countercyclical. Notice that, the IRFs of the inflation rate, the interest rates are all in annual terms.

i) the countercyclicality of the *number* of firms exiting the market; ii) the countercyclicality of the bank markup; iii) the countercyclicality of the loan spread. The counterfactual on firms destruction implied by the model with exogenous exit, depends exclusively on having assumed an exogenous and constant exit probability. The model with exogenous exit also implies an exogenous and constant banks' markup.

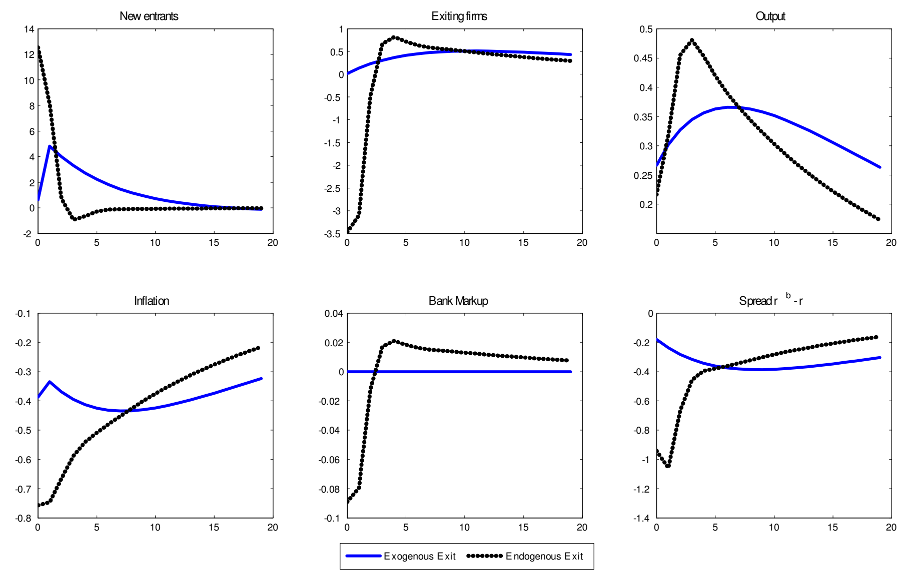

Figure 3: IRFs to a positive productivity shock. Endogenous versus exogenous exit model

4.2.2 Impulse Response Functions: Efficient versus Inefficient Banks

Figure 4 shows the IRFs in response to a positive shock to the level of productivity in the model with endogenous exit, comparing two alternative models: the baseline model with monopolistic banks and the model with efficient banks. Notice that, the first difference between the two models is that in the model with monopolistic banks the banks' markup is countercyclical. This leads to a stronger amplification mechanism of the shock than in the model with efficient banks. The reason is the following. In the model with inefficient banks firms anticipate the positive effect of their expected death probability on the loan rate, and discount less their future profits.

This in turn implies that firms set lower prices and produce more output. The stronger reduction of inflation is then followed by a stronger decrease of the policy rate and by an even stronger fall in the loan rate, so that the loan spread decreases. This leads to an increase in firms profits, higher entry and lower exit and thus to a further increase in output with respect to the model with efficient banks.

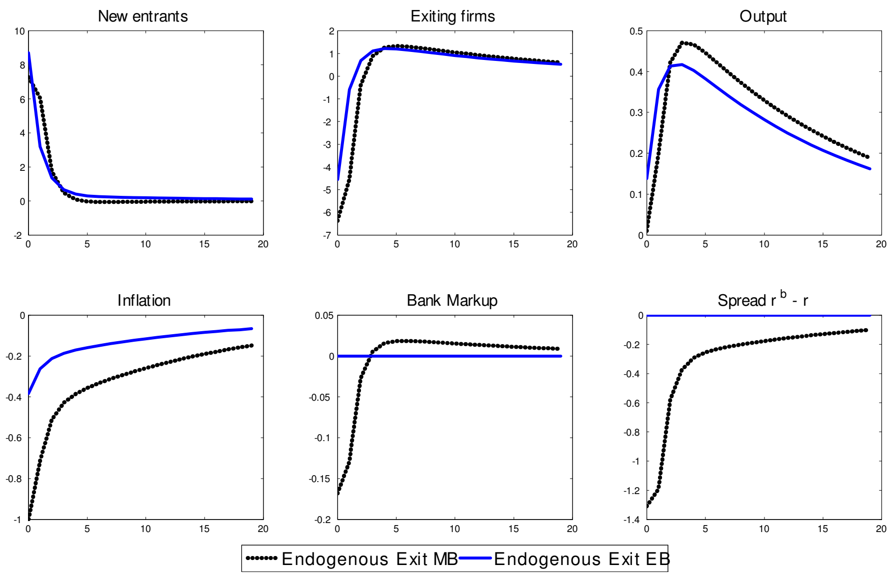

Figure 4: IRFs to a positive productivity shock. Endogenous exit model with monopolistics banks (dotted lines) versus endogenous exit model with efficient banks (solid lines).

4.3 Uncertainty Shocks

We now show the IRFs to an uncertainty shock, which is a shock to the volatility of the aggregate productivity. To examine the dynamic effects of the uncertainty shock, we solve the model using third-order approximations to the equilibrium conditions around the steady state. We follow the procedure suggested by Fernandez-Villaverde et al. (2011) to compute the impulse responses.[23] .

[23] In particular, using Dynare, we first simulate the model (using a third-order approximations to the decision rules) for 2,096 periods, starting from the deterministic steady state. We the drop the first 2,000 periods to avoid dependence on initial conditions and

Figure 5 compares the performance of our baseline model (as before labeled as *Endogenous exit MB*) with the endogenous exit model with efficient banks (labeled as *Endogenous Exit EB).*

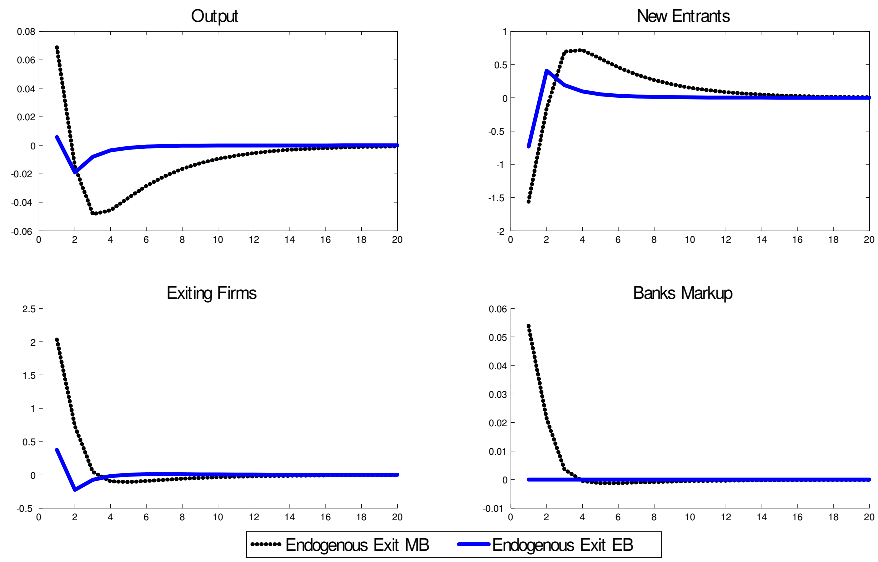

Figure 5: IRFs in response to an uncertainty shock. Baseline model with monopolistics banks (dotted lines) versus endogenous exit model with efficient banks (solid lines).

Notice that in both models an uncertainty shock is followed by an increase in firms destruction and a decrease in firms entry, together with a reduction in output. The recessionary effects are stronger in the model with monopolistic banks (at least in the medium run), which is indeed characterized by an increase in the banks' markup. This makes the cost of loans higher, thus reducing firms profits opportunities by a greater amount with respect to the model with efficient banks.

we use the remaining 96 periods to compute the ergodic mean of each variable. Then, starting from the ergodic means, we run two different simulations of 20 periods each, one with an uncertainty shock (i.e. a one-standard-deviation increase in uncertainty in the first period) and the other with no shocks. Finally, we compute the IRFs as the percentage differences between these two simulations.

5 Empirical Evidence on Uncertainty Shocks

To provide evidence on the relevance of uncertainty shocks, we now estimate a small BVAR model and show the impulse responses to orthogonalized shocks to macroeconomic uncertainty. As a proxy for the aggregate macroeconomic uncertainty we use the CBOE Volatility Index (VIX) downloaded from FRED database. Data on Real GDP, Inflation, firms' Births and Deaths, and Bank Markup are the same used in Section 2. Given the sample size of the series of Births and Deaths we estimate a BVAR using the sample: 1993Q3-2015Q1. Against the short sample background we choose to estimate the model with Bayesian techniques, this avoids sampling errors in estimating error bands for the impulse responses that may occur when estimating a highly over parameterized model (see Sims and Zha, 1998). The BVAR model has the following form:

$$Y_t = c + B_1 Y_{t-1} + ... + B_p Y_{t-p} + \epsilon_t, \qquad \text{where } \epsilon_t \sim N(0, \Sigma),$$

where $Y_t = [VIX, Inflation, \Delta RGDP, \Delta Births, \Delta Deaths, BankMarkup]$ is the vector of the variable used in the BVAR, i.e.: the CBOE Volatility Index (VIX), the Inflation rate, the logarithm of real GDP, the logarithm of firms Births and Deaths, the proxy of the Bank Markup. For each variable Δx represents the first difference of x. $B_1, B_2 ... B_p$ are autoregressive matrix and Σ is the variance-covariance matrix. We estimate a BVAR(1) and for the prior distribution of the parameters we choose a Normal-Diffuse prior of 0.8 on the autoregressive coefficient of the first lag. Following Leduc and Liu (2016), among many others,[24] we choose a lower triangular Cholesky identification, ordering the VIX index first, such that on impact shocks to the uncertainty index affect the other variables, while shocks to the other variables do not affect the VIX index on impact.

Figure 6 shows the impulse responses to a VIX shock. The median responses of the endogenous variables to one-standard-deviation increase in the innovations to uncertainty are depicted by solid lines, while shaded areas represent 84 percent credible intervals. Notice that, uncertainty shocks have a substantial impact on the other endogenous variables. While real GDP declines by 0.1 percent and remains below zero for first 12 quarters, establishments births declines by almost 1 percent and stays below zero for 6 quarters, similarly the number of establishments deaths increases by 0.5 percent. The bank markup increases by 0.5 percent points on impact and

[24] This ordering has been largerly used in the literature (see for example., Bloom, 2009).

shows a very persistent pattern. Notice, that the credible intervals of the markup are large, even though they are always above zero. Importantly, all these responses are qualitatively similar to the responses implied by our baseline model. In particular, the response of establishments deaths show a very similar pattern of its model equivalent, i.e. firms exit. They both increase on impact and then the slightly go below zero for few time The response of inflation is positive on impact and becomes negative from the second periods, in line with the results find by Leduc and Liu (2016), among others.

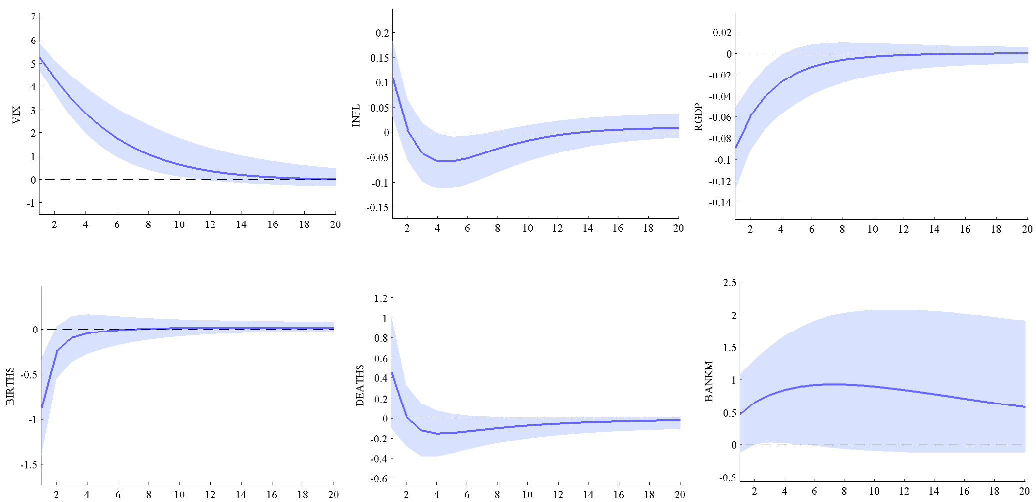

Figure 6. IRFs to an orthogonalized shock to the CBOE Volatility Index (VIX). Using Normal Diffuse Priors.

Finally, Figure 7 shows the IRFs obtained estimating the same BVAR using Minnesota priors with a 0.8 on the first lag. Notice that, all the results are confirmed. Importantly, the credible intervals for establishments deaths and those for banks' markup are narrow and the responses on impact are more significant.

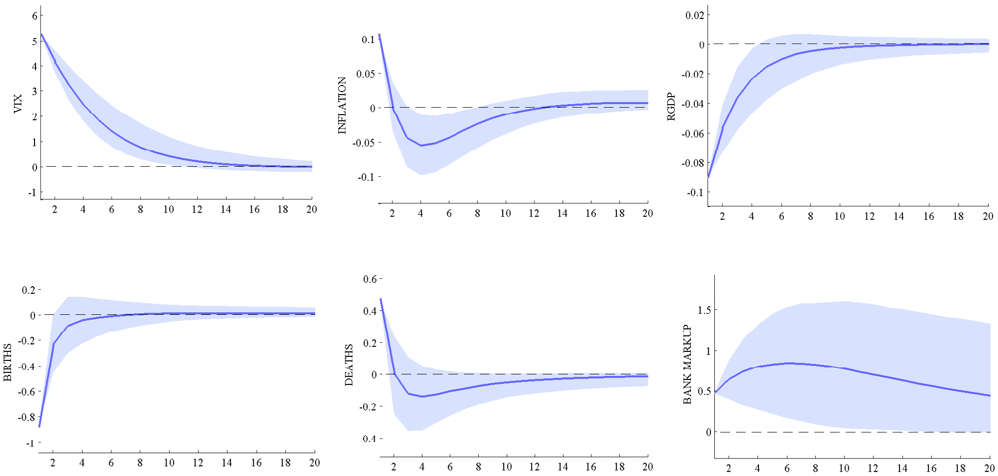

Figure 7. IRFs to an orthogonalized shock to the CBOE Volatility Index (VIX). Using Minnesota Priors.

6 Conclusion

We develop a NK-DSGE model with inefficient banks, together with endogenous firms' exit and entry decisions. We analyze the relationship between firms dynamics and banking in response to a shock to the level of the aggregate productivity as well as to a shock to the volatility of the aggregate productivity, i.e. to an uncertainty shock. We find the following results. First, in response to a shock to level of the aggregate productivity, economies characterized by endogenous firms exit present higher volatility of both real and financial variables than those implied by a standard BGM (2012) model with inefficient banks. Second, the endogenous exit margin implies countercyclical exit of the number of firms along with countercyclical banks' markups, thus being in line with the empirical evidence. Third, uncertainty shocks are recessionary and imply a decrease in the number of new entrants, an increase in the number of firms default and an increase in the banks' markup. Furthermore, our baseline model presents a stronger and more prolonged recession in the medium run than a model with efficient banks, in face of an uncertainty shock. Finally, estimating a small BVAR, using the CBOE Volatility Index (VIX) as a proxy for the aggregate macroeconomic uncertainty, we find that our theoretical results are well supported by the empirical responses to the uncertainty shock.

This paper is only a first attempt to understand the interactions between firms dynamics, and in particular the dynamics of the exit margin and banking. We strongly believe that further investigation, both from a theoretical and an empirical point of view, is needed on this issue. In this respect, the model can be extended along several dimensions. First, considering a different borrowing mechanism, where firms can borrow against a collateral, might be interesting to investigate. The estimation of the model through Bayesian techniques is also a future step of our research. Finally, investigating the role of firms endogenous exit in affecting welfare and the optimal monetary prescriptions is also part of our agenda.

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