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Paolo Giudici
(Università di Pavia)

Peter Sarlin
(Hanken School of Economics)

Alessandro Spelta
(Università Catttolica di Milano)

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Via San Felice, 5
I-27100 Pavia
http://epmq.unipv.eu/site/home.html

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The multivariate nature of systemic risk: direct and common exposures

P. Giudici*
University of Pavia

P. Sarlin†
Hanken School of Economics and RiskLab Finland,
A. Spelta ‡
Catholic University and Complexity Lab in Economics, Milan, Italy

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Abstract

To capture systemic risk related to network structures, this paper introduces a measure that complements direct exposures with common exposures, as well as compares these to each other. Trying to address the interconnected nature of financial systems, researchers have recently proposed a range of approaches for assessing network structures. Much of the focus is on direct exposures or market-based estimated networks, yet little attention has been given to the multivariate nature of systemic risk, indirect exposures and overlapping portfolios. In this regard, we rely on correlation network models that tap into the multivariate network structure, as a viable means to assess common exposures and complement direct linkages. Using BIS data, we compare correlation networks with direct exposure networks based upon conventional network measures, as well as we provide an approach to aggregate these two components for a more encompassing measure of interconnectedness.

JEL: G01, C58, C63

Keywords: Bank of International Settlements data, Correlation networks, Exposure networks

*Department of Economics and Management, University of Pavia, Pavia, Italy. *E-mail:* paolo.giudici@unipv.it.

†Hanken School of Economics and RiskLab Finland. *E-mail:* peter@risklab.fi

‡Department of Economics and Finance*E-mail:* alessandro.spelta@unicatt.it

1 Introduction

The last few years have witnessed an increasing research literature on systemic risk (for a definition see, for example, Allen and Gale, 2000; Acharya, 2009), with the aim of identifying the most contagious institutions and their transmission channels. Specific measures of systemic risk have been proposed for the banking sector; in particular, by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015). On the basis of market prices, these authors calculate the quantiles of the estimated loss probability distribution of a bank, conditional on the occurrence of an extreme event in the financial market.

The above approach is useful to establish policy thresholds aimed, in particular, at identifying the most systemic institutions. However, it is a bivariate approach, which allows to calculate the risk of an institution conditional on another or on a reference market but, on the other hand, it does not address the issue of how risks are transmitted between different institutions in a multivariate framework.

Trying to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, that combine the rich structure of financial networks (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who propose measures of connectedness based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013) and Ahelegbey et al. (2015) extend the approach introducing stochastic graphical models.

While the literature on correlation networks has focused on dependence structure among market prices, the focus in this paper is on correlations in network structures. Correlation network models, that tap into the multivariate network structure, seem a viable alternative to classical network models, as discussed in the recent papers by Brunetti et al. (2015). In particular, they seem to hold promise for assessing common exposures and complement direct linkages, in line with the general approach of Cai et al. (2014). However, the previous literature has neither compared the two models on the same application nor combined the two types of interconnectedness. This paper aims in particular at shedding light on these two problems, in the context of national interbank markets.

The network structure of national interbank markets has been studied, at the global level, using the Bank of International Settlements (BIS) data set: Garratt et al. (2011), McGuire and Tarashev (2006), Minoiu and Reyes (2013). In particular, Minoiu and Reyes (2013) used confidential data representing cross-border bilateral financial flows intermediated by national banking systems, and found evidence of important structural changes in financial banking networks, following the occurrence of stress events. The same authors pointed out that their result should be interpreted with some caution because of the large amount of non-reporting countries (their sample contains 184 countries, of which only 15 report bilateral positions to the BIS). Giudici and Spelta (2015) extended Minoiu and Reyes (2013), using data on the total financial exposure of each country with respect to the rest of the world: a database that, besides being publicly available, is more reliable. Applying a correlation network model to such data, one can establish indirect bilateral links between countries, that can be used to understand which countries are most central and, therefore, most contagious (or subject to contagion).

The methodological contribution of this paper is to formally compare classical networks and correlation based networks, using appropriate comparison metrics, in the modelling of interbank market flows between countries. Using the correlation as a measure of proximity in a multivariate framework, we provide measures of funding composition and portfolio similarities. From an applied viewpoint, we shed further light on the interpretation of country bilateral financial flows data, contained in the BIS statistics. We also provide an approach to aggregate the direct and indirect components of countries' exposures for a more encompassing measure of interconnectedness. Finally, we combine these measures with banking crises, in a standard early-warning setup, in order to evaluate whether and to what extent they are related to the build-up of imbalances prior to crisis events.

We find that total funding show an increase up to the 2007 financial crisis, followed by an abrupt fall even if some countries do not follow this general trend. Moreover, we find that the proximity between the funding composition of a country with respect to the others, as well as the portfolio composition, is generally decreasing for most countries. In particular, in 2000 the funding sets were highly correlated, whereas at the end of the time sample each country exhibits a specific funding composition. This means that the funding composition of most countries has become more and more concentrated on a limited number of specific lenders. This seems to be an important risk factor that has increased after the financial crisis, and is still in place. Moreover, results also suggest the predictive power of direct linkages are clearly outperformed by the other ways of defining relationships. While linkages based upon common exposures and a combination of direct and common exposures perform equally well in forecasting crisis episodes, the predictive power obtained combining the two type of network is superior for a certain parameter range. This highlights the importance of common exposures. While the information contented in the two ways of defining exposures seems to be similar enough for the common and the mixed measures to perform on par, the results still point to the fact that the use of common exposures provide an added value in signaling crises. From an economic point of view, this clearly shows that common exposures, or so-called funding composition overlap, indeed are channels of contagion and should be accounted for when measuring systemic risk.

The paper is organized as follows. Section 2 introduces our methodological proposal. Section 3 describes the empirical results obtained with the application of both models to the Bank of International Settlement cross-border financial flow data. Section 4 compares our exposure measures in terms of performance in signaling a crisis through the RiskRank. Finally, Section 5 contains some concluding remarks and future research directions.

2 Measuring common exposure through correlation models

Systemic risk concerns the risks posed by balance sheet relationships and interdependencies among players in a system or market, where the failure of a single entity can cause a cascading failure, which could potentially bring down the entire system or market. These balance sheet linkages can be represented by a network that describes the mutual relationships between the different economical agents involved.

A network can be represented by means of a graph $G = (V; E)$ that consists of a set V of n vertices and a set E of m edges. A weight w_{ij}, $i, j = 1, ..., m$ is possibly associated to each edge $(i; j)$ and, if this is the case, a weighted (or valued) graph is defined.

In the wake of the recent crisis it has been argued that network theory can enrich the understanding of financial systems, systemic risk, and the comprehension of the factors causing failures in financial markets. Usually, researchers approached financial systems through the study of connections among financial institutions exploring banking liabilities and claims because credit inter-linkages play a crucial role in propagating, absorbing or magnifying shocks. However, despite the fact that the topology of a network is known to play a major role in robustness against shocks, the lack of bilateral data have prevented the systematical investigation of the topological properties of the international financial network. Fortunately, whenever the data are missing or confidential correlation based networks seem a viable alternative to classical network models. While the literature has focused on dependence structures among market prices, the focus in this paper is on correlations in network structures.

Correlation networks are suitable for analyzing the structure of pairwise correlations among a set of N time series. The proximity measure:

$$d_{ij} = 2 - \sqrt{2\,(1 - C_{ij})} \tag{1}$$

where C_{ij} is the correlation coefficient between two time series $s(i)$ and $s(j)$ computed along a given time window, can be shown to satisfy all the metric axioms (Mantegna and Stanley, 2007). Therefore, it may be used to develop the topological analysis of the international financial network. To this end, we proceed as follows.

We associate different time series with different nodes of a network. Each pair of nodes can be thought to be connected by an edge, with a weight that can be related to the correlation coefficient between the two corresponding time series. A proximity network of n nodes can be derived by its associated $n \times n$ matrix of proximities $\mathbf{D}$, a weighted adjacency matrix, with elements $d_{i,j}$ described by equation (1). Having obtained the network, a natural further step is to describe each node centrality with an interconnectedness summary measure. This task is necessary, not only from a descriptive viewpoint, but also to provide an indicator that can act as a measure of systemic importance.

In the present study the set of nodes represent countries, while the set of edges depends on the definition or the meaning of a link. The aim of the paper is, in fact, to study and to compare networks of direct flows between countries' banking sectors with common exposure networks based on correlations between streams of loans. A link between two countries in a direct network represents a flow of funds, in millions of dollars, between a borrower and a lender. A link in a common exposure network, instead, measures the similarity between the funding composition or between the portfolio allocations of two countries, depending on whether in-flows or out-flows are used to compute the correlations. While in a direct network the links are directional, from lender to a borrower, in common exposure networks they are undirected, and they are computed starting from the correlation between the in-flows (out-flows) of a country with respect to all other countries.

To exemplify, we can describe each country by means of two vectors $1 \times N$ encompassing loans from and to all other countries (in- and out-flows). If we define with $In^i \in \mathbb{R}^{1\times N}$ the vector that represents the quantity each country invests in i, then the scalar In^i_j is the quantity invested in country i by country j. Analogously if we let $Out^i \in \mathbb{R}^{1\times N}$ be the vector that represents the quantity country i invests in all other countries, Out^i_j is the scalar that describes the amount invested by country i in country j.

In an in-flow common exposure network, that we called *funding composition similarity network*, the weighted link d^{In}_{ij} between two countries is the similarity between the two vectors In^i and In^j, that contain the amounts invested by all other countries, respectively in i and j. It represents the proximity between the funding composition sets of the two countries:

$$d^{In}_{ij} = 2 - \sqrt{2\left(1 - C_{In^i,In^j}\right)} \tag{2}$$

where C_{In^i,In^j} is the correlation between the two funding composition sets. A high value of d^{In}_{ij} means that the total funding that the two countries receive from the investors has the same composition, and therefore they have similar funding risk.

Differently, in an out-flow common exposure network, the so-called *portfolio similarity network*, the weighted link d^{Out}_{ij} between two countries is the similarity between the two vectors, Out^i and Out^j, that contain the amounts invested by countries i and j in all other countries:

$$d^{Out}_{ij} = 2 - \sqrt{2\left(1 - C_{Out^i,Out^j}\right)} \tag{3}$$

where C_{Out^i,Out^j} represents the correlation between the two portfolio composition sets. A high value $d^{Out}_{i,j}$ means that i and j invest similar proportions of funds in all other countries and, therefore, they have overlapped portfolios, and similar credit risk.

One of the problems that has received much attention in the study of financial networks has been determining interconnections among institutions with the aim of evaluating the impact that an institution's bilateral exposures has on other institutions within the system. In this literature, interconnectedness is related to the detection of the most central players in the network. The simplest way of measuring the centrality of a node is by counting the number of neighbours it has. Or, in the weighted case, summing the weights of the links associated to a node.

In the case of direct networks, we can define two local measures of centrality, the in- and the out-strength, defined as follows. Let $\mathbf{W_t}$ be a weighted adjacency matrix such that $w_{i,j,t}$ is the quantity lent from j to i at time t. The in-strength of country i in a direct (real) network R, at time t is defined as:

$$S_{i,t}^{I,R} = \textstyle\sum_j w_{i,j,t} \tag{4}$$

and symmetrically the out-strength is defined as:

$$S_{j,t}^{O,R} = \textstyle\sum_i w_{i,j,t}. \tag{5}$$

In other words, the in-strength of a country in a given period, represents the total funding that such country receives from other countries in that period. The out-strength, on the other hand, represents the total portfolio of that country invests in all others.

For the common exposure network the in-strength for country i in t can be defined as the sum of the similarities between the funding composition set of a country and those of all other countries in period t:

$$S_{i,t}^{I,C} = \textstyle\sum_j d_{i,j,t}^{in} \tag{6}$$

Symmetrically, the out-strength can be defined as the sum of the similarities between the portfolio allocation of that country and the portfolio allocation of all other countries:

$$S_{i,t}^{O,C} = \textstyle\sum_i d_{i,j,t}^{out} \tag{7}$$

The higher $S_{i,t}^{I,C}$ the higher the similarity of the composition of the funding of country i with respect to all other countries. In other words, country i has a set of investors that invest amounts in all other countries in a proportional way. A low value of $S_{i,t}^{I,C}$ instead means that country i has a set of investors that is specific to that country. Similar considerations can be done looking at $S_{i,t}^{O,C}$ in terms of portfolio allocations.

Summarizing, while $S_{i,t}^{I,R}$, $S_{i,t}^{O,R}$ describe the total funding a country receives from the others or the total investment in other countries; $S_{i,t}^{I,C}$, $S_{i,t}^{O,C}$ describe the similarity of the funding composition of that country with respect to the others, or the similarity of portfolio allocations of that country with respect all others.

Having introduced, and compared, direct and correlation networks, it is quite natural to aggregate them in a measure of systemic risk that uses both the direct and the common exposure networks. To achieve this aim, for each time period we perform the following steps.

The first step normalizes the elements of each weighted adjacency matrix registered in a period subtracting their mean and dividing by their standard deviation:

$$\hat{w}_{i,j,t} = \frac{w_{i,j,t} - \langle w_{i,j,t} \rangle}{\sqrt{\left\langle w_{i,j,t}^2 \right\rangle - \langle w_{i,j,t} \rangle^2}} \tag{8}$$

$$\hat{d}_{i,j,t} = \frac{d_{i,j,t} - \langle d_{i,j,t} \rangle}{\sqrt{\left\langle d_{i,j,t}^2 \right\rangle - \langle d_{i,j,t} \rangle^2}} \tag{9}$$

In such a way the obtained elements represent two z-scores associated to each link. They indicate whether, for each pair of countries, the weight is above or below the mean and by how many standard deviations. Note that this step depure the series from the trend component. For the direct network a positive z-score associated to a link means that the flows between the two countries are greater than the mean (a negative z-score less than the mean). For the common exposure network a positive z-score means that two countries funding compositions are more similar than the mean (a negative z-score less than the mean).

The second step deals with the creation of a *combined matrix* for each time period. At a given t, each element of this object is obtained as a linear combination of the corresponding elements of the normalized direct and common exposure networks. The weights of the linear combination are the normalized singular values of the matrix obtained by aligning the two vectorized adjacency matrices. Notice that the weights change over time but not over nodes, and represent the strength of the two effects (direct vs common exposure) in the composition of the combined effect. More formally, the generic elements i, j of the combined matrix at time t is:

$$m_{i,j,t} = \alpha_t \hat{w}_{i,j,t} + (1 - \alpha_t)\, \hat{d}_{i,j,t} \tag{10}$$

where $\hat{w}_{i,j,t}$ and $\hat{d}_{i,j,t}$ are the normalized links between country i and country j at time t produced by the directed and by the common exposure matrix respectively. The parameter α_t governs the strength of the two components in generating the mixed links $m_{i,j,t}$ at time t.

3 Empirical network analysis

This section provides empirical analysis on both direct and common exposure networks.

3.1 Data

The Bank of International Settlements (BIS) produces statistics on international banking activities. The International Banking Statistics comprises consolidated banking statistics (CBS), which measure worldwide consolidated claims of banks headquartered in reporting countries, including claims of their own foreign affiliates but excluding interoffice positions. These statistics build on measures used by banks in their internal risk management systems, and include data on off-balance sheet exposures, such as risk transfers, guarantees and credit commitments.

We employ the consolidated banking statistics on an ultimate risk basis that are based on the country where the ultimate risk or obligor resides, after taking into account risk transfers. Note that, since the statistics capture banks' worldwide consolidated positions, the CBS reporting area is not synonymous of the location of the banking offices participating in the data collection. That is, a reporting country should consolidate the positions of all banking entities owned or controlled by a parent institution located in the reporting country, thus including banking entities which are actually domiciled elsewhere.

Reporting institutions are financial institutions whose business is to receive deposits, or close substitutes for deposits, and to grant credits or invest in securities on their own account. Thus, the community of reporting institutions should include not only commercial banks but also savings banks, credit unions or cooperative credit banks, and other financial credit institutions. Unfortunately a number of countries do not report their statistics on the asset side (out-flows). In our available dataset there are only 15 fully reporting countries and more than 240 that do not report. In addition, for historical reasons among others the time series contain varying starting dates, as well as a number of missing values. To address the above data quality issues we split the analysis of the in- and out-flows in two different databases. More precisely, for what concerns the funding side, we restrict the analysis to the 33 largest economies (for which the received loans sum up to last 100000 billion dollars for the period from 1998 to 2013). The considered time period starts from the third quarter of 1998 (Q3–1998) to the last quarter of 2013 (Q4–2013). On the other hand, for the investment side we are forced to use only 15 reporting countries, from the third quarter of 1998 (Q3–1998) to the last quarter of 2013 (Q4-2013)[1,2]. Notice that the proposed strategy is consistent with Basel III regulation that look separately at the lending and borrowing sides of banks' balance sheet to evaluate their systemic importance (Basel Committee on Banking Supervision, 2013).

3.2 In-strength: funding risk

This section encompasses the results from the application of the direct and of the common exposure networks to BIS data. In particular it refers to in-strength; the funding risk of the countries.

Figure 1 shows the two strength measures for each country: in blue we report the in-strength of each country in the direct network, and in green the strength for the funding composition similarity network [3]. Plots are on two different scales; as the left y-axis refers to in-strength of the direct network and the right y-axis refers to the in-strength

[1] Countries selected for in-flows analysis: AT = Austria, AU = Australia, BE = Belgium, BR = Brazil, CA = Canada, CH = Switzerland, CN = Cina, CZ = Czech Republic, DE = Germany, DK = Denmark, ES = Spain, FI = Finalnd, FR = France, GB = Great Britain, GR = Greece, HK = Hong Kong, IE = Ireland, IN = India, IT = Italy, JP = Japan, KR = South Korea, KY = Cayman Islands, LU = Luxemburg, MX = Mexico, NL = The Netherlands, NO = Norway, NZ = New Zeland, PL = Poland, PT = Portugal, RU = Russia, SE = Sweden, SG = Singapore, US = United State.

[2] Countries selected for out-flows analysis: AT = Austria, AU = Australia, BE = Belgium, CH = Switzerland, DE = Germany, DK = Denmark, ES = Spain, FR = France, GB = Great Britain, IE = Ireland, JP = Japan, NL = The Netherlands, SE = Sweden, TW = Taiwan, US = United Stat.

[3] Remember that the similarity matrices are symmetric.

of the funding composition similarity network. Each series has been normalized dividing it by the number of all possible $n-1$ peers a country has. The three colored vertical bars distinguish between the pre-crisis phase 2005–07, the first wave of the crisis 2007–09 and the second wave 2009–11. In the title of each subplot, which represents the name of the considered country, we also report the correlation coefficient between the two in-strength measures.

FIGURE 1 APPROXIMATELY HERE

From Figure 1, two general trends appear for the direct and for the funding composition similarity strengths. Regarding the former, that measures total funding, some countries show an increase of the in-strength up to the 2007 financial crisis, followed by an abrupt fall; these countries are BE, ES, FR, GR, GB, IE, IT, NL, PT in Europe, and the US. However, some countries do not follow this general trend: for instance, AU, CA, BR, BE, KR, LU, RU and the Baltic countries, whereas CN, HK, CH, JP, KY, MX, SG, IN show an ever increasing in-strength. Regarding the latter, the strength, that reflects the proximity between the funding composition of a country with respect to the others, is generally decreasing for most countries, except for DE, BR, IN and MX. In 2000 the funding sets were highly correlated, whereas at the end of the time sample each country exhibits a specific funding composition. This means that the funding composition of most countries has become more and more concentrated on a limited number of specific lenders. This seems to be an important concentration risk factor that has increased after the financial crisis, and is still in place. To further understand this behaviour, Figure 2 shows the change over time of the standard deviation of the funding composition of each country, as a measure of funding diversification.

FIGURE 2 APPROXIMATELY HERE

Figure 2 shows that the diversification of the funding composition of most countries decreases after the financial crisis, especially for countries such as US, GB and IT. Thus, reading jointly Figures 1 and 2, the funding concentration started piling up before the crisis but increased considerably afterwards. Before the crisis, we observe an increase in the total funding of countries, given by specific investments in those countries and not by a generalized increase in the overall system funding. After the crisis, we observe a decrease in total funding which does not directly correspond to a higher diversification but rather to a further concentration.

Note that Germany (DE) shows a remarkable positive correlation (0.67) between the in-strengths calculated starting from the direct and from the common exposure networks. This can be explained noting that, before the crisis, fundings of DE was increasing in a diversified way, differently from other countries: DE has attracted new investors that were previously investing in other countries: a flight to quality effect. After the crisis, instead, the investors set of DE has become more country-specific, like that of other countries.

3.3 Out-strength: exposure risk

We now consider the out-strength; the portfolio allocation risk of the countries. The out-strength calculated for the direct network measures the total out-flows of a country, which is nothing else than the sum of its investments in all other countries. On the other hand, the strength for the portfolio similarity network measures the average distance of a country's portfolio composition with respect to those of the other countries. Thus it function as a measure of credit risk.

Figure 3 shows the strength for each country. In blue we report the results for the out-strength of the direct network, while in green for the out-strength of the portfolio similarity network. Plots are on different scale (left real and right proximity), and each series is normalized as in the case of the in-strength. The three colored vertical bars distinguish between 2007–08, 2008–09, 2009–10. In the title that represents the name of the country, we also show the correlation coefficient between the two series for that country.

From Figure 3 we can see that the crisis affects the direct out-strength of many core European countries: CH, FR, NL, GB but not that of non EU-countries such as US, JP, AU and TW. This can be explained by the sudden decrease in the financial trust among EU countries that piled up after 2007. The trend of the out-strength of the portfolio similarity networks is decreasing for most of the countries as for the funding composition. Looking at the correlation between the two series, it is negative for most of the countries. Economically, while European countries have decreased their investment flows after their crisis, the contrary has occurred outside Europe. In both cases however the portfolio concentration has increased.

FIGURE 3 APPROXIMATELY HERE

3.4 Mixed strength measure

Figure 4 shows the dynamic of the weights used to mix the two types of networks according to equation (10). Due to the higher availability of data for the in-flow network case, we construct the mixed network starting from the dataset regarding the funding side of the countries.

From Figure 4 it clearly emerges that the direct component has the largest impact on the combined network; the gap between the two increases until the 2007 financial crisis where it stabilizes. In any case, the two weights have a very similar impact throughout.

FIGURE 4 APPROXIMATELY HERE

Below, in Figure 5 we report the in-strength of the combined network. The same strategy can be easily applied also to the out-strength but due to the low number of countries and observations we prefer to show the result only for the in-strength. Notice that a high positive strength could mean either that the total funding of a country is high or that the funding composition has a low concentration or both: in all cases, the higher the in-strength the lower the risk. On the contrary, the lower the in-strength the higher the risk.

Looking at Figure 5 it seems that the financial crisis works as a tipping point for most countries, after which a regime switch happens. On one hand, many countries present a fall of the mixed in-strength measure during the financial crisis. Some of them, mostly European countries: AT, BE, CZ ES, GR, GB, IT, PT, PL, DK, IE NO and JP have not yet recovered. Others, such as FR, IE, BR, FI, LU, NL, SE together with US, instead have.

On the other hand, another group of countries have not been affected at all by the crisis, and maintain a low risk profile throughout: CH, CN, DE, HK, KY, LU, NL, SG and DK. This group includes off-shore countries (HK, LU, KY) flight to quality countries (CH, DE) and emerging countries (IN, KR, MX). Investment in all this countries seems to be safe during distress periods. Finally, the remaining countries show a more volatile in-strength measure.

FIGURE 5 APPROXIMATELY HERE

4 Predictive performance

This section compares the use of direct exposures with common exposures in a predictive models of systemic banking crises. In short, we let vulnerability pass-through networks, where links are defined as direct and common exposures, and compare their predictive performance. To start with, we introduce crisis signaling as a task as well as the evaluation exercises around the task. Then, we move ahead to an empirical investigation of European crises.

4.1 The task of crisis signaling and evaluations

In order to test to what extent direct and indirect exposures proxy pass-through effects of stress in the banking sector, we employ standard approaches from crisis signaling. Early-warning models are concerned with differentiating between vulnerable (i.e., pre-crisis) and tranquil economies, which forms a standard classification problem. Generally speaking, we are aiming for a model that separates vulnerable and tranquil classes to discriminate between them by estimating the probability of being in a vulnerable state. For backtesting, however, the time-series dimension needs to be taken into account when testing the predictive power. We conduct recursive real-time out-of-sample tests to assess performance. This implies deriving a new model at each quarter using only information available up to that time point. By accounting for publication lags and using information in a realistic manner, this enables testing whether a measure would have ex ante provided means for predicting crisis events.

TABLE 1 APPROXIMATELY HERE

Following the standard evaluation framework for early-warning models in Sarlin (2013), we aim at mimicking an ideal leading indicator $C_n(h) \in \{0, 1\}$ for observation n (where $n = 1, 2, \ldots, N$) and forecast horizon h. This implies nothing else than a binary indicator that is one during vulnerable periods and zero otherwise. For detecting events C_n, we need a continuous measure indicating membership in a vulnerable state $p_n \in [0, 1]$, which is then turned into a binary prediction B_n that takes the value one if p_n exceeds a specified threshold $\tau \in [0, 1]$ and zero otherwise. The correspondence between the prediction B_n and the ideal leading indicator C_n can then be summarized into a so-called contingency matrix, as described in Table 1. In terms of the elements of the contingency matrix, we can differentiate between two different types of classification errors that a decision maker may be concerned with: missing crises and issuing false alarms. To formulate the concepts of usefulness and relative usefulness as measures of classification performance in Sarlin (2013), we define type I errors as the share of missed crises to the frequency of crises, i.e. $T_1 = FN/(FN+TP)$, and type II errors as the share of issued false alarms to the frequency of tranquil periods, i.e. $T_2 = FP/(TN + FP)$. Further, we need two terms: policymakers' relative preference between type I and II errors (μ) to account for the potentially imbalanced costs of errors and the unconditional probabilities of crises P_1 and tranquil periods P_2 to account for the potential difference in the size of the two classes. Based on these values, we can define the loss function as:

$$L(\mu) = \mu T_1 P_1 + (1 - \mu) T_2 P_2. \tag{11}$$

Further, based on this loss function, the absolute usefulness of the prediction model can be specified by comparing it to using the best guess of a policymaker (always or never signaling depending on class frequency and preferences):

$$U_a(\mu) = \min(\mu P_1, (1 - \mu) P_2) - L(\mu). \tag{12}$$

Finally, we compute relative usefulness, U_r to compare the absolute usefulness of the model to the absolute usefulness of a model with perfect performance ($L(\mu) = 0$). Additionally, to assess predictive performance, we also calculate standard measures from the classification and machine learning literature, in particular the area under the receiver operating characteristic (ROC) curve (AUC). These techniques provide both measures tailored to the preferences of a policymaker as well as more general-purpose measures to assess model performance. Other performance measures to be used in assessing the model include: (i) precision of signals $TP/(FP + TP)$, i.e. the share of correct signals to the frequency of signals; (ii) precision of tranquil predictions $TN/(FN + TN)$, i.e. the share of correct silence in tranquil times to the frequency of predicting tranquil time; (iii) recall of signals $TP/(FN + TP)$, i.e. the share of correct signals to the frequency of crisis times; (iv) recall of tranquil predictions $TN/(FP+TN)$, i.e. the share of correct silence in tranquil times to the frequency of tranquil times; (v) accuracy of the model $TP + TN/(FN + FP + TN + TP)$, i.e. the share of correct classifications.

4.2 Comparing direct and indirect exposures with RiskRank

In this section, we compare direct and indirect exposures with the RiskRank measure (Mezei and Sarlin, 2015). RiskRank uses two inputs: individual risk for a set of economies measured by an early-warning model and interconnectedness across these economies. Essentially, this allows measuring the vulnerability of an individual economy, accounting for both domestic risk as well as risk stemming from exposures to other economies. In this exercise, the early-warning model follows the approach in Holopainen and Sarlin (2015). For an early-warning model, we need two types of data: crisis events and vulnerability indicators. The crisis events are based upon the IMF database by Laeven and Valencia (2008), while the vulnerability indicators used include most common measures of widespread imbalances, such as excessive credit growth, excessive increases in stock and house prices, GDP growth, loans to deposits and debt service ratio, as well as more structural indicators, such as government debt, current account deficits and inflation. We use a standard logit model with 14 macro-financial indicators for 15 European economies and a forecast horizon of 5–12 quarters prior to crisis events, as is common in the literature. The network dimension is measured with BIS International Banking Statistics in three ways: direct, indirect (common exposures) and combined exposures. This provides ample means to compare the three types of networks.

RiskRank provides thus a risk measure that combines measures of individual risk and interconnectedness. In principle, it is nothing else than an aggregation operator for each entity c that aggregates over node values (i.e., individual risk) over link values (i.e., interconnectedness). Thus, we can write RiskRank as follows:

$$RR_c(x_1,\ldots,x_n,x_c) = \underbrace{x_c}_{\text{Individual effect of } c} + \sum_{i=1}^{n} \underbrace{\left(v\left(c_i\right) - \frac{1}{2}\sum_{j\neq i} I(c_i,c_j)\right)x_i}_{\text{Direct effect of } i \text{ on c}} \tag{13}$$

$$+ \underbrace{\sum_{i}^{n}\sum_{j\neq i}^{n} I(c_i,c_j)\prod(x_i,x_j)}_{\text{Indirect effect of } j \textit{ via } i \text{ on c}}$$

where c is the evaluated node and x_c is its associated node value. In this case, the nodes are countries and their values crisis probabilities. Further, $I(c_i,c_j)$ stands for the link between nodes i and j and $v(c_i)$ stands for the Shapley-index (average contribution of fixed element x_i in any subset).

Tables 2–5 summarize the performance of the RiskRank measure when defining linkages as direct versus common exposures. First, Table 2 summarizes the performance in signaling banking crises when a standard early-warning model without network effects is employed. Second, Table 3 summarizes performance of the early-warning model complemented with direct exposures. Third, Table 4 provides a summary of the performance with common exposures. Finally, Table 5 shows the performance of RiskRank when defining linkages across entities to be a combination of both direct and common exposures.

In line with the literature, the evaluated performance indicates that the models are beyond the best-guess of a policymaker and hence provide positive Usefulness. However, as expected (cf. Sarlin, 2013), the RiskRank measures provide Usefulness only for a policymaker more concerned with missing crises than issuing false alarms.

A comparison of the performance of RiskRank with these three definitions of linkages allows one to conclude the following:

- Direct linkages are clearly outperformed by the other ways of defining linkages.
- Linkages based upon common exposures and a combination of direct and common exposures perform equally well.
- While the difference between the common and the combined exposures is minor, the combined exposures outperform in the case of $\mu = [0.1, 0.4]$.

In our view, this highlights the importance of common exposures. While the information content in the two ways of defining exposures seems to be similar enough for the common and the mixed measures to perform on par, the results still point to the fact that the use of common exposures provide an added value in signaling crises. From an economic point of view, this clearly shows that common exposures, or so-called portfolio overlap, indeed is a channel of contagion and should be accounted for when measuring systemic risk.

TABLE 2 APPROXIMATELY HERE

TABLE 3 APPROXIMATELY HERE

TABLE 4 APPROXIMATELY HERE

TABLE 5 APPROXIMATELY HERE

5 Conclusions

Measuring portfolio similarity is a central task when modeling systemic risk and interconnectedness in financial systems, particularly for complementing measures based upon direct exposures. In this contribution we have shown that correlation network models that aim at capturing the multivariate network structure provide suitable means for representing the indirect dimension of systemic risk through common exposures. Moreover, we have provided an approach for combining direct exposures and correlations into one measure of systemic risk.

We have applied our proposed methods to the Bank of International Settlements consolidated banking statistics, with the aim of identifying central and important countries in the context of interconnectedness of the banking sector. This is particularly relevant in the case of banking sector distress and abrupt changes in liquidity and funding.

From an economical view point our empirical findings give two main results. Before the crisis, the total funding of most of the countries had increased via specific funders' investments and not by a generalized increase. After the crisis, we observe a decrease in total funding which does not correspond to a higher diversification but, rather, to a further concentration. Moreover, the evidence from the mixed network suggests that the financial crisis worked as a tipping point for most countries: many of them increased their risk profile during the financial crisis but just some of them recovered afterwards. Besides that, off-shore and flight to quality countries maintain a low risk profile throughout.

We have finally combined our proposed measures with banking crises, in order to assess whether and to what extent they are related to the build-up of imbalances prior to crisis events. The exercise clearly shows that common exposures are an important channel of contagion and should be accounted for when measuring systemic risk.

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Tables and Figures

		Actual Class	
		Pre-crisis period	Tranquil period
Predicted Class	Signal	Correct call *True Positive (TP)*	False alarm *False Positive (FP)*
	No Signal	Missed crisis *False Negative (FN)*	Correct silence *True Negative (TN)*

Table 1: A contingency matrix.

μ	τ	$\mathbf{U_a}$	$\mathbf{U_r}$	AUC	TN	TP	FN	FP	PP	RP	PN	RN	Acc.
0.0	1.00	0.00	NaN	0.92	747	0	76	1	0.00	0.00	0.91	1.00	0.91
0.1	1.00	0.00	-0.12	0.92	747	0	76	1	0.00	0.00	0.91	1.00	0.91
0.2	1.00	0.00	-0.05	0.92	747	0	76	1	0.00	0.00	0.91	1.00	0.91
0.3	0.99	0.00	-0.01	0.92	745	6	70	3	0.67	0.08	0.91	1.00	0.91
0.4	0.99	0.00	0.02	0.92	745	6	70	3	0.67	0.08	0.91	1.00	0.91
0.5	0.96	0.00	0.07	0.92	734	19	57	14	0.58	0.25	0.93	0.98	0.91
0.6	0.96	0.01	0.13	0.92	734	19	57	14	0.58	0.25	0.93	0.98	0.91
0.7	0.79	0.02	0.28	0.92	641	67	9	107	0.39	0.88	0.99	0.86	0.86
0.8	0.78	0.04	0.54	0.92	635	69	7	113	0.38	0.91	0.99	0.85	0.85
0.9	0.72	0.06	0.77	0.92	593	76	0	155	0.33	1.00	1.00	0.79	0.81
1.0	0.00	0.00	NaN	0.92	0	76	0	748	0.09	1.00	NaN	0.00	0.09

Table 2: Predictive performance without network. The table reports recursive out-of-sample performance for the direct network with a forecast horizon of 5-12 quarters. The table reports in columns the following measures to assess the overall performance of the models: preferences (μ), optimal threshold (τ), absolute (U_a) and relative (U_r) usefulness, and AUC = area under the ROC curve (TP rate to FP rate), TP = True positives, FP = False positives, TN= True negatives, FN = False negatives, Precision of positives (PP) = TP/(TP+FP), Recall of positives (RP) = TP/(TP+FN), Precision of negatives (PN) = TN/(TN+FN), Recall of negatives (RN) = TN/(TN+FP), Accuracy (Acc.) = (TP+TN)/(TP+TN+FP+FN).

μ	τ	U_a	U_r	AUC	TN	TP	FN	FP	PP	RP	PN	RN	Acc.
0.0	0.94	-0.02	NaN	0.94	732	34	42	16	0.68	0.45	0.95	0.98	0.93
0.1	0.94	-0.01	-1.45	0.94	732	34	42	16	0.68	0.45	0.95	0.98	0.93
0.2	0.94	-0.01	-0.39	0.94	732	34	42	16	0.68	0.45	0.95	0.98	0.93
0.3	0.94	0.00	-0.04	0.94	732	34	42	16	0.68	0.45	0.95	0.98	0.93
0.4	0.94	0.00	0.13	0.94	732	34	42	16	0.68	0.45	0.95	0.98	0.93
0.5	0.93	0.01	0.24	0.94	728	38	38	20	0.66	0.50	0.95	0.97	0.93
0.6	0.91	0.02	0.33	0.94	718	45	31	30	0.60	0.59	0.96	0.96	0.93
0.7	0.86	0.03	0.47	0.94	692	60	16	56	0.52	0.79	0.98	0.93	0.91
0.8	0.86	0.04	0.61	0.94	692	60	16	56	0.52	0.79	0.98	0.93	0.91
0.9	0.73	0.06	0.74	0.94	598	73	3	150	0.33	0.96	1.00	0.80	0.81
1.0	0.00	0.00	NaN	0.94	0	76	0	748	0.09	1.00	NaN	0.00	0.09

Table 3: Predictive performance of the direct network. The table reports recursive out-of-sample performance for the direct network with a forecast horizon of 5-12 quarters. The table reports in columns the following measures to assess the overall performance of the models: preferences (μ), optimal threshold (τ), absolute (U_a) and relative (U_r) usefulness, and AUC = area under the ROC curve (TP rate to FP rate), TP = True positives, FP = False positives, TN= True negatives, FN = False negatives, Precision of positives (PP) = TP/(TP+FP), Recall of positives (RP) = TP/(TP+FN), Precision of negatives (PN) = TN/(TN+FN), Recall of negatives (RN.) = TN/(TN+FP), Accuracy (Acc.) = (TP+TN)/(TP+TN+FP+FN).

μ	τ	U_a	U_r	AUC	TN	TP	FN	FP	PP	RP	PN	RN	Acc.
0.0	0.92	-0.03	NaN	0.97	722	45	31	26	0.63	0.59	0.96	0.97	0.93
0.1	0.92	-0.02	-2.49	0.97	722	45	31	26	0.63	0.59	0.96	0.97	0.93
0.2	0.92	-0.01	-0.78	0.97	722	45	31	26	0.63	0.59	0.96	0.97	0.93
0.3	0.92	-0.01	-0.21	0.97	722	45	31	26	0.63	0.59	0.96	0.97	0.93
0.4	0.92	0.00	0.08	0.97	722	45	31	26	0.63	0.59	0.96	0.97	0.93
0.5	0.89	0.01	0.28	0.97	713	56	20	35	0.62	0.74	0.97	0.95	0.93
0.6	0.85	0.03	0.47	0.97	695	71	5	53	0.57	0.93	0.99	0.93	0.93
0.7	0.84	0.04	0.65	0.97	690	74	2	58	0.56	0.97	1.00	0.92	0.93
0.8	0.84	0.06	0.78	0.97	690	74	2	58	0.56	0.97	1.00	0.92	0.93
0.9	0.82	0.07	0.89	0.97	675	76	0	73	0.51	1.00	1.00	0.90	0.91
1.0	0.00	0.00	NaN	0.97	0	76	0	748	0.09	1.00	NaN	0.00	0.09

Table 4: Predictive performance of the common exposure network. The table reports recursive out-of-sample performance for the direct network with a forecast horizon of 5-12 quarters. The table reports in columns the following measures to assess the overall performance of the models: preferences (μ), optimal threshold (τ), absolute (U_a) and relative (U_r) usefulness, and AUC = area under the ROC curve (TP rate to FP rate), TP = True positives, FP = False positives, TN= True negatives, FN = False negatives, Precision of positives (PP) = TP/(TP+FP), Recall of positives (RP) = TP/(TP+FN), Precision of negatives (PN) = TN/(TN+FN), Recall of negatives (RN) = TN/(TN+FP), Accuracy (Acc.) = (TP+TN)/(TP+TN+FP+FN).

μ	τ	U_a	U_r	AUC	TN	TP	FN	FP	PP	RP	PN	RN	Acc.
0.0	0.95	-0.02	NaN	0.96	734	32	44	14	0.70	0.42	0.94	0.98	0.93
0.1	0.95	-0.01	-1.24	0.96	734	32	44	14	0.70	0.42	0.94	0.98	0.93
0.2	0.95	-0.01	-0.32	0.96	734	32	44	14	0.70	0.42	0.94	0.98	0.93
0.3	0.95	0.00	-0.01	0.96	734	32	44	14	0.70	0.42	0.94	0.98	0.93
0.4	0.95	0.01	0.14	0.96	734	32	44	14	0.70	0.42	0.94	0.98	0.93
0.5	0.88	0.01	0.28	0.96	709	60	16	39	0.61	0.79	0.98	0.95	0.93
0.6	0.88	0.02	0.45	0.96	709	60	16	39	0.61	0.79	0.98	0.95	0.93
0.7	0.85	0.04	0.58	0.96	692	68	8	56	0.55	0.89	0.99	0.93	0.92
0.8	0.85	0.05	0.71	0.96	692	68	8	56	0.55	0.89	0.99	0.93	0.92
0.9	0.83	0.07	0.82	0.96	677	70	6	71	0.50	0.92	0.99	0.91	0.91
1.0	0.00	0.00	NaN	0.96	0	76	0	748	0.09	1.00	NaN	0.00	0.09

Table 5: Predictive performance of the combined network. The table reports recursive out-of-sample performance for the direct network with a forecast horizon of 5-12 quarters. The table reports in columns the following measures to assess the overall performance of the models: preferences (μ), optimal threshold (τ), absolute (U_a) and relative (U_r) usefulness, and AUC = area under the ROC curve (TP rate to FP rate), TP = True positives, FP = False positives, TN= True negatives, FN = False negatives, Precision of positives (PP) = TP/(TP+FP), Recall of positives (RP) = TP/(TP+FN), Precision of negatives (PN) = TN/(TN+FN), Recall of negatives (RN) = TN/(TN+FP), Accuracy (Acc.) = (TP+TN)/(TP+TN+FP+FN).

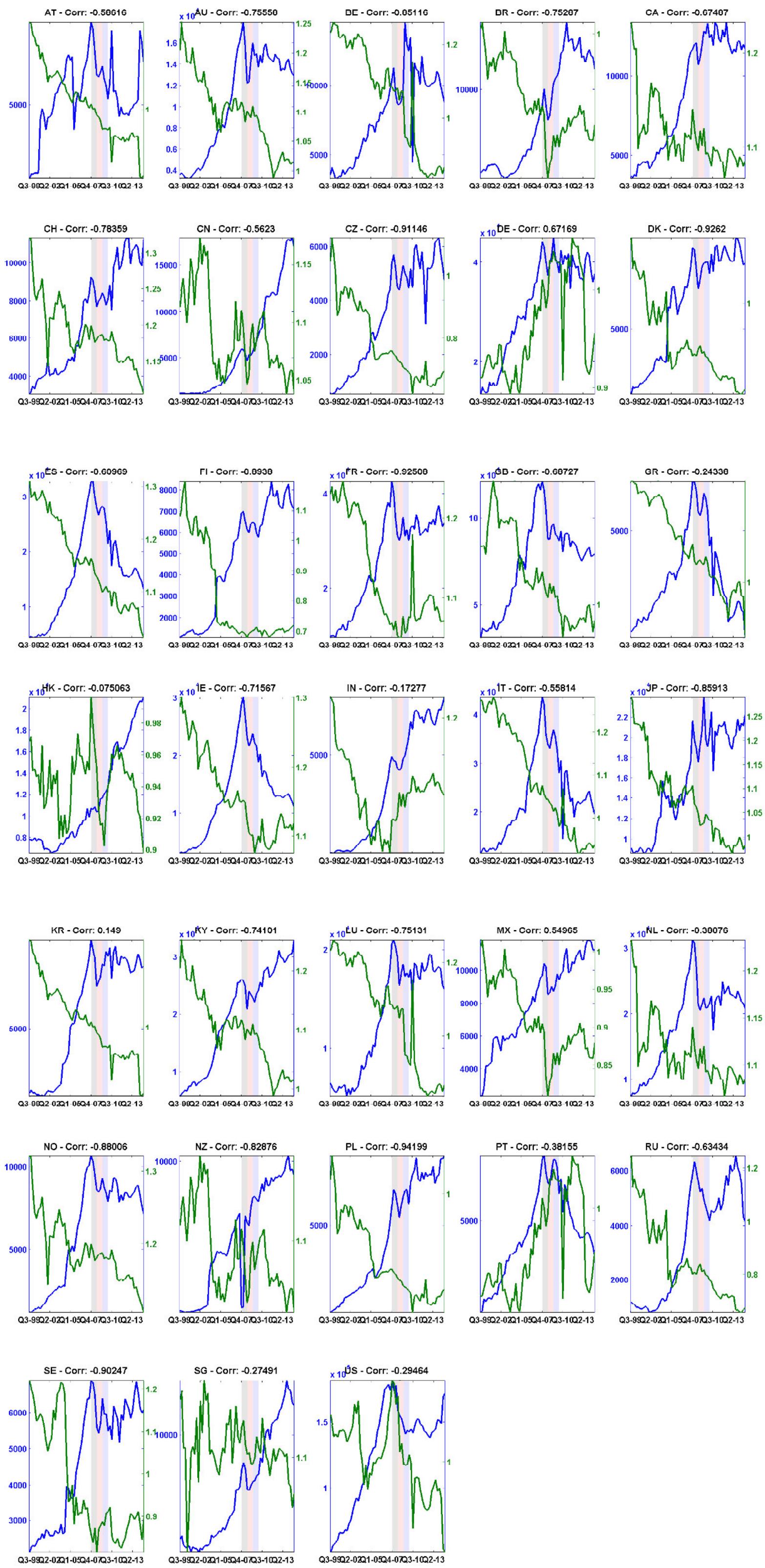

Figure 1: In-strength of each country for the direct (blue) and for the funding composition similarity (green) networks, near the name of each country we also report the correlation coefficients between the two measures. The three vertical bars emphasize the crisis periods (2005-07; 2007-09 and 2009-11).

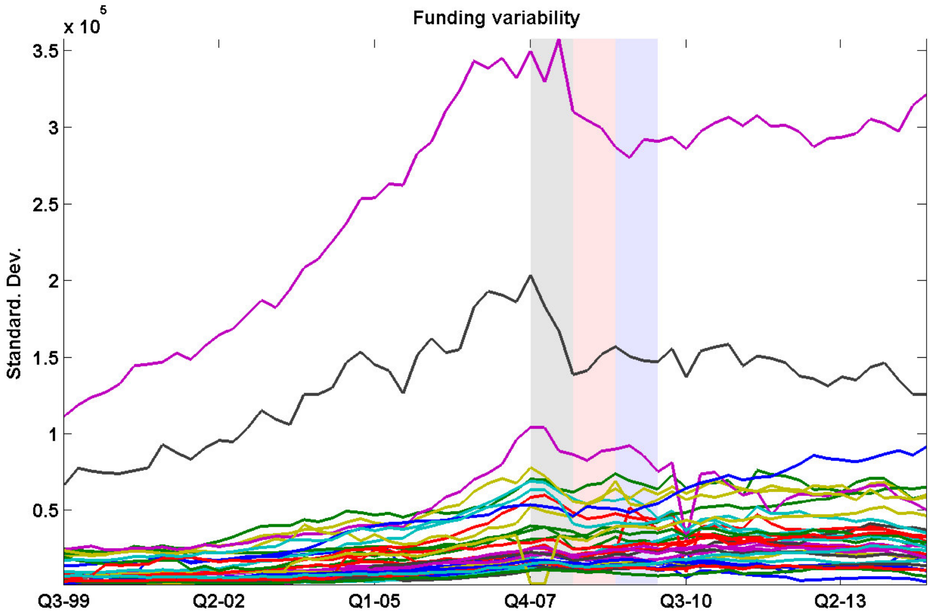

Figure 2: Change over time of the standard deviation of the funding composition of all countries.

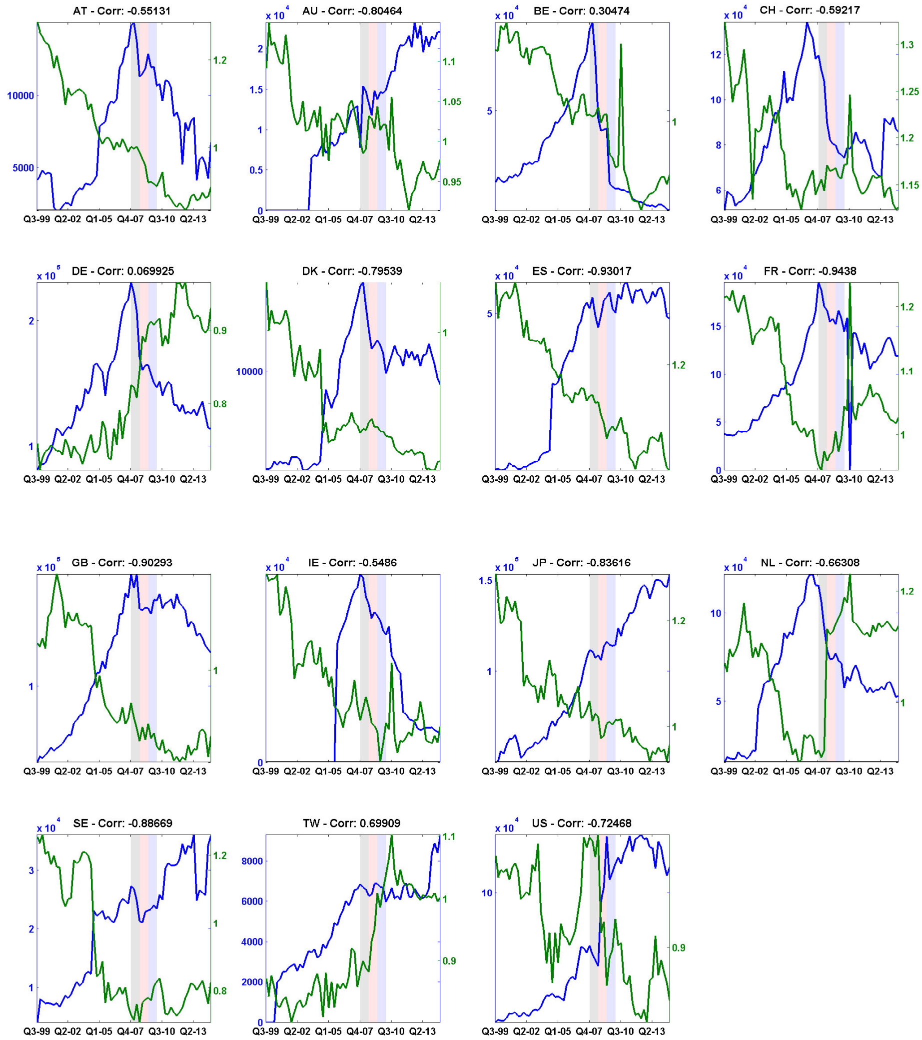

Figure 3: Out-strength of each country for the direct (blue) and for the portfolio similarity (green) networks, near the name of each country we also report the correlation coefficients between the two measures. The three vertical bars emphasize the crisis periods.

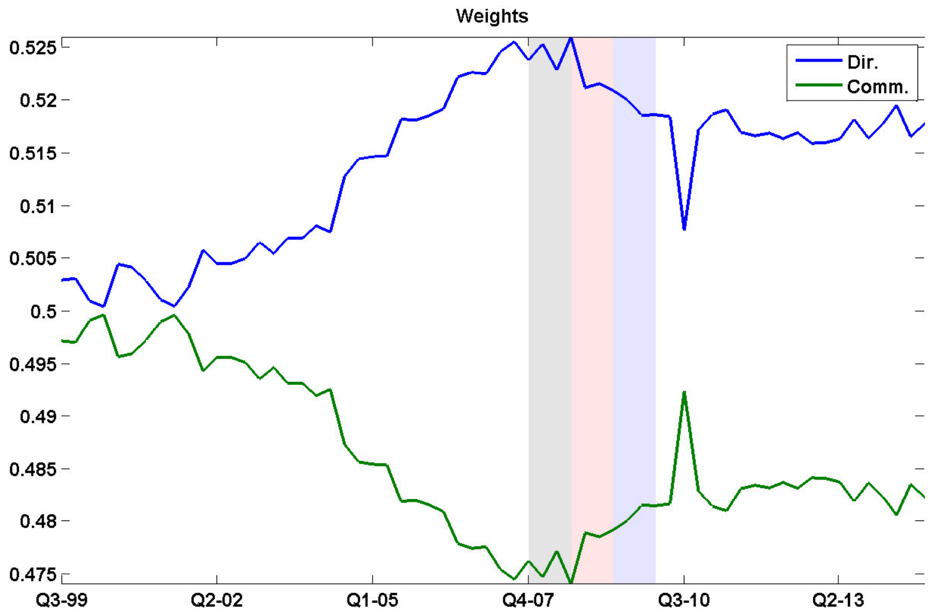

Figure 4: Weights associated to the direct (green) and to the funding composition similarity (blue) component of the mixed network. Before the crisis the real part gains in the mixed matrix while the correlation component decreases, whereafter the two series stay approximately at the same level.

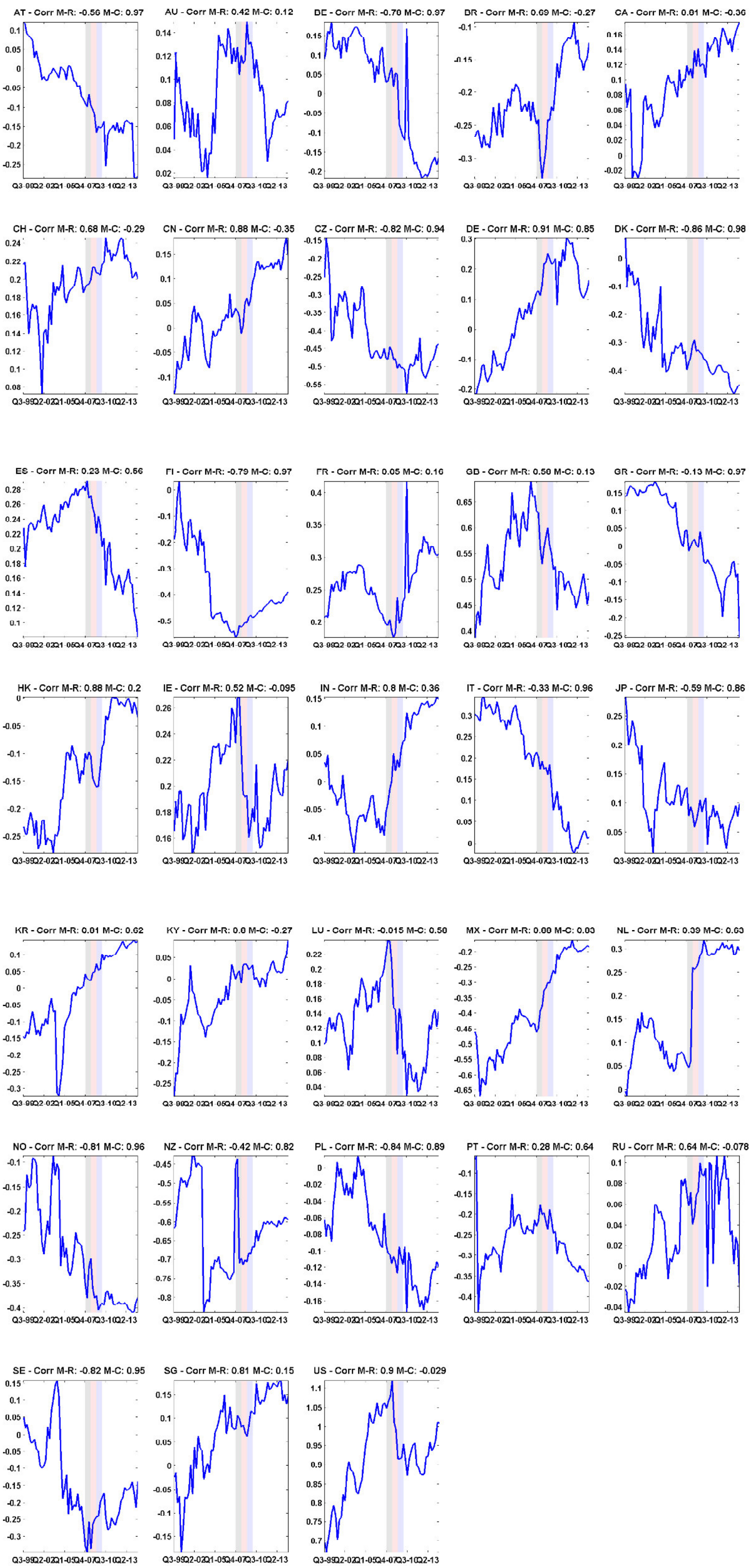

Figure 5: In-strength of the combined adjacency matrix, near the name of each country we also report the correlation coefficients between the combined and the direct in-strength (M-R) and the strength of the funding composition similarity network (M-C). The three vertical bars emphasize the crisis periods (2007–08; 2008–09 and 2009–10).