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**CoRisk: measuring systemic risk through
default probability contagion**

Paolo Giudici
(Università di Pavia)

Laura Parisi
(Università di Pavia)

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Via San Felice, 5
I-27100 Pavia
<http://epmq.unipv.eu/site/home.html>

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CoRisk: measuring systemic risk through default probability contagion *

Paolo Giudici and Laura Parisi
University of Pavia and NYU Stern School of Business
lparisi@stern.nyu.edu

Abstract

We propose a novel systemic risk measurement model, based on stochastic processes, correlation networks and conditional probabilities of default.

For each country we consider three different spread measures, one for each sector of the economy (sovereigns, corporates, banks), and we model each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. We then build a partial correlation network model, and by combining it with the spread measures we derive the conditional default probabilities of each sector. Comparing them with the unconditional ones, we obtain the CoRisk, which measures the variation in the probability of default due to contagion effects.

Our measurement model is applied to understand the time evolution of systemic risk in the economies of the European monetary union, in the recent period. The results show that, overall, the sovereign crisis has increased systemic risks more than the financial crisis. In addition, peripheral countries turn out to be exporters, rather than importers of systemic risk, and, conversely, core countries.

Keywords: correlation networks, default probabilities, systemic risk, stochastic processes.

1 Introduction

1.1 Background

The last few years have witnessed an increasing research literature on systemic risk (for a definition see, for example, Allen and Gale, 2000; Acharya, 2009), with the aim of identifying the most contagious institutions and their transmission channels, and of studying the impact of monetary policies on default probabilities, especially during crisis periods (see, for example, Chong et al., 2006; Longstaff, 2010; Shleifer and Vishny, 2010).

Specific measures of systemic risk have been proposed for the banking sector; in particular, by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015) who, on the basis of market share prices, calculate the quantiles of the estimated loss probability distribution of a bank, conditional on the occurrence of an extreme event in the financial market. A similar approach has been applied to sovereign systemic risk, using bond interest rates, by Popescu and Turcu (2014).

The above approach is useful to establish policy thresholds aimed, in particular, at identifying the most systemic institutions. However, it is a bivariate approach, which allows to calculate the risk of an institution conditional on another or on a reference market but, on the other hand, it does not address the issue of how risks are transmitted between different institutions in a multivariate framework.

Trying to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, that combine the rich structure of financial networks (see, e.g., Lorenz

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et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who propose measures of connectedness based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013) and Ahelegbey et al. (2015) extend the approach introducing stochastic graphical models, while Das (2015) introduces the decomposition of a systemic risk measure into individual and network contributions.

Correlation network models are very useful to identify the most important channels of contagion in a cross-sectional perspective. However, similarly to bivariate measures, they can not be used as predictive models in a time-varying context. This is the main focus of econometric causal methods, as the ones proposed by Duffie et al. (2000), Lando and Nielsen (2010), Koopman et al. (2012) and Betz et al. (2014).

Both correlation networks and econometric models explain whether the default probability of a bank, a country, or of a company, depends on that of the others, or on a set of exogenous systematic risk factors. A different stream of research developed, among others, by Bartram et al. (2007), Ang and Longstaff (2012), Battiston et al. (2012) and Brownlees et al. (2014), models systemic risk in terms of univariate stochastic processes, that may also depend on systematic factors which are, however, endogenously determined. A further advantage of stochastic processes is that they are non-linear and time-dependent, and, therefore, can produce powerful early warning indicators.

We propose multivariate stochastic processes, whose interrelationships can be investigated by means of correlation network models: doing so, we combine the advantages of econometric models (predictive capability) with those of correlation networks (identification of channels of contagion) and of stochastic process models (endogeneity and non-linearity). To achieve our aim, we significantly extend the approach of Ang and Longstaff (2012) and Brownlees et al. (2014) by employing a multiple set of linear combinations of two stochastic processes (a systematic and an idiosyncratic one), rather than a single process.

In more detail, we consider three spread measures based on publicly available data: (a) the spread between the cost of debt for countries (interest rates on 10-years maturity government bonds) and a benchmark rate, which gives a measure of sovereign risk; (b) the spread between the cost of debt for corporates (interest rates on bank lendings) and a benchmark rate, which gives a measure of corporate risk; (c) the spread between the funding cost of the banking system (interest rates on deposits of non-financial corporates and households) and a benchmark rate, which gives a measure of bank risk.

We define three stochastic processes on the three spread measures: a probability of default can then be deduced from the estimated spreads, unconditionally for each sector and country. We then estimate a correlation network model, based on partial correlations, within each economic sector and across different countries and time, as suggested, although in a different modelling framework, by Gray et al. (2013), Ramsay and Sarlin (2015) and Schwaab et al. (2015). A new set of default probabilities can thus be calculated, conditionally on the dependence structure between countries and sectors defined by the estimated correlation networks.

The difference between the unconditional and the conditional probability of default can be employed to assess the effect of systemic contagion, introducing an appropriate novel measure, that will be named *CoRisk*. The introduction of conditional probabilities of default will also enable the calculation of the aggregate default probability for an entire country, thereby disentangling different risk sources: institution-specific and systemic, for each of the sovereign, corporate and bank sector.

In more detail, we will propose two different kinds of *CoRisk*: *CoRisk_{in}*, which measures how a sector of a country is influenced by the default probability of its neighbours in the network, thus providing a measure of its vulnerability; *CoRisk_{out}*, which measures to what extent each economic sector of a country influences its neighbours in the graph, thus providing a measure of its systemic importance.

We remark that our proposal emphasises the difference between systematic and systemic risks: *CoRisk*, in fact, incorporates both of them, with the former deriving from the benchmark rate in the spread measures and the latter deriving from contagion effects obtained through correlation networks.

We also remark that our methodology makes explicit, using a clear probability metric, what suggested in Das (2015): a measure of systemic risk that can be decomposed in an individual node plus a network component. We also remark that a similar approach has been recently proposed by Mezei and Sarlin (2015), who define an aggregation operator in order to jointly estimate the importance of each single node as well as contagion effects deriving from links with other nodes in the graph. We improve both approaches by calculating node default probabilities for three different economic sectors in each country and by deriving link measures of contagion through

partial correlations between linear combinations of stochastic processes: in such a way we can (a) allow for non-linear effects through stochastic differential equations, (b) allow for contagion effects, not only between, but also within each country, (c) disentangle the idiosyncratic and the systematic, as well as the institution-specific and the systemic components for the three economic sectors in each country. In addition, our *CoRisk* measure is allowed to be both positive or negative, meaning that the individual default probability of each economic sector or country can be increased or decreased according to the sign of partial correlations: from an economic viewpoint, this means that, when a country is negatively related to troubled countries, its final default probability decreases because it is perceived as a flight-to-quality haven.

Our proposed model will be applied and compared to data that concern four time windows: the pre-crisis period (2003-2006), the financial-crisis period (2007-2009), the sovereign-crisis period (2010-2012) and the post-crisis period (2013-2015), for the countries belonging to the Euro area.

Our main economic findings can be summarized as follows. First, in the Euro area the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis. Second, the contribution of the bank and the sovereign sectors to systemic risk increased, in all countries, respectively during the financial and the sovereign crisis, with core economies mainly affected by contagion effects and peripheral countries characterised by high idiosyncratic *PDs*, exacerbated by within peripheral cluster contagion effects. Third, peripheral (core) countries are more exporter (importer) rather than importer (exporter) of systemic risks.

The paper is structured as follows: Section 2 describes the proposed models, with Section 2.1 introducing multivariate linear combinations of interest rate spread models and partial correlation networks and Section 2.2 defining default probabilities and *CoRisk*. Section 3 describes the application of the proposed models, with Section 3.1 presenting data and descriptive statistics, Section 3.2 presenting the empirical evidence obtained from multivariate stochastic processes and partial correlation networks and Section 3.3 presenting the obtained default probabilities and *CoRisk* measures. Finally, Section 4 concludes with some final remarks.

2 Proposal

2.1 Multivariate Stochastic Processes

For each country we consider the aggregate financial liabilities of three economic sectors: sovereign, (non-financial) corporates and banks.

For each given sector, and independently from the others, we assume that the time dynamics of the liabilities of each country, expressed by the evolution of the associated interest rate, can be described by a linear combination of two stochastic processes: a common systematic process and an idiosyncratic process. More formally, for each country $i = 1, \dots, N$:

$$Z_t^i = \alpha^i y_t^i - \beta^i S_t, \quad (2.1)$$

where S_t stands for the systematic process, while y_t^i represents the idiosyncratic process referred to country i ; the parameter β^i measures the weight of the systematic process, while α^i measures the weight of the idiosyncratic process, both on the general, complete process Z_t^i , that describes the resulting time-evolution of the interest spread.

We remark that the previous equation assumes that the systematic process is the same for all countries, but it differently influences each country-specific process Z_t^i , through the weight β^i . From an economic viewpoint, the above formulation expresses Z_t^i as the difference between the cost of a long term debt and the cost of liquidity.

Both the systematic and the idiosyncratic processes can be modelled as CIR processes (Cox, Ingersoll and Ross, 1985), as follows:

$$\begin{cases} dS_t = (a - vS_{t-1}) dt + b\sqrt{S_{t-1}} dB_t, \\ dy_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i) dt + \theta_3^i \sqrt{y_{t-1}^i} dW_t, \end{cases} \quad (2.2)$$

where dB_t and dW_t are two independent Brownian motions.

We then assume the following correlation structure:

$$\begin{cases} \text{Corr}[y_t^i, y_t^j] = \rho^{ij}, \\ \text{Corr}[S_t, y_t^j] = \gamma^j. \end{cases} \quad (2.3)$$

Note that the first equation is consistent with the assumptions used in the formulation of multidimensional CIR processes (see e.g. Kalogeropoulos et al., 2011); the second one introduces an innovation in the literature, assuming a correlation between each idiosyncratic process and the systematic process S_t .

In this way we obtain a more general process able: (a) to capture both the systematic and the idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries and sectors, by means of graphical network models, as we shall see later.

We now show the resulting expression of the instantaneous covariance matrix, for our multivariate linear combination of CIR processes.

First define:

$$P = \begin{bmatrix} 1 & \rho^{12} & \dots & \rho^{1N} \\ \rho^{21} & 1 & \dots & \rho^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1} & \rho^{N2} & \dots & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \gamma^1 \\ \vdots \\ \gamma^i \\ \vdots \\ \gamma^N \end{bmatrix}, \quad (2.4)$$

where each element in P is the correlation coefficient between the idiosyncratic processes of any two countries, while each element of Γ is the correlation coefficient between any idiosyncratic process and the systematic process, as defined in (2.3).

The instantaneous covariance matrix A can then be shown to be:

$$A = \Phi \cdot \Theta^T, \quad (2.5)$$

where

$$[\Phi]^i = [\beta^i b \sqrt{S_0}, \quad \beta^i, \quad \alpha^i \sqrt{S_0 y_0^i} b \theta_3^i [\Gamma]^i, \quad \alpha^i \sqrt{y_0^i} \theta_3^i \sqrt{[P]^i}],$$

$$[\Theta^T]^j = \begin{bmatrix} \beta^j b \sqrt{S_0} \\ \alpha^j \sqrt{S_0 y_0^j} b \theta_3^j [\Gamma]^j \\ \beta^j \\ \alpha^j \sqrt{y_0^j} \theta_3^j \sqrt{[P]^j} \end{bmatrix}.$$

The parameters of the proposed process can be estimated extending results available for univariate stochastic processes (see e.g. Iacus, 2008), based on the maximization of the log-likelihood function.

To estimate the weights of the idiosyncratic (α^i) and the systematic (β^i) processes, we consider a method of moments estimation procedure. Let y^i and s be the observed time vectors of (country specific) interest rates and of systematic rates. Let then $d^i = y^i - s$ be the observed vector of spreads. The weights can then be estimated as:

$$\begin{cases} \alpha^i = \text{Corr}(d^i, y^i), \\ \beta^i = \text{Corr}(d^i, s). \end{cases} \quad (2.6)$$

From an economic viewpoint, it is important to understand the meaning of the weight coefficients α^i and β^i : if they are both positive, it means that both the correlation between d^i and y^i and that between d^i and s are positive, which means that the idiosyncratic process component of d^i increases faster than the systematic one: $|\frac{\partial y^i}{\partial t}| > \frac{\partial s}{\partial t}$. If the two coefficients are both negative, the systematic component, instead, changes faster than the idiosyncratic one: $|\frac{\partial s}{\partial t}| > \frac{\partial y^i}{\partial t}$.

The linear combination of stochastic processes proposed in (2.1) can be extended to allow for dependence between sovereign, corporate and bank spreads. More formally, suppose that all

these measures, indicated respectively with the indexes $\{1, 2, 3\}$, are determined by the difference between an idiosyncratic and a common, systematic component, for each country $i = 1, \dots, N$:

$$\begin{cases} Z_{t,1}^i = \alpha_1^i y_{t,1}^i - \beta_1^i S_t, \\ Z_{t,2}^i = \alpha_2^i y_{t,2}^i - \beta_2^i S_t, \\ Z_{t,3}^i = \alpha_3^i y_{t,3}^i - \beta_3^i S_t. \end{cases} \quad (2.7)$$

In (2.7) the systematic component follows a univariate CIR process, while all idiosyncratic processes are modeled as a multivariate CIR:

$$\begin{cases} dS_t = (a - vS_{t-1}) dt + b\sqrt{S_{t-1}} dB_t, \\ dy_{t,\{1,2,3\}}^i = [(\theta_1)_{\{1,2,3\}}^i - (\theta_2)_{\{1,2,3\}}^i y_{t-1,\{1,2,3\}}^i] dt + (\theta_3)_{\{1,2,3\}}^i \sqrt{y_{t-1,\{1,2,3\}}^i} dW_t. \end{cases}$$

We then assume the following correlation structure:

$$\begin{cases} \text{Corr}[y_t^m; y_t^n] = \rho^{mn}, \\ \text{Corr}[y_t^m; S_t] = \gamma^m, \end{cases} \quad (2.8)$$

where $\{m, n\} \in (V \times W)$, with $V = \{1, \dots, N\}$ for the countries, and $W = \{1, 2, 3\}$ for the sectors of the economy.

From the above assumptions, the instantaneous covariance matrix of the new process turns out to be the same as that in (2.5), albeit with a different dimensionality, being a $3N \times 3N$ rather than a $N \times N$ matrix.

The estimated covariance matrix \hat{A} can be employed to build a correlation network model between countries and economic sectors. However, such covariances can be misleading because they take into account only bivariate (marginal) relationships between interest spreads. We can obtain conditional covariances, that can adjust bivariate relationships by the presence of other variables. Conditional covariances can then be normalized to obtain conditional (partial) correlations.

Formally, let

$$\hat{A}^{-1}$$

be the inverse of the covariance matrix, with elements a^{mn} . The partial correlation between variables Z^m and Z^n , conditional on the remaining variables in $V \times W$, $\rho_{mn|VW}$ can be obtained as:

$$\rho_{mn|VW} = \frac{-a^{mn}}{\sqrt{a^{mm}a^{nn}}}. \quad (2.9)$$

The estimated partial correlations can be employed to build a correlation network model, as shown in Giudici and Spelta (2015). Before doing so we add a further explanation of the partial correlation coefficient, and of its difference with respect to the (ordinary) marginal coefficient.

For $\{m, n\} \in (V \times W)$, let $S = (V \times W) \setminus \{m, n\}$. Suppose to express the dependence between spread measures through multiple linear equations in the following way:

$$\begin{cases} Z^m = a_m + \sum_{n \neq m} a_{mn|S} Z^n; \\ Z^n = a_n + \sum_{m \neq n} a_{nm|S} Z^m. \end{cases} \quad (2.10)$$

It can be shown that the partial correlation coefficient between Z^m and Z^n , given all the other $3N - 2$ spread measures, can be interpreted as the (signed) geometric average between the multiple linear coefficients introduced in (2.10):

$$|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{a_{mn|S} \cdot a_{nm|S}}. \quad (2.11)$$

Note that if we had only two components ($S = \emptyset$), equation (2.10) becomes:

$$\begin{cases} z^m = a + a_{mn} Z^n, \\ z^n = a + a_{nm} Z^m; \end{cases} \quad (2.12)$$

from which the (ordinary) marginal correlation ρ_{mn} can be derived as the geometric average between the coefficients of the univariate linear models of equation (2.12): $|\rho_{mn}| = |\rho_{nm}| = \sqrt{a_{mn} \cdot a_{nm}}$.

We can thus build a correlation network based on partial correlations, rather than on marginal correlations. To achieve this aim we proceed as follows.

Let $G = (P, E)$ be an undirected graph, with vertex set $P = V \times W = \{1, \dots, 3N\}$, edge set $E = P \times P$, and a binary matrix, with elements e_{mn} , that describes whether pairs of vertices are (symmetrically) linked between each other ($e_{mn} = 1$) or not ($e_{mn} = 0$). An edge between two nodes m and n will then be present in the network if and only if the corresponding partial correlation $\rho_{mn|S}$ is significantly different from zero.

A simple way to detect partial correlations significance consists in using ordinary pairwise statistical t-tests, as partial correlations can also be interpreted as correlations between regression residuals. Alternatively, a more complex search over possible graphical models could be run, as described in Giudici and Spelta (2015).

2.2 Default probabilities and CoRisk

For each country and economic sector $m \in V \times W$, the probability of default, PD_t^m , can be obtained considering the expected dynamic of debt:

$$D_{t+1}^m = (1 - PD_t^m) e^{\alpha^m y_t^m} D_t^m, \quad (2.13)$$

where D_{t+1}^m (D_t^m) is the total debt at time $t + 1$ (t).

Note that the analogous dynamic of risk-free debt is the following:

$$D_{t+1}^m = e^{\beta^m S_t} D_t^m. \quad (2.14)$$

Assuming to be in an arbitrage-free context, we can equate (2.13) with (2.14) and obtain PD_t^m :

$$PD_t^m = 1 - e^{-Z_t^m}. \quad (2.15)$$

From the above equation, it is clear that if Z_t^m decreases, the probability of default decreases, consistently with the definition of the process Z_t^m as the spread between country specific interest rates and benchmark risk free rates.

We now aim at extending this definition of probability of default, which will be named institution-specific PD , to a notion that takes contagion into account. To this aim, we introduce TPD^m , the Total Probability of Default of an economic agent m , built as a function of both an idiosyncratic (PD^m) and a systemic ($CoRisk$) component.

In order to derive TPD and, consequently, $CoRisk$, we combine the interpretation of partial correlation coefficients in (2.10) with the default probabilities derived in (2.15). Formally, let m be the economic entity for which we want to measure the contagion effect, and n be any other economic entity which may have an effect on m . By substituting Z^m with $\ln\left(\frac{1}{1-TPD^m}\right)$ and Z^n with $\ln\left(\frac{1}{1-TPD^n}\right)$ in the first equation of (2.10), it can be shown that:

$$TPD^m = 1 - (1 - PD^m) \cdot \prod_{n \in ne(m)} (1 - PD^n)^{\rho_{mn|S}}, \quad (2.16)$$

where $ne(m)$ indicates all the first-order neighbours of m .

Let us define $CoRisk_{in}^m$ as a function of the survival probability of the neighbours n connected to m , as follows:

$$CoRisk_{in}^m = 1 - \prod_{n \in ne(m)} (1 - PD^n)^{\rho_{mn|S}}. \quad (2.17)$$

It can be shown that, if we assume that $TPD > 0$ (a rather obvious request), $CoRisk_{in}^m$ can be interpreted as the percentage variation of the complement of the default probability (the survival probability) due to contagion effects:

$$CoRisk_{in}^m = \frac{(1 - PD^m) - (1 - TPD^m)}{1 - PD^m}. \quad (2.18)$$

Economically, $CoRisk_{in}^m$ measures the change in the survival probability of an agent m when contagion deriving from its neighbours $n \in ne(m)$ is included.

Similarly, we can measure the outgoing contagion effects, more precisely we can calculate to what extent institution m affects its set of neighbours $ne(m)$. Formally, we can define $CoRisk_{out}^m$ as follows:

$$CoRisk_{out}^m = 1 - \prod_{n \in ne(m)} (1 - PD^m)^{\rho_{nm|S}} = 1 - (1 - PD^m)^{\sum_{n \in ne(m)} \rho_{nm|S}}. \quad (2.19)$$

Note that the two definitions (2.17) and (2.19) introduce asymmetric effects: even if the graph is not oriented and, thus, it is symmetric, the incoming and outgoing contagion effects are different, since each node is associated to a different default probability and, consequently, its contagion effect towards its neighbours is different from the effect it receives from them. More precisely, if the two measures coincide, then the default probability of node m is equal to the geometric average of the default probabilities of its neighbours: on the contrary, if $CoRisk_{out}^m > CoRisk_{in}^m$ ($<$), then the default probability of node m is bigger (lower) than the geometric average of the default probabilities of its neighbours.

As an example, consider the graphs in Figure 1, where each node is associated to its institution-specific PD and each pair of nodes is associated to the corresponding partial correlation coefficient $\rho_{mn|S}$.

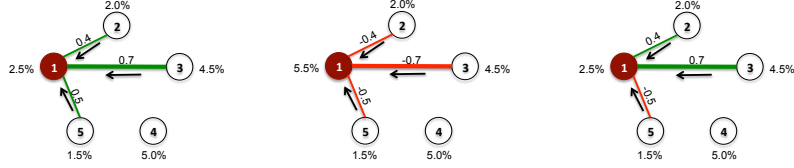


Figure 1: $CoRisk_{in}$, an illustrative example

In the first case, with all positive correlations, the final $CoRisk_{in}$ value is 0.047, meaning that contagion has decreased the survival probability of node 1 by 4.7%, bringing its default probability from $PD^1 = 2.9\%$ to $TPD^1 = 7.2\%$. In the second example, instead, all the correlation coefficients are negative, and the calculated $CoRisk_{in}$ becomes -0.049, meaning that contagion has increased the survival probability of node 1 by 4.9%. According to equation (2.16), the total TPD^1 has decreased, being equal to 0.87%. Note that the $CoRisk$ measure in this second case is not equal, in absolute value, to the one obtained in the previous example: this because the exponent ρ introduces non-linear effects in the relationship in (2.17). In the last example, where both positive and negative correlations appear, the calculated $CoRisk_{in}$ measure is equal to 0.032, meaning that contagion has decreased the survival probability of node 1 by 3.2%, reaching a total default probability $TPD^1 = 5.6\%$.

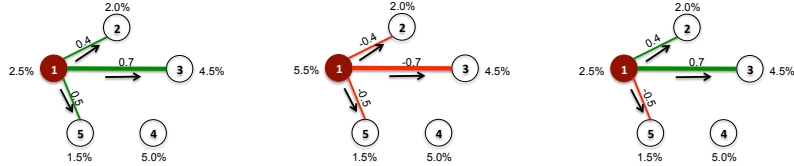


Figure 2: $CoRisk_{out}$, an illustrative example

Figure 2 reports the same graphs as in Figure 1, but we now concentrate on the outgoing effects in order to understand how node 1 affects its neighbours. In the first example, the overall $CoRisk_{out}$ is equal to 0.040: this result is lower with respect to the $CoRisk_{in}$ value because the incoming contagion is highly affected by the large default probability of node 3. Similarly, in the second situation the final $CoRisk_{out}$ is -0.095. This result is lower than the corresponding $CoRisk_{in}$ because, now, the default probability of node 1 is much bigger than the default probabilities of its neighbours: consequently, the contagion effect due to negative correlations is amplified, meaning that a negative relation with node 1 strongly decreases the default probability of the set $ne(1)$. In the last example the calculated $CoRisk_{out}$ measure is equal to 0.015, lower than $CoRisk_{in}$ as in the first example.

From a mathematical viewpoint, $CoRisk$ (both *in* and *out*) is expressed as a function of partial correlations and default probabilities. By remembering that $\rho_{mn|S} \in [-1, 1]$ and $PD \in [0, 1]$, $CoRisk$ is thus a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, in particular $f(x, y) : [-1, 1] \times [0, 1] \rightarrow (-\infty, 1]$.

In order to better interpret this measure, it is important to study its limit conditions. More precisely, $CoRisk$ is equal to zero when, for all the first-order neighbours, one of the two following conditions holds:

$$\begin{cases} PD^n = 0, & \forall n \in ne(m); \\ \rho_{mn|S} = 0, & \forall n \in ne(m). \end{cases} \quad (2.20)$$

This is consistent with the definition of *CoRisk*, meaning that the contribution to the default probability of a country m that derives from contagion effects is null (a) if all its neighbours have zero default probability, or (b) if country m is not partially related to any other country.

Secondly, *CoRisk* reaches its highest value 1 if $\exists n \in ne(m)$ s.t. $PD^n = 1$, meaning that the highest contribution to systemic risk of country m occurs when at least one of its neighbours n is in default.

Finally, it is interesting to observe that contagion risk is negative when partial correlations prevail: in particular, $CoRisk \rightarrow -\infty$ if $\exists n \in ne(m)$ s.t. the two following conditions simultaneously hold:

$$\begin{cases} PD^n \rightarrow 1, \\ \rho_{mn|S} \rightarrow -1. \end{cases} \quad (2.21)$$

The total default probabilities introduced in (2.16) are referred to index m , and are thus defined for each economic sector within each country. However, it is interesting to calculate the total default probability of different countries, obtained by aggregating the default probabilities of their economic sectors: it is reasonable to assume that a country will default if at least one of its economic sector defaults. In order to achieve this objective, each TPD^m can be considered as a conditional probability with respect to all the other neighbours $ne(m)$. In particular, denoting with A_1^i , A_2^i and A_3^i the sets of defaults for, respectively, the sovereign, corporate and bank sectors of country i , we are interested in deriving $P(\bigcup_{j \in W} A_j^i | S^i)$, where $S^i = \{A^m; \forall m \in V \times W, m \in ne(i, j), m \neq (i, j)\}$.

It can be shown that the aggregate total default probability of country i is the following:

$$TPD_{country}^i = 1 - [1 - Pr(A_2^i | A_1^i, A_3^i, S^i)] \cdot [1 - Pr(A_3^i | A_1^i, S^i)] \cdot [1 - Pr(A_1^i | S^i)], \quad (2.22)$$

where the three probabilities in (2.22) are the TPD derived through (2.16) by considering, respectively, all neighbours, all neighbours but the corporate sector of country i , all neighbours but the corporate and bank sectors of country i .

3 Application

We focus on eleven european countries: Austria, Belgium, Finland, France, Germany and the Netherlands (core countries); Greece, Ireland, Italy, Portugal and Spain (peripheral countries).

For each country we consider three idiosyncratic components, for sovereign, corporate and bank risk: (a) interest rates on government bonds, (b) aggregate interest rates on bank loans to non-financial corporates, (c) aggregate interest rates on bank deposits from non-financial corporates and households. Concerning the common systematic component, there are many choices for a benchmark rate: we suggest a rate that reflects the impact of the European Central Bank monetary policy, such as the 3-months Euribor.

In order to evaluate the evolution of the resulting $N = 11 \times 3$ -dimensional system of interest rate spreads, we have considered four different time windows: (a) the pre-crisis period (2003-2006), (b) the financial crisis period (2007-2009), (c) the sovereign crisis period (2010-2012) and (d) the post-crisis period (2013-2015).

All data are publicly available and have been selected with a monthly frequency.

3.1 Descriptive statistics

A summary statistics of the data is shown in Table 1: for each of the three economic sectors, data have been grouped in four time windows, and means, standard deviations as well as correlations with the Euribor interest rate are reported.

From Table 1 note that interest rates on loans have the highest correlation coefficients with Euribor interest rates, during all time-windows and in almost all countries. The same correlations vary for interest rates on government bonds: low during the pre-crisis period and higher afterwards (with the exception of Greece). The correlations of bank interest rates with the Euribor follow a similar pattern, being very low until 2012 in almost all countries, and strongly positive afterwards.

The time evolution of the interest rate processes for the sovereign sector can be observed in Figure 3.

Figure 3 shows that interest rates on government bonds were initially very similar, while in 2010 they started diverging: decreasing in core countries and increasing in peripheral countries.

Pre-crisis Period									
Country	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	3.866	0.368	-0.033	4.096	0.289	0.371	3.248	0.189	-0.463
Bel	3.894	0.366	-0.041	4.525	0.225	0.171	4.117	0.251	-0.415
Fin	3.845	0.381	0.009	3.640	0.312	0.791	2.664	0.225	0.202
Fra	3.859	0.352	-0.020	4.351	0.159	0.399	3.669	0.102	-0.389
Ger	3.806	0.352	0.008	4.982	0.189	-0.065	3.142	0.274	-0.540
Gre	4.045	0.343	0.109	5.659	0.264	0.880	0.402	0.117	0.606
Ire	3.826	0.378	-0.002	4.675	0.372	0.634	2.564	0.202	0.806
Ita	4.027	0.349	0.096	4.538	0.307	0.654	3.131	0.258	0.140
Net	3.843	0.362	-0.015	4.693	0.207	0.272	3.971	0.269	-0.074
Por	3.919	0.358	0.071	4.548	0.321	0.929	3.033	0.252	0.301
Spa	3.850	0.362	-0.019	3.619	0.324	0.780	2.487	0.174	0.144

Financial-crisis Period									
Country	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	4.198	0.298	0.752	4.531	0.943	0.967	3.352	0.179	0.383
Bel	4.216	0.322	0.834	4.754	0.671	0.986	4.009	0.156	0.427
Fin	4.107	0.350	0.847	4.378	1.103	0.980	2.968	0.355	0.838
Fra	4.063	0.370	0.852	4.710	0.587	0.956	3.565	0.059	0.353
Ger	3.808	0.502	0.840	4.961	0.579	0.986	2.688	0.047	0.726
Gre	4.826	0.437	-0.417	6.326	0.777	0.972	1.386	0.523	-0.175
Ire	4.686	0.481	-0.668	5.321	1.274	0.990	2.427	0.457	0.748
Ita	4.494	0.268	0.644	5.208	1.062	0.968	3.182	0.533	0.972
Net	4.067	0.352	0.846	4.751	0.797	0.994	3.758	0.053	0.153
Por	4.385	0.292	0.605	5.416	1.031	0.949	3.058	0.407	0.930
Spa	4.218	0.278	0.756	4.873	0.789	0.904	2.717	0.234	0.528

Sovereign-crisis Period									
Country	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	2.972	0.587	0.586	2.845	0.232	0.934	2.297	0.104	0.256
Bel	3.565	0.660	0.877	3.460	0.169	0.920	3.182	0.177	-0.136
Fin	2.634	0.642	0.499	2.450	0.269	0.971	2.138	0.129	-0.232
Fra	2.992	0.482	0.618	3.318	0.144	0.823	3.150	0.080	0.023
Ger	2.282	0.697	0.383	3.837	0.191	0.933	2.564	0.095	0.500
Gre	15.780	6.526	0.011	5.649	0.553	0.313	2.491	0.312	-0.265
Ire	7.171	2.136	0.832	3.323	0.281	0.869	1.939	0.399	-0.182
Ita	4.984	0.891	0.305	3.505	0.332	0.221	2.784	0.441	-0.470
Net	2.638	0.622	0.469	3.436	0.192	0.991	3.801	0.139	0.026
Por	8.728	2.953	0.426	4.264	0.676	0.203	2.511	0.456	-0.172
Spa	5.179	0.815	-0.008	3.532	0.260	0.404	2.486	0.282	0.051

Post-crisis Period									
Country	$y_{t,1}$ (Sov, %)			$y_{t,2}$ (Corp, %)			$y_{t,3}$ (Bank, %)		
	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur	Mean	SD	Cor-Eur
Aus	1.430	0.608	0.829	2.323	0.097	0.969	1.681	0.206	0.837
Bel	1.676	0.740	0.854	2.851	0.190	0.950	2.697	0.222	0.847
Fin	1.357	0.564	0.864	1.870	0.110	0.964	1.587	0.330	0.786
Fra	1.589	0.650	0.856	2.725	0.194	0.856	2.864	0.124	0.835
Ger	1.091	0.521	0.857	3.085	0.204	0.911	2.015	0.192	0.833
Gre	8.943	1.881	-0.293	5.442	0.366	0.921	2.161	0.971	0.891
Ire	2.485	1.158	0.816	3.095	0.071	-0.496	1.892	0.280	0.630
Ita	3.014	1.133	0.801	3.511	0.237	0.947	2.896	0.368	0.578
Net	1.387	0.612	0.850	2.908	0.174	0.953	3.560	0.170	0.820
Por	4.205	1.708	0.718	4.108	0.338	0.919	2.730	0.434	0.952
Spa	3.045	1.266	0.728	3.129	0.360	0.935	2.258	0.348	0.800

Table 1: Summary statistics for interest rates on government bonds ($y_{t,1}^i$), interest rates on loans to non-financial corporates ($y_{t,2}^i$) and interest rates on deposits to families and non-financial corporates ($y_{t,3}^i$), for the four time-windows

Greece, Ireland and Portugal present the highest volatility, corresponding to their sovereign crisis in 2010-2011, followed by Italy and Spain and, to a lesser extent, Belgium.

The time evolution of the interest rate processes for the corporate sector can be observed in Figure 4.

From Figure 4 one can notice that interest rates on loans to non-financial corporates differ across the main european countries in a manner that is consistent across time. In particular, Greece and Portugal have the highest values while Finland and Austria present the lowest ones. The interest curves of corporates do not show substantial overlaps: they all increase during the financial crisis of 2008 and, to a lesser extent, during the sovereign crisis of 2011. All rates show positive correlations with the Euribor dynamics. Overall, the scale of variation of corporate rates is much smaller than that of sovereign rates, especially in peripheral countries.

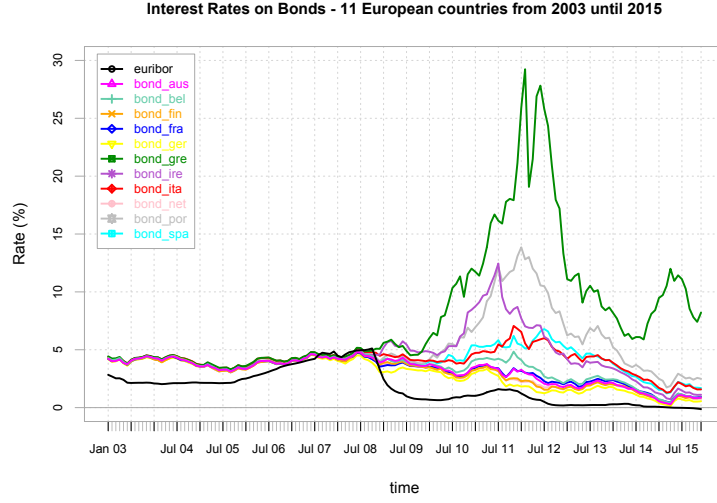


Figure 3: Monthly time evolution of 10-years maturity bond interest rates and of the 3-months Euribor, from January 2003 until December 2015

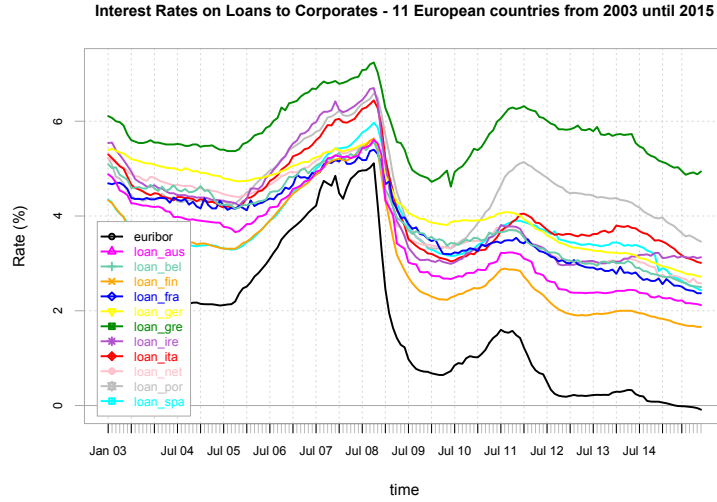


Figure 4: Monthly time evolution of interest rates on loans to non-financial corporates and of the 3-months Euribor, from January 2003 until December 2015

The time evolution of the interest rate processes for the bank sector can be observed in Figure 5.

Figure 5 shows an interest rate pattern substantially different with respect to sovereigns and non-financial corporates. The highest rates can be detected in France, Belgium and the Netherlands consistently through time, while the curves of the other countries do overlap: this is especially true for peripheral countries, affected not only by the financial crisis but also by the sovereign crisis. In addition, while France, Belgium, the Netherlands and Germany show low correlations with the Euribor rate across time, the other countries increase their correlations, especially after 2012. Overall, the scale of variation of bank rates is slightly lower than that of corporate ones.

3.2 Multivariate stochastic processes

The first step in model estimation consists in deriving the coefficients in (2.2), for the three sectors of each country (and for the common systematic process). The Appendix contains such estimated coefficients, along with the estimates of the relative weights of the idiosyncratic and systematic

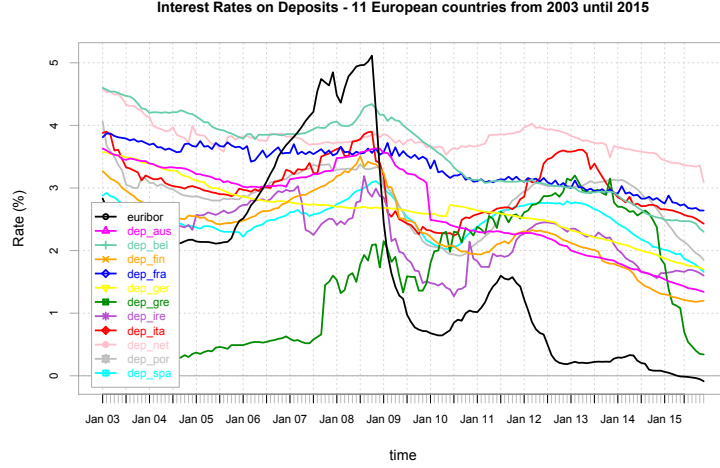


Figure 5: Monthly time evolution of interest rates on deposits and of the 3-months Euribor, from January 2003 until December 2015

processes.

From Table 6 note that, during the two crisis periods, all the parameters (drift and volatility terms) of the three processes are sensibly higher in peripheral countries. In the post-crisis period, the drift returns to the initial values (with the exception of Greece), but the volatility remains quite high.

From Table 7 note that, during the pre-crisis period, both weights α_1 and β_1 are negative for the sovereign and bank sectors in all countries: the systematic component, therefore, changes faster than the idiosyncratic one. On the contrary, during the post-crisis years, almost all weights are positive, meaning that the idiosyncratic component changes faster than the systematic one, consistently with the current situation of almost zero monetary rates. Overall, the weights of the processes change over time, but such changes are homogeneous and similar across the eleven considered countries through time.

We now derive, according to (2.9), the network models obtained for the sovereign, corporate and bank spreads. To achieve this aim it is necessary to calculate, within each sector j , the inverse correlation matrix of the spreads $Z_{t,j}^i$ for the eleven countries i and in each time period t , the resulting partial correlation coefficients, and the p -values associated to the significance t-test of each partial correlation. As a general rule, a connection between two countries will be removed when the p -value is greater than $\alpha = 0.10$.

We can thus derive the partial correlation network for each sector and for the four different time windows. The results are shown in Figure 6, in which the green lines stand for positive significant partial correlations, while red lines indicate negative significant partial correlations; moreover, the thicker the line, the stronger the connection.

Comparing the sovereign partial correlation networks in Figure 6 note that their pattern has substantially changed over the years: in the pre-crisis period the overall number of significant correlations is quite high; during the financial crisis the number of significant correlations decreases; during the sovereign crisis correlations further decreases, and a "clustering effect" that separates core and peripheral economies in two quite distinct subgraphs emerges. Last, in the post-crisis period the partial correlation pattern returns to the pre-crisis situation, however with a persisting clustering effect, emphasized not only by positive within subgraph correlations, but also by negative ones between the two subgraphs.

Bank partial correlation networks, similarly to sovereign ones, are quite connected in the first two periods, and become sparser afterwards. In this case, the clustering effect becomes evident in the last, rather than in the third period. This time delay may also be due to the different kind of data used for banks with respect to sovereigns: the latter are market-based data, characterized by quick reactions to the economic perspectives of a country; the former, instead, depend upon banks' decisions and are characterized by a viscosity degree with respect to the external environment.

By analyzing the corporate partial correlation networks in Figure 6, note that also in this case the partial correlation pattern has substantially changed over the years. During the pre-crisis period the overall number of significant correlations is quite high, similarly to the sovereign

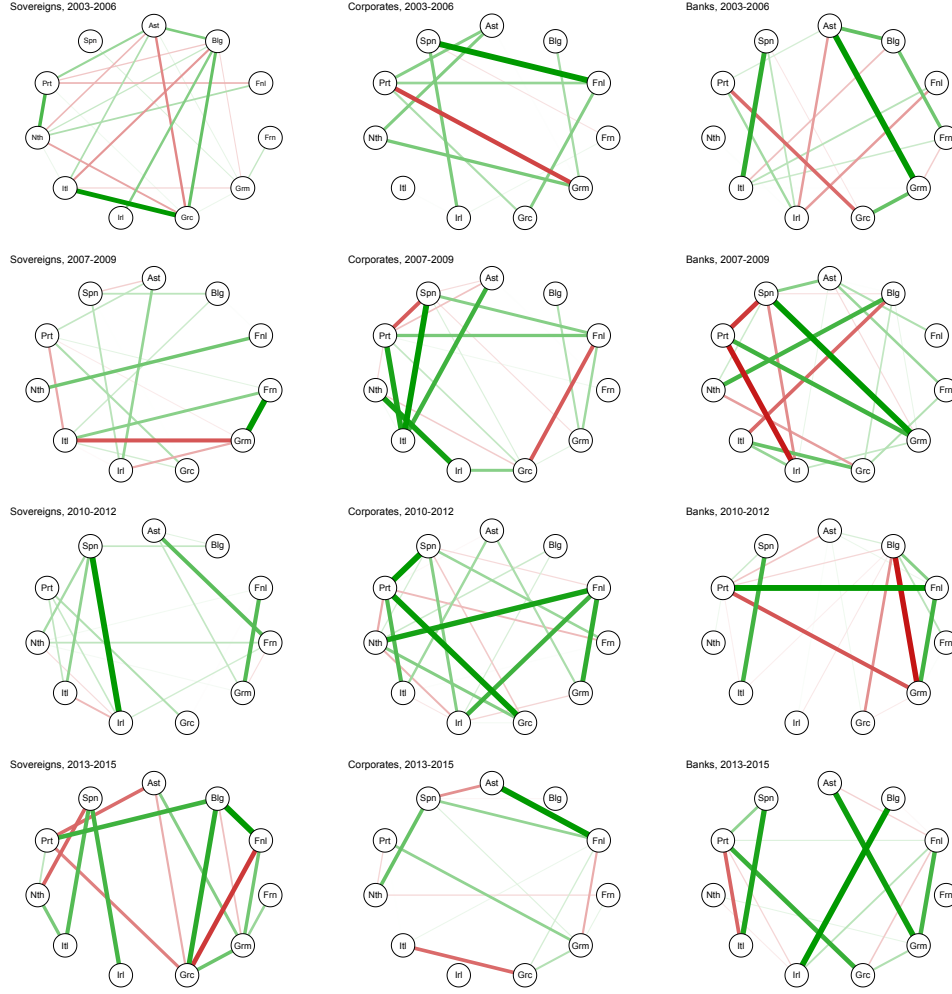


Figure 6: Network graphs for the eleven european countries considered in the sample, based on $Z_{t,1}^i$ (left), $Z_{t,2}^i$ (middle) and $Z_{t,3}^i$ (right), for the pre-crisis (first row), financial-crisis (second row), sovereign-crisis (third row) and post-crisis (fourth row) periods

and bank ones. During the financial crisis the number of significant correlations substantially decreases; during the sovereign crisis significant correlations increase again in number, and they drop in the last period, characterized by low growth and close-to-zero Euribor interest rates. Differently from what observed in the other two economic sectors, a clustering effect between core and peripheral countries is not evident: a possible explanation is that corporate interest rates are highly and constantly correlated with Euribor rates across time and, thus, clustering effects become less significant.

3.3 Default probabilities and CoRisk

After having estimated all the process parameters and the partial correlation networks, we are now able to calculate the institution-specific probability of default of each sovereign ($PD_{t,1}^i$), corporate ($PD_{t,2}^i$) and bank ($PD_{t,3}^i$) sector in each country i , based respectively on the spread measures $Z_{t,1}^i$, $Z_{t,2}^i$ and $Z_{t,3}^i$ according to equation (2.15). By using such PD s and the correlation networks, we can calculate the $CoRisk_{in}$ measures and, through them, the total default probability of each economic sector in each country $TPD_{t,\{1,2,3\}}^i$ as in (2.16).

Summary statistics of $CoRisk_{in}$ for the different time windows are shown in Table 2. The corresponding time evolution is shown in Figure 7.

Let us firstly consider the sovereign graphs in Figure 7. By looking at the single institution-specific PD (top graphs), it is clear that Greece presents the most critical situation, with the highest PD values. Portugal has similar, but lower results. Ireland presents an anticipated

Pre-crisis Period												
Country	<i>CoRisk_{sov}</i> (%)				<i>CoRisk_{corp}</i> (%)				<i>CoRisk_{bank}</i> (%)			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.36	0.30	1.09	2.23	1.91	0.09	1.78	2.11	1.90	0.18	1.68	2.31
Bel	1.42	0.32	1.14	2.34	0.49	0.09	0.26	0.56	0.78	0.28	0.44	1.49
Fin	-0.01	0.01	-0.03	0.01	4.28	0.32	3.97	5.16	0.52	0.13	0.39	0.84
Fra	0.60	0.14	0.47	1.01	0.92	0.09	0.73	1.07	1.11	0.31	0.84	2.01
Ger	0.44	0.13	0.32	0.79	2.01	0.23	1.56	2.33	2.20	0.55	1.75	3.87
Gre	0.96	0.17	0.82	1.45	0.99	0.18	0.83	1.51	-0.37	0.08	-0.64	-0.30
Ire	0.89	0.20	0.71	1.46	1.21	0.13	1.08	1.54	2.09	0.25	1.65	2.56
Ita	0.91	0.16	0.78	1.36	0.92	0.07	0.83	1.10	1.90	0.15	1.78	2.30
Net	0.95	0.22	0.75	1.57	1.97	0.17	1.67	2.23	0.60	0.09	0.52	0.85
Por	0.58	0.14	0.44	0.98	1.15	0.26	0.94	1.87	-0.04	0.15	-0.21	0.30
Spa	0.51	0.12	0.40	0.86	1.25	0.12	1.10	1.54	1.52	0.30	1.25	2.34

Financial-crisis Period												
Country	<i>CoRisk_{sov}</i> (%)				<i>CoRisk_{corp}</i> (%)				<i>CoRisk_{bank}</i> (%)			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	1.86	0.78	0.97	3.52	-0.04	0.16	-0.54	0.13	6.86	1.56	4.08	8.98
Bel	4.82	1.73	2.46	8.19	-0.20	0.36	-0.75	0.14	4.17	0.55	3.07	4.98
Fin	3.98	1.40	2.02	6.62	2.05	1.61	0.11	5.35	3.60	0.91	1.98	4.83
Fra	6.66	2.65	3.76	12.13	1.26	0.52	0.43	2.10	5.32	1.40	2.84	7.11
Ger	-2.18	0.97	-4.11	-1.00	5.94	2.49	2.02	9.40	7.79	1.76	4.32	10.06
Gre	3.19	1.34	1.71	6.00	2.12	1.28	0.58	4.63	3.19	1.15	1.24	4.58
Ire	1.28	0.69	0.34	2.29	5.44	2.34	2.56	10.83	-1.78	0.63	-2.52	-0.62
Ita	-0.06	0.67	-0.92	0.94	7.37	3.43	2.38	13.57	2.86	0.51	2.13	4.01
Net	3.10	1.10	1.32	4.40	2.06	1.30	-0.01	3.85	3.77	1.18	1.77	5.27
Por	1.96	0.71	0.74	2.80	2.70	1.25	0.75	4.79	-1.80	0.34	-2.36	-1.28
Spa	1.79	0.71	0.94	3.25	2.14	1.14	0.35	3.65	-1.07	1.05	-2.19	0.59

Sovereign-crisis Period												
Country	<i>CoRisk_{sov}</i> (%)				<i>CoRisk_{corp}</i> (%)				<i>CoRisk_{bank}</i> (%)			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	2.86	0.53	1.73	3.62	3.21	0.48	2.40	3.98	2.45	0.62	1.35	3.44
Bel	2.59	0.28	2.17	3.15	0.44	0.30	-0.16	0.87	-1.63	0.34	-2.19	-1.11
Fin	1.47	0.39	0.77	2.09	5.04	0.86	3.60	6.42	4.29	0.88	2.86	5.74
Fra	4.02	0.89	2.65	6.08	0.66	0.21	0.23	0.95	1.95	0.30	1.43	2.45
Ger	1.78	0.45	0.97	2.44	-0.40	0.28	-0.97	-0.04	-2.02	0.21	-2.40	-1.74
Gre	3.61	1.22	1.71	5.71	1.55	0.31	1.05	2.00	-1.41	0.23	-1.76	-0.97
Ire	4.99	1.29	2.78	6.72	1.70	0.28	1.32	2.17	-0.48	0.03	-0.53	-0.42
Ita	2.82	1.19	1.45	4.83	1.47	0.31	1.05	2.00	0.43	0.17	0.18	0.71
Net	1.64	0.36	0.73	2.35	2.90	0.50	2.04	3.68	0.57	0.12	0.43	0.77
Por	10.80	3.06	5.70	16.10	4.36	0.53	3.51	5.12	0.07	0.07	-0.05	0.18
Spa	8.81	1.71	6.46	12.69	3.72	0.68	2.68	4.86	1.84	0.37	1.36	2.45

Post-crisis Period												
Country	<i>CoRisk_{sov}</i> (%)				<i>CoRisk_{corp}</i> (%)				<i>CoRisk_{bank}</i> (%)			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Aus	-4.60	0.82	-5.88	-3.18	2.78	0.11	2.58	2.99	0.64	0.06	0.52	0.74
Bel	6.78	1.08	4.87	8.42	0.31	0.02	0.26	0.34	2.60	0.21	2.21	2.90
Fin	-3.75	1.31	-6.79	-1.81	6.14	0.39	5.40	6.52	0.65	0.19	0.42	1.04
Fra	0.43	0.20	0.05	0.76	0.51	0.06	0.41	0.59	0.07	0.00	0.07	0.08
Ger	5.23	0.95	3.64	6.60	7.20	0.40	6.52	7.76	5.62	0.78	4.29	6.80
Gre	-0.94	0.21	-1.21	-0.53	-0.72	0.07	-0.81	-0.59	0.89	0.16	0.55	1.12
Ire	1.64	0.67	0.68	2.81	-1.26	0.05	-1.37	-1.20	0.93	0.19	0.69	1.28
Ita	1.80	0.64	0.85	2.87	-1.57	0.07	-1.72	-1.47	0.52	0.13	0.35	0.78
Net	0.82	0.24	0.46	1.19	0.62	0.11	0.45	0.79	0.18	0.24	-0.11	0.62
Por	-4.01	0.98	-5.70	-2.44	1.61	0.08	1.44	1.72	1.42	0.61	0.25	2.10
Spa	2.63	1.15	0.91	4.24	1.78	0.07	1.65	1.85	4.15	0.46	3.43	4.90

Table 2: Summary statistics of $CoRisk_{in}$ measures, for the sovereign, corporate and bank sectors, in the four time-windows

increase in its default probability because of its specific sovereign crisis in 2011, but in the following years it starts performing quite well until reaching very low PD values in 2015. Italy and Spain show similar values, while core countries behave quite similarly to each other, with the lowest PD s across time.

The $CoRisk_{in}$ pattern can be understood by looking at the networks in Figure 6: countries with high positive correlations with peripheral economies, characterized by high PD s, have a high $CoRisk_{in}$: this is the case, for example, of France and Belgium in the second period, strongly connected, respectively, with Italy and Portugal, and with Italy and Spain. Similarly, Spain presents a high $CoRisk_{in}$ contribution during the sovereign-crisis period, due to its strong positive link with Ireland, a particularly troubled country in such years. On the other hand, countries which are negatively or not connected with peripheral ones (such as, for instance, Germany in the second period and Finland and Austria in the last years) have close to zero or negative $CoRisk_{in}$

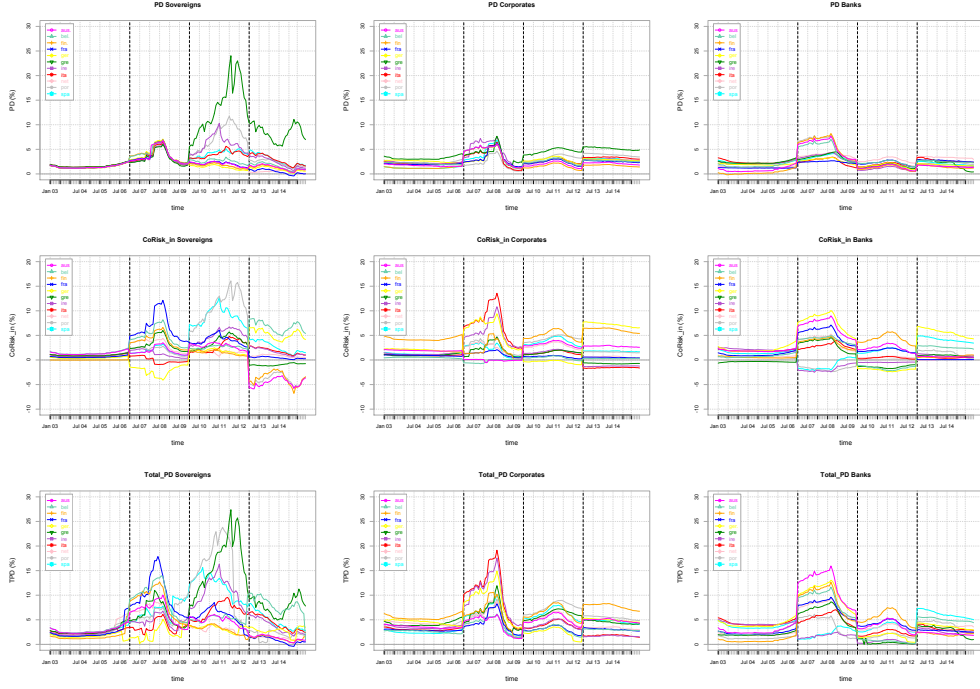


Figure 7: Institution-specific default probabilities $PD_{t,\{1,2,3\}}^i$ (top), $CoRisk_{in}$ measures (middle) and total default probabilities $TPD_{t,\{1,2,3\}}^i$ (bottom) from 2003 until 2015, for the sovereign (left), corporate (middle) and bank (right) sectors

measures.

The final TPD time-evolution is obviously a mix between the institution-specific PD and the $CoRisk_{in}$ contribution, with the former seeming to prevail. In addition, further interesting conclusions emerge. In peripheral economies, characterized by high institution-specific PDs , the $CoRisk_{in}$ contribution should be very low; the creation of two distinct clusters, however, creates a sort of "loop", because peripheral economies start being positively connected only between each other, and negatively connected with core ones. For this reason their total default probability TPD is strongly influenced not only by its corresponding institution-specific PD , but also by high $CoRisk_{in}$ values. For the same reason, core economies preserve low total PD , even after the inclusion of contagion effects: the only one exception is France, which presents an extremely high $CoRisk_{in}$ during the financial crisis due to a positive connection with Italy. Germany lies in an intermediate situation, with its $CoRisk_{in}$ growing in the recent years along with positive connections with the periphery, in the light of its increasing leading role in the Euro area.

The corporate graphs in Figure 7 show institution-specific PDs less volatile than sovereign ones, across both countries and time. They all peak during the financial crisis and decrease afterwards, remaining almost constant during the following years. In recent times, the ranking of countries reflects the PD situation observed for sovereign risk, with Greece presenting much higher values than all the other countries, and core economies having the lowest ones. This means that, in Europe, sovereign risk has become the main risk driver behind portfolio allocation.

The $CoRisk_{in}$ pattern shows that almost all countries suffered contagion effects during the financial crisis and, to a lesser extent, during the sovereign crisis. More precisely, Italy presents the highest $CoRisk_{in}$ values because of its strong positive relationships with Portugal and Spain (see Figure 6).

Differently from what has been observed in the sovereign case, $CoRisk_{in}$ is the prevailing effect in the calculation of the total default probability of the corporate sector (with the exception of Germany for the last two periods, because of its very low institution-specific PD values): such a conclusion is supported by Figure 6, which shows that partial correlations are much higher in number and in value, and that a clustering effect is not present.

The bank graphs in Figure 7 reveal that, differently from the sovereign and corporate sectors, the institution-specific PDs of all countries have been influenced only by the financial crisis. Consistently with Figure 5, Belgium, Finland, Portugal and Spain present the highest peaks

during 2008, because of their high average values of interest rates on deposits.

The $CoRisk_{in}$ pattern shows both positive and negative contagion effects during the second time-period, with the former regarding core countries and the latter regarding peripheral countries. More precisely, Germany has a positive contagion effect because of its positive relations with Portugal and Spain, Austria and France because of their positive links to both peripheral and core countries; on the other hand, Ireland, Portugal and Spain are characterized by negative correlations with each other and with the remaining peripheral economies, and are thus characterized by negative $CoRisk_{in}$ values. In the post-crisis period, when two distinct clusters start emerging as for the sovereign case, $CoRisk_{in}$ increases again both in core and peripheral economies, because of highly positive partial correlations within each cluster.

Similarly to the corporate sector, $CoRisk_{in}$ is the prevailing effect in the composition of the total default probability in all time-periods and in almost each country.

In order to understand to what extent a country as a whole is influenced by its neighbours, the aggregate total default probability has been proposed in (2.22): this measure enables us to synthesize contagion effects deriving from different economic sectors into a unique default probability at the country level. The aggregation takes into account intra country contagion effects, that are sterilized. Such results are shown in Figure 8.

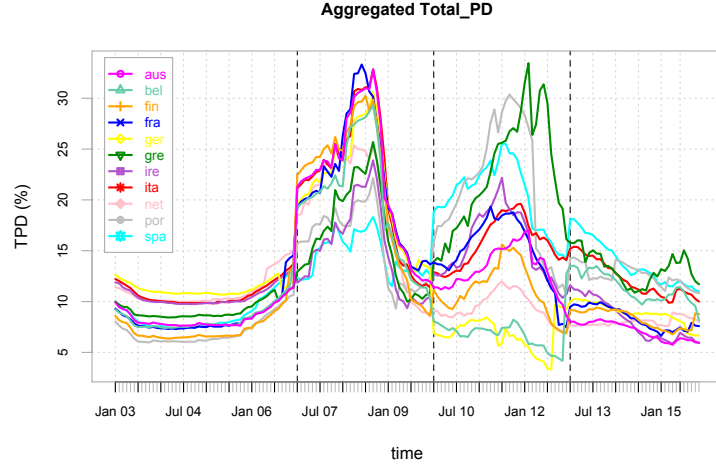


Figure 8: Aggregated total default probabilities $TPD_{country}^i$ from 2003 until 2015

The analysis of Figure 8 shows how the aggregated total default probability of each country has been influenced by the financial and the sovereign crisis. Two main considerations emerge. First, the financial crisis had a more homogenous impact across countries than the sovereign one: all the aggregated TPD strongly increased during 2008, while in the following time-window a clear distinction between peripheral and core countries appears, with the former characterized by higher values and the latter by lower ones (even decreasing for Germany and Belgium). Two particular countries need a deeper understanding: France, which presents high values mainly because of its positive correlations with peripheral countries, during both the financial and the sovereign crisis; Ireland, characterized by a deep sovereign crisis in 2011, worsen by positive links with peripheral countries (Spain) until 2012, but now performing well, with very low TPD values and positive relations with core economies. Second, the pre- and post- crisis periods appear to be substantially different: during the pre-crisis years, in fact, default probabilities were almost constant and stable across time, and very homogenous across countries; but after the sovereign crisis the situation has become more heterogenous both from a dynamic and a cross-sectional perspective, with high volatilities in all countries and a clear distinction between peripheral (Greece, Spain, Portugal, Italy) and core (Belgium, Finland, the Netherlands, France, Germany, Austria, Ireland) economies. This effect, consistently with Figure 7, means that the sovereign crisis has had a stronger and more persistent impact, that has made sovereign risk the main risk driver. A possible explanation of this effect lies in the different ways peripheral and core economies reacted to the financial crisis, depending on their "sovereign space": peripheral countries were not in "good health" even before the crisis, and even if interest rates on government bonds did not reflect it, the financial crisis triggered problems to emerge.

Having derived the aggregated default probability at the country level, it is important to understand the evolution of its composition over time. The total PD of a country, in fact, has been calculated as a function of the total default probabilities of its three economic sectors; furthermore, the TPD of each economic sector has been obtained as a function of two contributions: its institution-specific PD and the $CoRisk_{in}$ measure. By applying a log transformation it becomes possible to disentangle the final default probability of a country into six components: three deriving from the economic sectors, and two deriving from the distinction between institution-specific PD and $CoRisk_{in}$ contribution. Since we are interested in analyzing these results for macro-prudential policy purposes, in this context negative $CoRisk_{in}$ contributions have been set up to zero, and mean normalized values in the four time-windows have been derived in order to compare histograms rather than continuous time series. The results are shown in Figure 9.

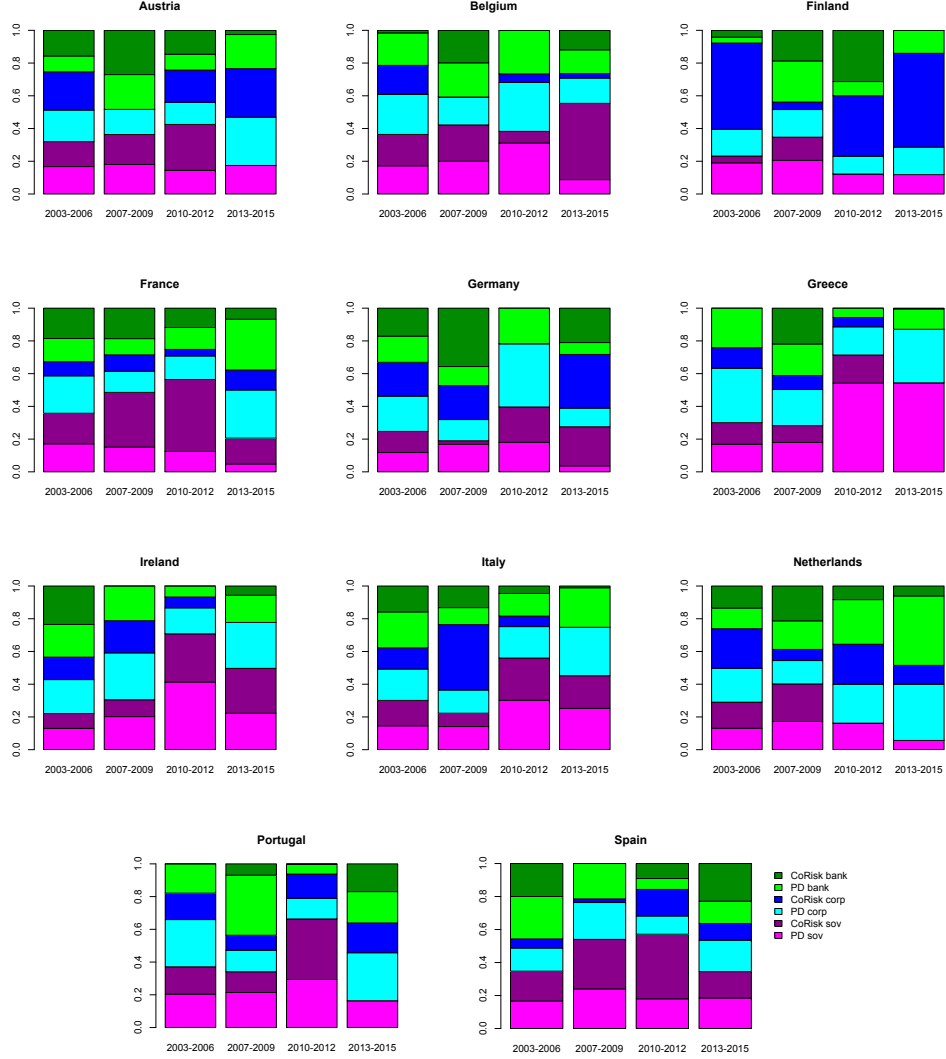


Figure 9: Aggregated total default probabilities contributions: $CoRisk_{in}$ and PD components for the three economic sectors in the four time-windows

From Figure 9 one can observe that, for all time periods, the sovereign contribution is larger in peripheral countries than in core ones; furthermore, in core economies the main component of sovereign risk is due to contagion effects, while in peripheral countries the institution-specific PD component is much higher. Moreover, peripheral economies show high $CoRisk_{in,sov}$ contributions because of loop effects, which means positive correlations and, thus, contagion effects between each other. In almost all countries the corporate contribution is stronger during "normal" times, such as before the financial crisis and in the last period, depending on institution-specific PD for

peripheral economies and on contagion effects in core economies (Austria, Finland, Germany, the Netherlands). Finally, core economies suffered a substantial improvement in contagion effects for the bank sector during the sovereign crisis through their exposition to peripheral banks, while peripheral economies witnessed an increase in their institution-specific bank default probabilities.

Overall, the contribution of the bank and the sovereign sectors to systemic risk increased, in all countries, respectively during the financial and the sovereign crisis, with prevailing contagion effects in core economies, and higher institution-specific PDs and contagion effects deriving as a consequence of clustering in peripheral countries. Note also that the distribution of risk in its six components looks quite homogenous before the financial crisis, while after it the situation is not back to normality, because strong contagion risks persist in core economies, while institution-specific default probabilities are still high and even worsened by clustering effects in peripheral ones.

For comparative purposes, we have also considered the eigenvector centrality (see e.g. Furfine, 2003; Billio et al., 2012) and the weighted degree, calculated as the sum of all the significant partial correlations, and we have compared the results with the $CoRisk_{in}$ measure. The degrees of connectivity and the eigenvectors are reported in the Appendix, Table 8, while a comparison between the rankings obtained with our methodology ($CoRisk_{in}$) and the degree of connectivity and eigenvector centrality is shown in Table 3.

Period	Sovereign			Corporate			Bank		
	$CoRisk$	DC	$Eigen.$	$CoRisk$	DC	$Eigen.$	$CoRisk$	DC	$Eigen.$
2003-2006	Bel	Net	Gre	Fin	Fin	Fin	Ger	Aus	Ger
	Aus	Gre	Ita	Ger	Net	Por	Ire	Ita	Aus
	Gre	Ita	Por	Net	Ger	Spa	Ita	Ger	Gre
	Net	Por	Spa	Aus	Aus	Gre	Aus	Bel	Bel
	Ita	Bel	Ire	Spa	Ire	Ire	Spa	Fra	Fin
	Ire	Fra	Aus	Ire	Por	Ita	Fra	Spa	Ita
	Fra	Ger	Ger	Por	Gre	Aus	Bel	Ire	Fra
	Por	Ire	Fra	Gre	Spa	Net	Net	Net	Spa
	Spa	Spa	Bel	Ita	Ita	Ger	Fin	Por	Ire
	Ger	Aus	Net	Fra	Fra	Fra	Por	Fin	Net
2007-2009	Fin	Fin	Fin	Bel	Bel	Bel	Gre	Gre	Por
	Fra	Fra	Ger	Ita	Ita	Por	Ger	Ger	Spa
	Bel	Spa	Fra	Ger	Ire	Ita	Aus	Aus	Net
	Fin	Aus	Net	Ire	Ger	Gre	Fra	Fra	Aus
	Gre	Gre	Spa	Por	Fin	Ire	Bel	Net	Bel
	Net	Fin	Fin	Spa	Gre	Ger	Net	Gre	Fra
	Por	Net	Por	Gre	Por	Fin	Fin	Fin	Ger
	Aus	Bel	Aus	Net	Fra	Spa	Gre	Bel	Fin
	Spa	Ire	Gre	Fin	Spa	Fra	Ita	Ita	Gre
	Ire	Por	Ire	Fra	Net	Net	Spa	Spa	Ita
2010-2012	Ita	Ita	Bel	Aus	Bel	Aus	Ire	Ire	Ire
	Ger	Ger	Ita	Bel	Aus	Bel	Por	Por	Por
	Por	Fra	Fin	Fin	Fin	Por	Fin	Fin	Por
	Spa	Spa	Ger	Por	Aus	Ire	Aus	Aus	Spa
	Ire	Aus	Net	Spa	Spa	Gre	Fra	Spa	Ita
	Fra	Ger	Aus	Aus	Fra	Fin	Spa	Fra	Net
	Gre	Net	Bel	Net	Net	Spa	Net	Net	Gre
	Aus	Bel	Fra	Ire	Ita	Net	Ita	Ita	Fin
	Ita	Por	Spa	Gre	Por	Bel	Por	Por	Aus
	Bel	Ita	Por	Ita	Ire	Aus	Ire	Ire	Fra
2013-2015	Ger	Gre	Ita	Fra	Gre	Fra	Gre	Gre	Ire
	Net	Fin	Gre	Bel	Ger	Ita	Bel	Ger	Ger
	Fin	Ire	Ire	Ger	Bel	Ger	Ger	Bel	Bel
	Bel	Bel	Por	Ger	Fin	Fin	Ger	Ger	Ger
	Ger	Ger	Net	Fin	Ger	Aus	Spa	Spa	Aus
	Spa	Ita	Fin	Aus	Aus	Net	Bel	Bel	Spa
	Ita	Fra	Bel	Spa	Spa	Spa	Por	Por	Gre
	Ire	Spa	Ita	Por	Por	Ita	Ire	Ire	Por
	Net	Ire	Ire	Net	Net	Bel	Gre	Gre	Ita
	Fra	Por	Ger	Fra	Fra	Ire	Fin	Fin	Fin

Table 3: Rankings obtained with $CoRisk_{in}$, degree of connectivity and eigenvector centrality measures, ordered from the highest to the lowest

Table 3 shows three kinds of ranking, one for each measure of connectivity, for the three economic sectors and the four time periods: in each list, countries are listed in descendent order. In order to compare the rankings obtained with the $CoRisk_{in}$ measure to the other two, a Spearman

non-parametric correlation test has been applied: the results are shown in Table 4.

Period	Sovereign		Corporate		Bank	
	<i>DC</i>	<i>Eigen.</i>	<i>DC</i>	<i>Eigen.</i>	<i>DC</i>	<i>Eigen.</i>
2003-2006	0.436	0.136	0.936	0.373	0.764	0.245
2007-2009	0.582	0.064	0.809	0.811	0.936	0.518
2010-2012	0.136	-0.736	0.691	0.655	0.982	0.345
2013-2015	0.736	0.018	0.927	0.245	0.982	0.573

Table 4: Correlation coefficients between $CoRisk_{in}$ rankings and rankings based on, respectively, degree centrality (DC) and eigenvector centrality ($Eigen.$), for the three economic sectors and the four time-windows

Table 4, which shows correlation coefficients between the rankings obtained with the $CoRisk_{in}$ measure and those obtained with the degree of centrality and the eigenvector centrality, reveals that, overall, the $CoRisk_{in}$ ordering is quite similar to the one obtained with the degree of centrality: the difference between the two lies in the fact that the former weights each link in the graph considering not only partial correlations, but also the default probability of neighbours. On the other hand, eigenvector centrality does not take into account weights deriving from the default probability of neighbours and, in addition, it considers the importance of each node in the graph by looking at its relations with other central nodes, so that a node becomes much more important if it is connected to important ones. This mechanism, performed without considering the impact of each node on the basis of its default probability, amplifies the "error", or the distance between $CoRisk_{in}$ and the eigenvector centrality measure. This effect is particularly evident during the sovereign-crisis period in the sovereign sector.

As we remarked in the methodological section, $CoRisk_{out}$ measures how each node in the graph affects its neighbours and, thus, provides an estimation of the systemically importance of each economic sector in each country. The obtained results, compared with $CoRisk_{in}$ ones, are shown in Figure (10).

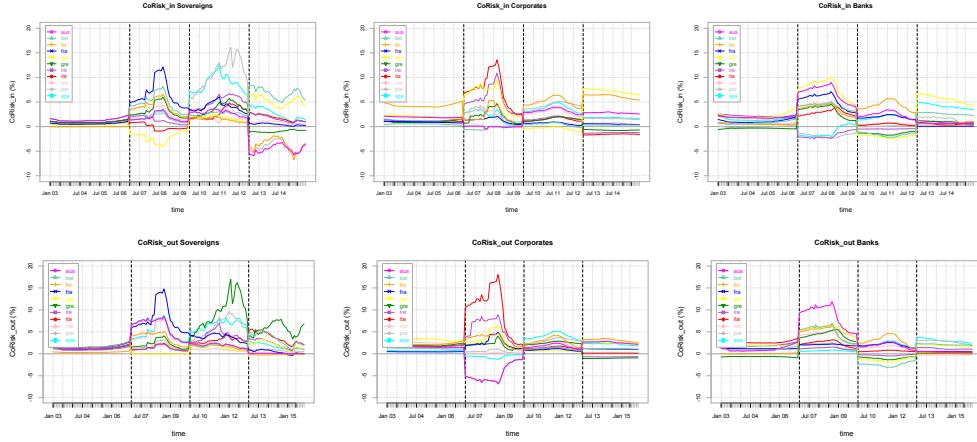


Figure 10: Comparison between $CoRisk_{in}$ (top) and $CoRisk_{out}$ (bottom), from 2003 until 2015 and for the sovereign (left), corporate (middle) and bank (right) sectors

By comparing the $CoRisk_{in}$ with the $CoRisk_{out}$ contributions for the sovereign sector, different conclusions can be deduced across countries and time. First, during the pre-crisis and the financial crisis periods, the two measures look very similar, while important differences start emerging during the sovereign crisis period, in which it appears clear that Greece is more an exporter rather than an importer of risk, while the situation is reversed for Portugal and Spain. In most recent years, all peripheral countries have the highest, even if decreasing, $CoRisk_{out}$ contributions, since their institution-specific PD is significantly higher than that of core economies. It is interesting to observe that there are not negative $CoRisk_{out}$ measures for the sovereign sector, meaning that all european countries overall contribute to increase the default probability of their neighbours.

The incoming and outgoing contributions for the corporate sectors emphasize, once again, the difference between core and peripheral countries, with the latter characterized by higher $CoRisk_{out}$ and the former by higher $CoRisk_{in}$. Moreover, as in the previous case, one can notice that the two

CoRisk contributions are very similar during the first two time-periods, while they start diverging afterwards. Same results can be observed for the bank sector. Overall, peripheral (core) countries are more exporter (importer) rather than importer (exporter) of systemic risk; moreover, during the first two periods $CoRisk_{in}$ and $CoRisk_{out}$ are very similar for almost the entire sample, meaning that in those years default probabilities were much more homogenous across european countries than afterwards. This result can be once more explained by the emerging of clustering effects starting from the third period.

4 Conclusions

In this work we have proposed a new systemic risk measurement model, based on multivariate stochastic processes, correlation networks and default probabilities. The model has been applied to the economies of the European monetary union. For each country we have considered three spread measures (sovereign spread, corporate spread, bank spread), and we have modelled each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. We have introduced a correlation network model between all countries, within each sector and across them, thus deriving a statistical representation of the transmission mechanism of systemic risk that correctly takes into account interdependence effects. We have then derived the probability of default for each country and sector, both unconditionally and conditionally on the network structure: the comparison between them allows the definition of a novel risk indicator, the *CoRisk*, that explicitly measures the contagion effect on the probability of default, including both systematic and systemic components.

From an applied viewpoint, our proposed methodology seems quite effective and efficient, particularly when compared to alternative network based measures, such as the weighted degree and the eigenvector centrality. The main findings deriving from the application of our methodology to the Euro area can be summarized as follows.

Overall, the contribution of the bank and sovereign sectors to systemic risk increased in all countries, respectively during the financial and the sovereign crisis. Sovereign risk is larger in peripheral countries than in core ones: while the main component of sovereign risk is due to contagion effects (*CoRisk*) in core economies, the institution-specific *PD* component is much higher in peripheral countries. In addition, peripheral economies show high sovereign risk, because of loop effects with each other deriving from clustering.

Corporate risk appears to be the most important source of risk in "normal" times: before the financial crisis and in the last, post-crisis period. It is mostly determined by contagion effects in core economies and institution-specific *PD* components in peripheral countries.

Bank risk for core economies suffered a substantial improvement of contagion effects during the sovereign crisis, through their exposition to peripheral banks. On the other hand, peripheral economies witnessed an increase in their institution-specific bank default probabilities and the creation of some loop effects.

The distribution of risk in its components looks quite homogenous before the financial crisis, while after the crisis the situation has not come back to normality, because of persisting contagion effects in core economies, and high institution-specific default probabilities, worsened by clustering effects, in peripheral ones.

To conclude, within the Euro area the sovereign crisis has had a larger impact on systemic risk with respect to the financial crisis. A possible explanation consists in different ways peripheral and core economies reacted to the financial crisis: peripheral countries, with high public debts, had little fiscal space and, therefore, the financial crisis triggered their imbalances to emerge in the subsequent sovereign crisis.

Finally, by comparing in and out contagion effects, peripheral (core) countries appear to be more exporter (importer) rather than importer (exporter) of systemic risk.

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6 Appendix

	a	v	b
2003-2006	0.014	0.001	0.056
2007-2009	0.899	0.355	0.569
2010-2012	0.405	0.262	0.149
2013-2015	0.008	0.002	0.090

Table 5: Estimated parameters of the systematic process S_t (3-months Euribor), for the pre-crisis, financial-crisis, sovereign-crisis and post-crisis periods

Period	Country	$y_{t,1}$ (Sov)			$y_{t,2}$ (Corp)			$y_{t,3}$ (Bank)		
		$(\theta_1)_1$	$(\theta_2)_1$	$(\theta_3)_1$	$(\theta_1)_2$	$(\theta_2)_2$	$(\theta_3)_2$	$(\theta_1)_3$	$(\theta_2)_3$	$(\theta_3)_3$
2003-2006	Aus	0.319	0.085	0.073	0.365	0.092	0.029	0.101	0.035	0.009
	Bel	0.351	0.093	0.075	0.481	0.108	0.034	0.143	0.039	0.015
	Fin	0.348	0.093	0.079	0.053	0.014	0.032	0.237	0.093	0.018
	Fra	0.388	0.103	0.077	0.828	0.191	0.041	0.998	0.285	0.033
	Ger	0.390	0.105	0.079	0.305	0.063	0.012	0.173	0.065	0.021
	Gre	0.433	0.109	0.076	0.006	0.001	0.028	0.012	0.001	0.035
	Ire	0.333	0.090	0.077	0.190	0.041	0.036	0.011	0.001	0.042
	Ita	0.392	0.099	0.074	0.252	0.057	0.030	0.075	0.041	0.039
	Net	0.379	0.101	0.080	0.366	0.079	0.018	0.500	0.075	0.032
	Por	0.374	0.097	0.076	0.004	0.001	0.035	0.728	0.248	0.034
	Spa	0.361	0.096	0.076	0.056	0.015	0.032	0.033	0.015	0.030
2007-2009	Aus	0.448	0.110	0.083	1.500	0.342	0.001	0.069	0.021	0.027
	Bel	0.333	0.082	0.080	1.497	0.324	0.001	0.070	0.019	0.025
	Fin	0.235	0.061	0.081	1.487	0.356	0.001	1.507	0.514	0.001
	Fra	0.238	0.062	0.081	1.500	0.216	0.001	0.522	0.022	0.515
	Ger	0.145	0.045	0.093	1.243	0.319	0.001	0.401	0.012	0.523
	Gre	0.760	0.150	0.108	1.518	0.320	0.002	1.096	0.022	1.480
	Ire	0.755	0.157	0.109	1.494	0.318	0.001	0.023	0.009	0.906
	Ita	0.635	0.143	0.074	1.507	0.347	0.001	0.001	0.011	0.568
	Net	0.231	0.061	0.080	1.483	0.351	0.001	0.001	0.006	0.620
	Por	0.796	0.183	0.091	1.507	0.288	0.001	1.497	0.099	0.001
	Spa	0.708	0.169	0.083	1.521	0.311	0.001	0.033	0.005	0.756
2010-2012	Aus	1.507	0.529	0.001	0.592	0.126	0.060	1.487	0.670	0.001
	Bel	1.492	0.435	0.001	0.349	0.106	0.034	0.356	0.118	0.012
	Fin	0.030	0.032	0.112	0.232	0.106	0.047	0.099	0.047	0.025
	Fra	0.073	0.039	0.115	0.691	0.073	0.345	1.271	0.406	0.037
	Ger	0.040	0.042	0.121	0.150	0.045	0.023	0.001	0.003	0.021
	Gre	1.835	0.102	0.593	0.362	0.058	0.046	0.407	0.148	0.094
	Ire	0.407	0.057	0.262	0.067	0.021	0.037	0.157	0.069	0.108
	Ita	0.489	0.095	0.158	0.093	0.023	0.030	0.035	0.001	0.043
	Net	1.477	0.597	0.001	0.001	0.035	0.027	0.212	0.054	0.021
	Por	0.624	0.061	0.234	0.156	0.029	0.037	0.027	0.001	0.027
	Spa	0.710	0.129	0.162	0.115	0.031	0.028	0.038	0.008	0.019
2013-2015	Aus	0.050	0.057	0.168	0.001	0.034	0.015	0.001	0.116	0.007
	Bel	0.038	0.047	0.157	0.001	0.058	0.021	0.001	0.078	0.014
	Fin	0.057	0.061	0.170	0.001	0.040	0.012	0.019	0.029	0.021
	Fra	0.047	0.053	0.154	0.057	0.028	0.036	0.250	0.091	0.035
	Ger	0.047	0.069	0.191	1.483	0.497	0.001	0.985	0.274	0.001
	Gre	1.023	0.124	0.286	0.020	0.009	0.024	1.493	0.653	0.001
	Ire	0.038	0.052	0.159	0.484	0.155	0.018	0.057	0.042	0.030
	Ita	0.014	0.030	0.134	0.001	0.002	0.022	0.104	0.048	0.034
	Net	0.039	0.049	0.165	0.001	0.046	0.029	1.486	0.430	0.001
	Por	0.104	0.051	0.188	0.001	0.070	0.015	0.001	0.107	0.026
	Spa	0.066	0.054	0.135	1.511	0.493	0.001	0.001	0.142	0.014

Table 6: Estimated parameters of the idiosyncratic processes $y_{t,1}^m$ (interest rates on 10-years maturity government bonds), $y_{t,2}^m$ (interest rates on loans to non-financial corporates) and $y_{t,3}^m$ (interest rates on deposits) for the pre-crisis, financial-crisis, sovereign-crisis and post-crisis periods

Period	Country	$Z_{t,1}$ (Sov)		$Z_{t,2}$ (Corp)		$Z_{t,3}$ (Bank)	
		α_1	β_1	α_1	β_1	α_1	β_1
2003-2006	Aus	-0.095	-0.772	0.572	0.267	-0.435	-0.950
	Bel	-0.089	-0.775	0.629	0.298	-0.113	-0.916
	Fin	-0.087	-0.749	0.230	-0.172	-0.677	-0.869
	Fra	-0.103	-0.782	0.401	-0.091	-0.346	-0.981
	Ger	-0.115	-0.775	0.720	0.415	-0.712	-0.939
	Gre	-0.040	-0.759	0.346	-0.550	-1.031	-0.958
	Ire	-0.096	-0.755	0.611	0.240	-0.950	-0.895
	Ita	-0.043	-0.758	0.486	0.025	-1.318	-0.812
	Net	-0.102	-0.772	0.577	0.170	-0.451	-0.888
	Por	-0.070	-0.754	0.201	-0.520	-0.442	-0.830
	Spa	-0.101	-0.774	0.268	-0.109	-1.015	-0.871
2007-2009	Aus	0.449	-0.990	0.298	-0.926	0.818	-0.994
	Bel	0.357	-0.960	0.298	-0.926	0.521	-0.996
	Fin	0.481	-0.991	0.318	-0.889	1.046	-0.988
	Fra	0.497	-0.990	0.286	-0.982	0.686	-0.995
	Ger	0.753	-0.980	0.226	-0.991	0.859	-0.978
	Gre	0.253	-0.980	0.050	-0.968	0.470	-0.889
	Ire	0.295	-0.986	0.052	-0.832	0.621	-0.963
	Ita	0.353	-0.990	0.103	-0.880	1.167	-0.947
	Net	0.496	-0.991	0.276	-0.984	0.511	-0.982
	Por	0.385	-0.988	-0.055	-0.871	0.991	-0.992
	Spa	0.443	-0.992	0.207	-0.926	0.646	-0.936
2010-2012	Aus	0.637	-0.167	0.373	-0.929	0.207	-0.969
	Bel	0.854	0.438	0.473	-0.971	0.438	-0.932
	Fin	0.584	-0.185	0.271	-0.930	0.150	-0.964
	Fra	0.593	-0.318	0.452	-0.966	0.418	-0.983
	Ger	0.498	-0.238	0.527	-0.963	0.287	-0.977
	Gre	0.938	-0.054	0.766	-0.432	0.309	-0.860
	Ire	0.968	0.753	0.501	-0.793	0.149	-0.785
	Ita	0.781	-0.178	0.542	-0.735	0.385	-0.851
	Net	0.567	-0.235	0.463	-0.994	0.526	-0.950
	Por	0.926	0.298	0.697	-0.399	0.357	-0.746
	Spa	0.738	-0.468	0.528	-0.802	0.321	-0.825
2013-2015	Aus	0.974	0.731	0.889	0.781	0.948	0.399
	Bel	0.982	0.788	0.984	0.694	0.972	0.478
	Fin	0.976	0.765	0.883	0.507	0.959	0.555
	Fra	0.830	0.769	0.968	0.394	0.935	-0.279
	Ger	0.969	0.754	0.981	0.597	0.952	0.325
	Gre	0.980	-0.315	0.996	0.832	0.991	0.859
	Ire	0.987	0.777	0.910	-0.932	0.942	0.238
	Ita	0.989	0.761	0.992	0.798	0.964	0.285
	Net	0.011	0.464	0.981	0.624	0.966	0.163
	Por	0.830	0.769	0.993	0.815	0.995	0.909
	Spa	0.985	0.681	0.993	0.857	0.974	0.596

Table 7: Weight coefficients of the two components of the general processes $Z_{t,1}^m$ (sovereign spread), $Z_{t,2}^m$ (corporate spread) and $Z_{t,3}^m$ (bank spread), for the pre-crisis, financial-crisis, sovereign-crisis and post-crisis periods

Period	Country	Sovereign		Corporate		Bank	
		DC	Eigen.	DC	Eigen.	DC	Eigen.
2003-2006	Aus	0.701	0.227	0.931	0.277	1.448	0.990
	Bel	0.971	0.158	0.161	0.000	0.995	0.356
	Fin	0.015	0.000	2.026	1.000	0.186	0.121
	Fra	0.916	0.196	0.227	0.000	0.761	0.000
	Ger	0.915	0.211	1.100	0.000	1.236	1.000
	Gre	1.161	1.000	0.597	0.520	-0.308	0.781
	Ire	0.877	0.231	0.791	0.378	0.707	0.000
	Ita	1.061	0.954	0.516	0.325	1.324	0.000
	Net	1.219	0.125	1.149	0.226	0.273	0.000
	Por	0.979	0.578	0.717	0.710	0.240	0.000
	Spa	0.873	0.340	0.573	0.694	0.739	0.000
2007-2009	Aus	1.064	0.210	-0.106	0.099	1.610	0.894
	Bel	0.721	0.010	-0.044	0.050	0.821	0.806
	Fin	0.916	0.307	0.712	0.298	0.962	0.244
	Fra	2.402	0.973	0.435	0.135	1.185	0.548
	Ger	-0.185	1.000	1.439	0.710	1.921	0.466
	Gre	1.019	0.189	0.691	0.794	1.038	0.000
	Ire	0.512	0.182	1.512	0.758	0.021	0.000
	Ita	0.156	0.000	2.383	0.938	0.784	0.000
	Net	0.887	0.585	0.242	0.118	1.093	0.927
	Por	0.451	0.277	0.639	1.000	-0.012	0.000
	Spa	1.309	0.423	0.276	0.210	0.154	1.000
2010-2012	Aus	1.170	0.388	1.379	0.000	1.379	0.000
	Bel	0.872	0.220	-1.070	0.174	-1.070	0.000
	Fin	0.651	1.000	2.592	0.698	2.592	0.000
	Fra	1.663	0.000	0.911	0.000	0.911	0.000
	Ger	1.009	0.862	-0.794	0.000	-0.794	0.000
	Gre	0.676	0.000	-0.632	0.836	-0.632	0.217
	Ire	0.514	0.000	-0.100	0.880	-0.100	0.000
	Ita	0.765	0.000	0.174	0.000	0.174	0.356
	Net	0.940	0.612	0.357	0.301	0.357	0.332
	Por	0.813	0.000	0.047	1.000	0.047	1.000
	Spa	1.614	0.000	1.204	0.516	1.204	0.482
2013-2015	Aus	0.069	0.000	1.381	0.650	0.125	0.728
	Bel	2.242	0.565	0.025	0.192	1.023	0.150
	Fin	-0.147	0.619	2.011	1.000	0.316	0.268
	Fra	0.784	0.000	0.213	0.000	-0.012	0.059
	Ger	1.883	0.000	1.872	0.000	2.773	1.000
	Gre	-0.705	0.000	-0.192	0.019	0.391	0.676
	Ire	0.571	0.068	-0.183	0.126	0.513	0.094
	Ita	1.286	0.312	0.036	0.216	0.189	0.353
	Net	-0.200	0.769	0.383	0.616	-0.232	0.002
	Por	0.540	1.000	0.479	0.000	0.878	0.635
	Spa	0.584	0.000	0.707	0.348	1.805	0.719

Table 8: Degree of connectivity (DC) and eigenvector centrality (Eigen.) measures for the three economic sectors and for the pre-crisis, financial-crisis, sovereign-crisis and post-crisis periods