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Modeling Systemic Risk with Correlated Stochastic Processes

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Modeling Systemic Risk with Correlated Stochastic Processes*

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Abstract

In this work we propose a novel systemic risk model, based on stochastic processes and correlation networks.

For each country we consider three different spread measures, one for each sector of the economy (sovereign, corporates, banks), and we model each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor.

We provide an estimation model for the parameters of the processes and, for each country, we derive the aggregate default probabilities of each sector. Systemic risk is then estimated by means of a network model based on the partial correlations between the estimated processes of all sectors and countries.

Our model is applied to understand the time evolution of systemic risk in the economies of the European monetary union, in the recent period. The results show that systemic risk has increased during the crisis years and that, after the crisis, a clear separation between core and peripheral economies has emerged, for all sectors of the economy.

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1 Introduction

1.1 Economical background

The last few years have witnessed an increasing research literature on systemic risk (for a definition see, for example, Allen and Gale, 2000; Acharya, 2009), with the aim of identifying the most contagious institutions and their transmission channels, and of studying the impact of monetary policies on default probabilities, especially during crisis periods (see, for example, Chong et al., 2006; Longstaff, 2010; Shleifer and Vishny, 2010).

Specific measures of systemic risk have been proposed for the banking sector; in particular, by Acharya et al. (2010), Adrian and Brunnermeier (2011), Brownlees and Engle (2012), Acharya et al. (2012), Dumitrescu and Banulescu (2014) and Hautsch et al. (2015) who, on the basis of market share prices, calculate the quantiles of the estimated loss probability distribution of a bank, conditional on the occurrence of an extreme event in the financial market. A similar approach has been applied to sovereign systemic risk, using bond interest rates, by Popescu and Turcu (2014).

The above approach is useful to establish policy thresholds aimed, in particular, at identifying the most systemic institutions. However, it is a bivariate approach, which allows to calculate the risk of an institution conditional on another or on a reference market but, on the other hand, it does not address the issue of how risks are transmitted between different institutions, in a multivariate framework.

Trying to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, that combine the rich structure of financial networks (see, e.g., Lorenz et al., 2009; Battiston et al., 2012) with a parsimonious approach based on the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who propose measures of connectedness based on Granger-causality tests and variance decompositions. Barigozzi and Brownlees (2013) and Ahelegbey et al. (2015) extend the approach introducing stochastic graphical models.

Correlation network models are very useful to identify the most important channels of contagion in a cross-sectional perspective. However, similarly to bivariate measures, they can not be used as predictive models in a time-varying context. This is the main focus of econometric causal methods, as the ones proposed by Duffie et al. (2000), Lando and Nielsen (2010), Koopman et al. (2012) and Betz et al. (2014).

Both correlation networks and econometric models explain whether the default probability of a bank, a country, or of a company, depends on that of the others, or on a set of exogenous systematic risk factors. A different stream of research developed, among others, by Bartram et al. (2007), Ang and Longstaff (2012), Battiston et al. (2012) and Brownless et al. (2014), models systemic risk in terms of univariate stochastic processes, that may also depend on systematic factors which are, however, endogenously determined. A further advantage of stochastic processes is that they are non-linear and time-dependent, and, therefore, can produce powerful early warning indicators.

Our aim is to propose multivariate stochastic processes, whose interrelationships can be investigated by means of correlation network models: doing so, we combine the advantages of econometric models (predictive capability) with those of correlation networks (identification of channels of contagion) and of stochastic process models (endogeneity and non-linearity). To achieve this aim, we significantly extend the approach of Ang and Longstaff (2012) and Brownless et al. (2014) by employing a multiple set of linear combinations of two stochastic processes (a systematic and an idiosyncratic one), rather than a single process.

In more detail, we consider three spread measures based on publicly available data: (a) the spread between the cost of debt for countries (interest rates on 10-years maturity government bonds) and a benchmark rate, which gives a measure of sovereign risk; (b) the spread between the cost of debt for corporates (interest rates on bank lendings) and a benchmark rate, which gives a measure of corporate risk; (c) the spread between the funding cost of the banking system (interest rates on deposits of non-financial corporates and households) and a benchmark rate, which gives a measure of bank risk.

We define three stochastic process on the three spread measures, and calculate sector-specific default probabilities, taking into account correlations between countries. We then introduce a correlation structure between the three sectorial spreads as suggested, although in a different modelling framework, by Gray et al. (2013), Ramsay and Sarlin (2015) and Schwaab et al. (2015). We are thus able to estimate a correlation network model, based on the partial correlations between each pair of sectorial spreads, within each country and across different ones.

1.2 Statistical background

Cox, Ingersoll and Ross (CIR, 1985) have introduced a class of stochastic differential equations (SDE), particularly useful to study the time-evolution of financial quantities.

In particular, the dynamics of an interest rate y_t can be evaluated starting from a general family of non-parametric, time-homogeneous and continuous SDE model, specified by the following (see, e.g., Chan et al., 1992):

$$\mathrm{d}\, y_t = (\theta_1 - \theta_2 y_{t-1})\,\mathrm{d}\, t + \theta_3 (y_{t-1})^{\beta}\,\mathrm{d}\, W_t, \tag{1.1}$$

where $\beta = 0.5$ corresponds to the CIR process, while $\beta = 0$ represents the Vasicek model (Vasicek, 1977), and $\mathrm{d}\, W_t$ is a standard geometric Brownian motion.

The previous process and, in particular, the CIR, can be applied to model the joint dynamic of the interest rates of a group of N countries, $Y_t = (y_t^1, ..., y_t^N)$, as follows:

$$\mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i)\,\mathrm{d}\, t + \theta_3^i \sqrt{y_{t-1}^i}\,\mathrm{d}\, W_t, \qquad \text{i = 1, ..., N} \tag{1.2}$$

where each parameter vector $\Theta^i = (\theta_1^i, \theta_2^i, \theta_3^i)$ is country-specific. Note that, in the previous formulation, $(\theta_1^i - \theta_2^i y_{t-1}^i)$ is the drift term, in which $\frac{\theta_1^i}{\theta_2^i}$ represents the mean long term level of the interest rate y^i, θ_2^i is the adjustment speed, and θ_3^i is the volatility.

The structure of (1.2) can be enriched by introducing correlation coefficients between the N stochastic processes, leading to a multivariate CIR, as in Kalogeropoulos et al. (2011):

$$\begin{cases} \mathrm{Corr}(\mathrm{d}\, y_t^i, \mathrm{d}\, y_t^j) = \rho^{ij}, & \text{for } i \neq j \\ \mathrm{Corr}(\mathrm{d}\, y_t^i, \mathrm{d}\, y_t^j) = 1 & \text{for } i = j. \end{cases} \tag{1.3}$$

The variance of each CIR process can be calculated, and the result can be shown to be:

$$Var\left[y_t^i | y_0^i\right] = y_0^i \left(\frac{\theta_3^i}{\theta_2^i}\right)^2 \left(e^{-\theta_2^i t} - e^{-2\theta_2^i t}\right) + \frac{\theta_1^i}{2} \left(\frac{\theta_3^i}{\theta_2^i}\right)^2.$$

The limit of the above variance can be calculated for an adjustment speed that tends to zero:

$$\lim_{\theta_2^i \to 0} Var\left[y_t^i | y_0^i\right] = y_0^i(\theta_3^i)^2 t. \tag{1.4}$$

Then, using the correlation coefficients defined in (1.3), the instantaneous covariance matrix can be derived as:

$$A = \begin{bmatrix} y_0^1(\theta_3^1)^2 & \rho^{12}\sqrt{y_0^1 y_0^2}\theta_3^1\theta_3^2 & \dots & \rho^{1N}\sqrt{y_0^1 y_0^N}\theta_3^1\theta_3^N \\ \rho^{21}\sqrt{y_0^2 y_0^1}\theta_3^2\theta_3^1 & y_0^2(\theta_3^2)^2 & \dots & \rho^{2N}\sqrt{y_0^2 y_0^N}\theta_3^2\theta_3^N \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1}\sqrt{y_0^N y_0^1}\theta_3^N\theta_3^1 & \rho^{N2}\sqrt{y_0^N y_0^2}\theta_3^N\theta_3^2 & \dots & y_0^N(\theta_3^N)^2 \end{bmatrix}$$

Our aim is to broaden Kalogeropoulos et al. (2011), extending their multivariate CIR in a more general process able: (a) to capture both the systematic and the idiosyncratic components that may affect interest rate spread dynamics, using linear combinations of stochastic processes; (b) to model the correlation structure of interest rate spreads across different countries and sectors, by means of graphical network models.

Our proposed model will be applied and compared to data that concern three time windows: the pre-crisis period (2003-2006), the crisis period (2007-2009) and the post-crisis period (2010-2014), for the countries belonging to the Eurozone.

The paper is structured as follows: Section 2 describes the proposed models, with Section 2.1 introducing multivariate linar combinations of interest rate spread models, within each economic sector and Section 2.2 extending the methodology to correlated sectorial spreads. Section 3 describes the application of the proposed models, with Section 3.1 presenting data and descriptive statistics, Section 3.2 presenting the empirical evidence obtained from sectorial spread models, and Section 3.3 presenting the additional evidence obtained from correlated sectorial models. Finally, Section 4 concludes with some final remarks.

2 Proposal

2.1 Sectorial Spreads

For each country we measure the aggregate financial liabilities of three economic sectors: sovereign, (non financial) corporates and banks.

For each given sector, and independently from the others, we assume that the dynamics of the liabilities of each country, expressed by the evolution of the associated interest rate, can be described by a linear combination of two stochastic processes: one can be considered as the common systematic process; the second can be considered as an idiosyncratic process. More formally, for each country $i = 1, \ldots, N$:

$$Z_t^i = \alpha^i y_t^i - \beta^i S_t, \tag{2.1}$$

where S_t stands for the systematic process, while y_t^i represents the idiosyncratic process referred to country i; the parameter β^i measures the weight of the systematic process, while α^i measures the weight of the idiosyncratic process: both on the general, complete process Z_t^i, that describes the resulting time-evolution of the interest spread.

We remark that the previous equation assumes that the systematic process is the same for all countries, but it differently influences each country-specific process Z_t^i, through the weight β^i. From an economic viewpoint, the above formulation expresses Z_t^i as the difference between the cost of a long term debt and the cost of liquidity.

Both the systematic and the idiosyncratic processes can be formulated as stochastic differential equations, through the CIR specification:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i)\,\mathrm{d}\, t + \theta_3^i \sqrt{y_{t-1}^i}\,\mathrm{d}\, W_t, \end{cases} \tag{2.2}$$

where $\mathrm{d}\, B_t$ and $\mathrm{d}\, W_t$ are two independent Brownian motions.

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[dy_t^i, dy_t^j] = \rho^{ij}, \\ \mathrm{Corr}[dS_t, dy_t^j] = \gamma^j. \end{cases} \tag{2.3}$$

Note that the first equation is consistent with the assumptions used in the formulation of multidimensional CIR processes (see e.g. Kalogeropoulos et al., 2011); the second one introduces a correlation between each idiosyncratic process and the systematic process S_t.

We now show the resulting expression of the instantaneous covariance matrix, for our multivariate linear combination of CIR process.

First define:

$$P = \begin{bmatrix} 1 & \rho^{12} & \dots & \rho^{1N} \\ \rho^{21} & 1 & \dots & \rho^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{N1} & \rho^{N2} & \dots & 1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma^1 \\ \vdots \\ \gamma^i \\ \vdots \\ \gamma^N \end{bmatrix}, \tag{2.4}$$

where each element in P is the correlation coefficient between the idiosyncratic processes of any two countries, while each element of Γ is the correlation coefficient between any idiosyncratic process and the systematic process, as defined in (2.3).

Following steps similar to those presented for the multivariate CIR model of the last Section, the instantaneous covariance matrix A can then be shown to be:

$$A = \Phi \cdot \Theta^T, \tag{2.5}$$

where

$$[\Phi]^i = \left[\beta^i b\sqrt{S_0}, \quad \beta^i, \quad \alpha^i \sqrt{S_0 y_0^i} b\theta_3^i [\Gamma]^i, \quad \alpha^i \sqrt{y_0^i}\theta_3^i \sqrt{[P]^i}\right],$$

$$[\Theta^T]^j = \begin{bmatrix} \beta^j b\sqrt{S_0} \\ \\ \alpha^j \sqrt{S_0 y_0^j} b\theta_3^j [\Gamma]^j \\ \\ \beta^j \\ \\ \alpha^j \sqrt{y_0^j}\theta_3^j \sqrt{[P]^j} \end{bmatrix}.$$

The parameters of the proposed process can be estimated extending results available for univariate stochastic processes (see e.g. Iacus, 2008) as follows.

Define the following variables:

$$c = \frac{2\theta_2}{\theta_3^2(1 - e^{-\theta_2 t})}, \quad u = cBR_t e^{-\theta_2 t}, \quad q = \frac{2\theta_1}{\theta_3^2} - 1, \quad v = cBR_{t+1}.$$

The log-likelihood function of the process can be shown to be:

$$\ln L(\Theta) = (N-1)\ln c + \sum_{j=1}^{N-1}\left[-u_{t_j} - v_{t_j} + \frac{q}{2} ln\left(\frac{v_{t_j}}{u_{t_j}}\right) + \ln[I_q(2\sqrt{u_{t_j} v_{t_j}})]\right], \tag{2.6}$$

where $I_q(2\sqrt{uv})$ is the modified Bessel function of order q. The estimated parameter vector $\hat{\Theta}$ is thus found by maximizing the log-likelihood function:

$$\hat{\Theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) = \arg\max_{\Theta} \ln L(\Theta). \tag{2.7}$$

To estimate the weights of the systematic (α^i) and the idiosyncratic (β^i) processes, we consider a method of moments estimation procedure. Let y^i and s be the observed time vectors of (country specific) interest rates and of systematic rates. Let then $d^i = y^i - S$ be the observed vector of spreads. The weights can then be estimated as:

$$\begin{cases} \beta^i = \mathrm{Corr}\big(d^i, y^i\big), \\ \\ \alpha^i = \mathrm{Corr}\big(d^i, s\big), \end{cases} \tag{2.8}$$

From an economic viewpoint, it is important to understand the meaning of the weight coefficients α^i and β^i: if they are both positive, it means that both the correlation between d^i and y^i and that between d^i and s are positive, which means that the idiosyncratic process part of d^i increases faster than the systematic component part: $|\frac{\partial y^i}{\partial t}| > \frac{\partial s}{\partial t}$. If the two coefficients are both negative, the systematic component changes instead faster than the idiosyncratic one: $|\frac{\partial s}{\partial t}| > \frac{\partial y^i}{\partial t}$.

Once the parameters are estimated, the proposed multivariate stochastic process model can be used to derive the aggregate sectorial default probabilities of each country, as follows.

For each country i, the probability of default, PD_t^i, can be obtained considering the expected dynamic of the debt:

$$D_{t+1}^i = (1 - PD_t^i)e^{\alpha^i y_t^i} D_t^i, \tag{2.9}$$

where D_{t+1}^i (D_t^i) is the total debt at time $t+1$ (t).

Note that the analogous dynamic of risk-free debt is the following:

$$D_{t+1}^i = e^{\beta^i S_t} D_t^i. \tag{2.10}$$

Assuming to be in an arbitrage-free context, we can equate (2.9) with (2.10) and obtain PD^i:

$$PD_t^i = 1 - e^{-Z_t^i}. \tag{2.11}$$

From the above equation, it is clear that if Z_t^i decreases, the probability of default decreases, consistently with the definition of the process Z_t^i as a spread between country specific interest rates and benchmark risk free rates.

2.2 Correlated Sectorial Spreads

The linear combination of stochastic processes proposed in (2.1), can be extended to allow for dependence between sovereign, corporate and bank spreads. More formally, suppose that all these measures, indicated respectively with the indexes $\{1, 2, 3\}$, are determined by the difference between an idiosyncratic and a common, systematic component, for each country $i = 1, \ldots, N$:

$$\begin{cases} Z_{t,1}^i = \alpha_1^i y_{t,1}^i - \beta_1^i S_t, \\ Z_{t,2}^i = \alpha_2^i y_{t,2}^j - \beta_2^i S_t, \\ Z_{t,3}^i = \alpha_3^i y_{t,3}^k - \beta_3^i S_t, \end{cases} \tag{2.12}$$

In (2.12) the systematic component follows a univariate CIR process, while all idiosyncratic processes are modeled as a multivariate CIR:

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_{t,\{1,2,3\}}^i = [(\theta_1)_{\{1,2,3\}}^i - (\theta_2)_{\{1,2,3\}}^i y_{t-1,\{1,2,3\}}^i]\,\mathrm{d}\, t + (\theta_3)_{\{1,2,3\}}^i \sqrt{y_{t-1,\{1,2,3\}}^i}\,\mathrm{d}\, W_t. \end{cases}$$

We then assume the following correlation structure:

$$\begin{cases} \mathrm{Corr}[y_t^m; y_t^n] = \rho^{mn}, \\ \mathrm{Corr}[y_t^m; S_t] = \gamma^m, \end{cases} \tag{2.13}$$

where $\{m, n\} \in (V \times W)$, with $V = \{1, ..., N\}$ for the countries, and $W = \{1, 2, 3\}$ for the sectors of the economy.

From the above assumptions, the instantaneous covariance matrix of the new process turns out to be the same as that in (2.5), albeit with a different dimensionality, being a $3N \times 3N$ rather than a $N \times N$

matrix. Consequently, parameter estimates and default probabiltiies can be derived similarly as in Section 2.1.

The estimated covariance matrix $\hat{A}$ can be employed to build a correlation network model between countries and economic sectors. However, such covariances can be misleading because they take into account only bivariate (marginal) relationships between interest spreads. We can obtain conditional covariances, that can adjust bivariate relationship by the presence of other variables. Conditional covariances can then be normalised to obtain conditional (partial) correlations.

Formally, let

$$\hat{A}^{-1}$$

be the inverse of the covariance matrix, with elements a^{mn}.The partial correlation between variables Z_m and Z_n, conditional on the remaining variables in VXW, ρ_{mnVW} can be obtained as:

$$\rho_{mnVW} = \frac{-\sigma^{mn}}{\sqrt{\sigma^{mm}\sigma^{nn}}}.$$

The estimated partial correlations can be employed to build a correlation network model, as shown in Giudici and Spelta (2015). Before doing so we add a further explanation of the partial correlation coefficient, and of its difference with respect to the (ordinary) marginal coefficient.

For $\{m, n\} \in (V \times W)$, let $S = (V \times W) \setminus \{m, n\}$. Suppose to express the dependence between spread measures through multiple linear equations in the following way:

$$\begin{cases} z^m = a_m + \sum_{n \neq m} a_{mn|S} z^n; \\ z^n = a_n + \sum_{m \neq n} a_{nm|S} z^m. \end{cases} \tag{2.14}$$

It can be shown that the partial correlation coefficient between Z^m and Z^n, given all the other $3N-2$ spread measures, is equivalent to the geometric average between the multiple linear coefficients introduced in (2.14):

$$\rho_{mn|S} = \rho_{nm|S} = \sqrt{a_{mn|S} \cdot a_{nm|S}}. \tag{2.15}$$

Note that if we had only two components ($S = \varnothing$), equation (2.14) becomes:

$$\begin{cases} Z^m = a + \alpha_{mn} Z^n, \\ Z^n = a + \alpha_{nm} Z^m; \end{cases} \tag{2.16}$$

from which the (ordinary) marginal correlation ρ_{mn} can be derived as the geometric average between the coefficients of the univariate linear models of equation (2.16): $\rho_{mn} = \rho_{nm} = \sqrt{a_{mn} \cdot a_{nm}}$.

For interpretational purposes, a relation between partial and marginal correlations can be shown. For simplicity, let us consider three elements $\{m, n, p\} \in (V \times W)$. It can be demonstrated that the partial correlation between elements i and j, given k, is the following:

$$\rho_{mn|p} = \rho_{nm|p} = \frac{\rho_{mn} - \rho_{mp}\rho_{np}}{\sqrt{(1 - \rho_{mp}^2)(1 - \rho_{np}^2)}}, \tag{2.17}$$

where ρ_{mn}, ρ_{mp} and ρ_{np} indicate marginal correlations.

Similarly, if we consider four elements $\{m, n, p, q\} \in (V \times W)$, the partial correlation between m and n, given p and q, is the following:

$$\rho_{mn|pq} = \frac{\rho_{mn|p} - \rho_{mq|p}\rho_{nq|p}}{\sqrt{(1 - \rho_{mq|p}^2)(1 - \rho_{nq|p}^2)}}. \tag{2.18}$$

The previous equation can be generalized for an arbitrary number of conditioning elements in a recursive way.

We can thus build a correlation network based on partial correlations, rather than on marginal correlations. To achieve this aim we proceed as follows.

Let $G = (V, E)$ be an undirected graph, with vertex set $V = \{1, ..., N\}$, and edge set $E = V \times V$, a binary matrix, with elements e_{mn}, that describes whether pairs of vertices are (symmetrically) linked between each other ($e_{mn} = 1$), or not ($e_{mn} = 0$). An edge between two nodes m and n will then be present in the network if and only if the corresponding partial correlation $\rho_{mn|S}$ is significantly different from zero.

A simple way to detect the significativity of partial correlations is to use ordinary pairwise statistical t-tests, as partial correlations can also be interpreted as correlations between regression residuals. Alternatively, a more complex search over possible graphical models could be run, as described in Giudici and Spelta (2015).

Once a correlation network is estimated, in terms of edge presence/absence, along with the estimated partial correlations, we can summarise it in measures that describe the contagion effect of each node (a sector of a country, in our context). We propose to calculate a weighted degree based on partial correlations, calculated as follows:

$$D^m = \sum_{n \neq m} \rho_{mn|S}, \qquad \{m, n\} \in (V \times W). \tag{2.19}$$

The above measure can be aggregated to compare systemic risks of different countries and/or of different economic sectors.

3 Application

We focus on seven european countries: France, Germany and the Netherlands (core countries); Greece, Italy and Spain (peripheral countries), and, finally, Ireland (in between the two groups, as we will demonstrate in this Section).

For each country we consider three idiosyncratic components, respectively for sovereign, corporate and bank risk: (a) interest rates on government bonds, (b) aggregate interest rates on loans to non-financial corporates, (c) aggregate interest rates on bank deposits from non-financial corporates and households. Concerning the common systematic component, there are many choices for a benchmark rate: for macroeconomic interpretation purposes, we suggest a rate that reflects the impact of the monetary policy, such as the 3-months Euribor.

In order to evaluate the evolution of the $N = 7 \times 3$ -dimensional system of interest rate spreads, we have considered three different time windows: (a) the pre-crisis period (2003-2006), (b) the crisis period (2007-2009) and (c) the post-crisis period (2010-2014).

All data are publicly available and have been selected with a monthly frequency.

3.1 Descriptive statistics

The time evolution of the interest rate processes for the sovereign sector can be observed in Figure 1.

From Figure 1 it is clear that the situation has radically changed after the financial crisis of 2008: until that year, in fact, interest rates on government bonds of different countries were very similar, while in 2009 they started radically changing, and precisely they started decreasing in core countries and increasing in peripheral countries.

Greek bond rates are characterized by the highest values for the whole period and by a very high volatility, with a strong peak during 2012. Ireland presents the highest values concentrated around

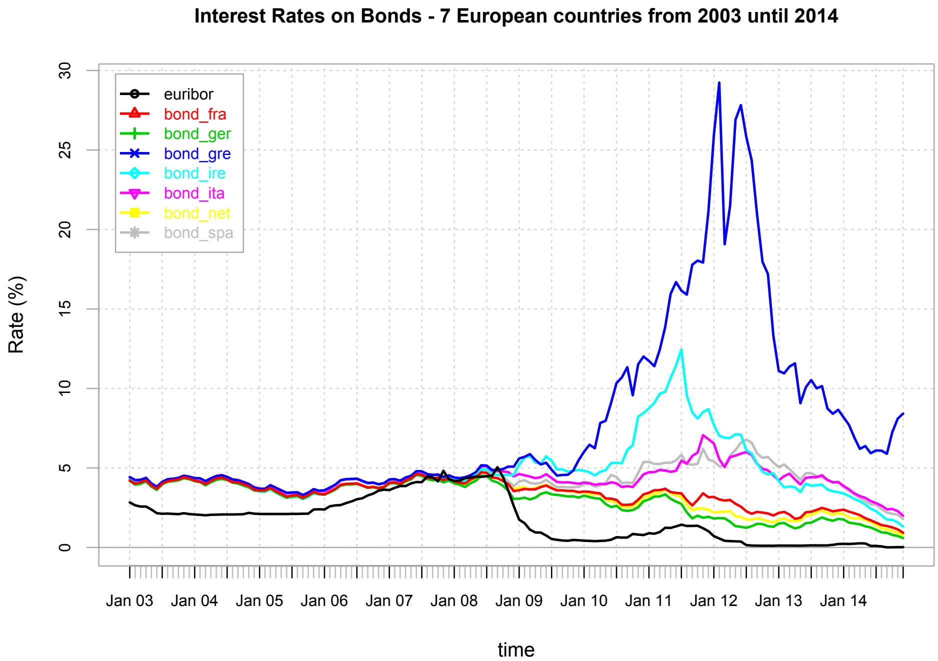

Figure 1: Monthly time evolution of 10-years maturity bond interest rates and of the 3-months Euribor, from January 2003 until December 2014

2011, followed by a remarkable drop during the next years. Spain and Italy seem to have very similar curves and, on the other hand, Germany, France and the Netherlands bond rates are quite homogeneous and definitely lower than the others. Note also that the Euribor rate dropped in 2008-2009, until reaching the actual situation of almost zero monetary rates.

The correlation matrices between the observed interest rates can be calculated, and they are reported in Table 1 (2003-2006), Table 2 (2007-2009) and in Table 3 (2010-2014).

From Table 1 note that all the correlation coefficients are positive, meaning that, during the crisis period, all interest rates vary together. On the other hand, the correlation coefficients between the Euribor and the bond interest rates are low, meaning that, during the same period, the link between monetary rates and sovereign bond rates is weak, as expected under stable economic conditions.

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.998	1.000						
Greece	0.984	0.987	1.000					
Ireland	0.997	0.997	0.984	1.000				
Italy	0.986	0.989	0.996	0.986	1.000			
Netherl.	0.997	0.996	0.978	0.995	0.984	1.000		
Spain	0.999	0.998	0.982	0.997	0.986	0.996	1.000	
Euribor	-0.012	0.015	0.114	0.007	0.099	-0.007	-0.011	1.000

Table 1: Correlation matrix between the interest rates on 10-years government bonds and the 3-months Euribor: pre-crisis period (2003-2006)

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.972	1.000						
Greece	-0.343	-0.528	1.000					
Ireland	-0.455	-0.611	0.850					
Italy	0.737	0.571	0.268	0.075	1.000			
Netherl.	0.991	0.945	-0.266	-0.390	0.795	1.000		
Spain	0.914	0.807	0.021	-0.136	0.916	0.945	1.000	
Euribor	0.862	0.868	-0.478	-0.713	0.590	0.848	0.739	1.000

Table 2: Correlation matrix between the interest rates on 10-years government bonds and the 3-months Euribor: crisis period (2007-2009)

From Table 2 one can notice that the situation changes. Firstly, correlation coefficients are strongly positive between France, Germany and the Netherlands. Secondly, there is a significant, negative link between France, Germany and the Netherlands on one side, and Greece and Ireland on the other side, because of the deep sovereign crisis in the latter two countries during the years 2007-2009. Thirdly, Spain and Italy appear to be positively related both to core countries (France, Germany and Netherlands) and to each other. Finally, it is interesting to note that during the crisis period core countries (France, Germany, the Netherlands) and Spain have positive and high correlations with monetary rates, while such coefficients for Greece and Ireland are negative. Thus, during the crisis period, core countries start diverging from peripheral ones, this being reflected by their grouping in terms of positive correlations, in the negative correlations between core and periphery and in the different impact of the monetary rate.

From Table 3 one can notice that the situation partially returns to the pre-crisis period: almost all correlation coefficients are positive.

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.908	1.000						
Greece	0.283	-0.066	1.000					
Ireland	0.805	0.647	0.563	1.000				
Italy	0.589	0.243	0.828	0.713	1.000			
Netherl.	0.947	0.987	0.030	0.708	0.353	1.000		
Spain	0.562	0.284	0.793	0.714	0.918	0.378	1.000	
Euribor	0.747	0.658	0.308	0.895	0.520	0.700	0.426	1.000

Table 3: Correlation matrix between the interest rates on 10-years government bonds and the 3-months Euribor: post-crisis period (20010-2014)

However, the most positive links are between France, Germany and the Netherlands on one side, and between Greece, Italy, and Spain on the other side. This reinforces the clustering effect started during the crisis period, with one cluster composed by core economies, and the other including peripheral economies (Spain, Italy and Greece), with Ireland in between these two groups. Indeed, the correlations of interest rates with the monetary rate are now all positive, and higher for core countries (including Ireland) than for peripheral ones (Greece, Italy and Spain), further emphasizing the clustering effect.

The time evolution of the interest rate processes for the corporate sector can be observed in Figure 2.

From Figure 2 one can notice that interest rates on loans to non-financial corporates differ across the main european countries: in particular, Greece has the highest interest rates; Spain has the lowest values until the financial crisis and France afterwards; Germany and the Netherlands have similar levels, especially since 2009. Overall, it seems that interest curves represented in the graph follow a similar path, with some correlation with the Euribor dynamics.

The correlation matrices between the observed interest rates are reported in Table 4 (2003-2006), Table 5 (2007-2009) and in Table 6 (2010-2014).

From Table 4 one can notice that almost all the correlation coefficients are strongly positive, meaning an important relationship between interest rates on loans to non-financial corporates during the pre-crisis period: this result is very similar to what has been observed for sovereign risk. In terms of correlations between the Euribor and interest rates on loans, the highest values are referred to peripheral countries, while core countries (France, Germany and the Netherlands)

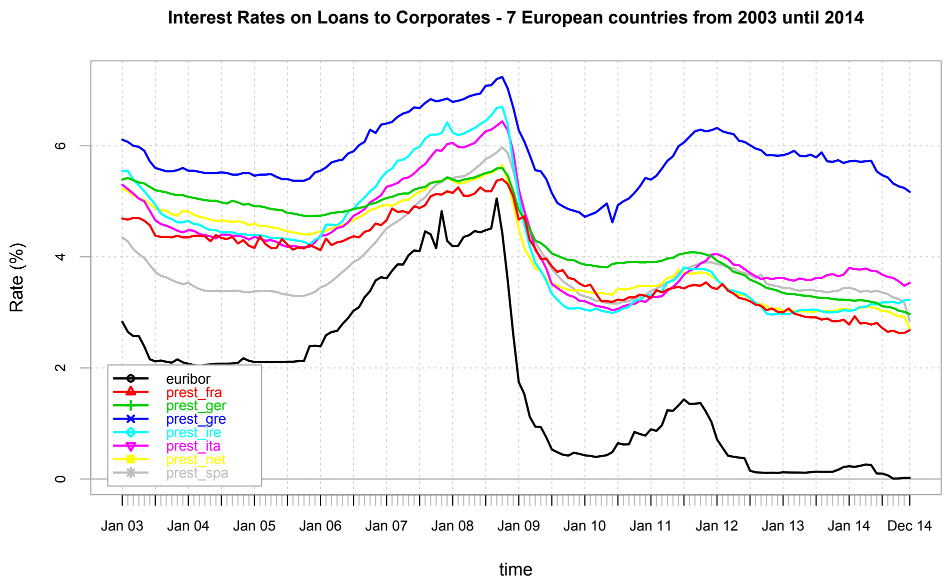

Figure 2: Monthly time evolution of interest rates on loans to non-financial corporates and of the 3-months Euribor, from January 2003 until December 2014

are characterized by low or negative values.

From Table 5 one can notice that, during the crisis, the correlation coefficients between interest rates remain strongly positive. In addition, all the considered countries, both core and peripheral, are positively related to the monetary rates.

Table 6 shows that after the financial crisis the relationships between interest rates of different countries has substantially changed. Firstly, there is a strong, positive link between core countries on one side (Germany, France and the Netherlands) and between peripheral countries on the other side (Italy, Greece and Spain). Secondly, core countries are characterized by high correlations with Euribor interest rates, while such values for peripheral countries are lower. This result suggests to divide the countries into two clusters, one composed by core economies, and the other one including peripheral economies (Spain, Italy and Greece), with Ireland in between these two groups.

The time evolution of the interest rate processes for the bank sector can be observed in Figure 3.

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.817	1.000						
Greece	0.751	0.399	1.000					
Ireland	0.906	0.711	0.908	1.000				
Italy	0.896	0.696	0.921	0.991	1.000			
Netherl.	0.913	0.936	0.681	0.898	0.889	1.000		
Spain	0.828	0.555	0.965	0.973	0.977	0.796	1.000	
Euribor	0.446	-0.024	0.896	0.666	0.683	0.310	0.805	1.000

Table 4: Correlation matrix between the interest rates on lendings to non-financial corporates and the Euribor: pre-crisis period (2003-2006)

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.988	1.000						
Greece	0.989	0.995	1.000					
Ireland	0.983	0.999	0.993	1.000				
Italy	0.993	0.995	0.995	0.991	1.000			
Netherl.	0.976	0.997	0.986	0.998	0.986	1.000		
Spain	0.969	0.951	0.961	0.941	0.975	0.933	1.000	
Euribor	0.930	0.968	0.949	0.974	0.940	0.980	0.857	1.000

Table 5: Correlation matrix between the interest rates on lendings to non-financial corporates and the Euribor: crisis period (2007-2009)

From Figure 3 it is clear that interest rates on deposits have changed during the years considered in this analysis, and their dynamics is quite heterogeneous between different countries. In general, such rates have significantly increased during the financial crisis of 2008, they dropped in the following two years and, in peripheral countries, they increased again during the sovereign crisis of 2012. France presents a peculiar behavior, since its interest rates on deposits have remained quite stable during the whole period: this may be due to the structure of its banking system which is indeed quite concentrated. Finally, it is interesting to observe that, in all the countries considered, interest rates on deposits have started decreasing in the last years, and such a decrease is stronger and faster for peripheral countries with respect to core countries.

The correlation matrices between the processes are reported in Table 7 (2003-2006), Table 8 (2007-2009) and in Table 9 (2010-2014).

From Table 7 one can notice that, with the exception of France, almost all the correlation coefficients are positive, meaning a positive

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.964	1.000						
Greece	0.165	0.051	1.000					
Ireland	0.625	0.631	0.529	1.000				
Italy	-0.125	-0.247	0.869	0.381	1.000			
Netherl.	0.916	0.959	0.186	0.779	-0.082	1.000		
Spain	0.538	0.437	0.867	0.718	0.709	0.554	1.000	
Euribor	0.796	0.864	0.204	0.828	-0.069	0.942	0.480	1.000

Table 6: Correlation matrix between the interest rates on lendings to non-financial corporates and the Euribor: post-crisis period (2010-2014)

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.594	1.000						
Greece	0.003	0.583	1.000					
Ireland	-0.012	0.528	0.972	1.000				
Italy	0.584	0.943	0.693	0.679	1.000			
Netherl.	0.509	0.914	0.760	0.750	0.961	1.000		
Spain	0.412	0.876	0.839	0.838	0.963	0.964	1.000	
Euribor	-0.022	0.544	0.984	0.986	0.682	0.746	0.841	1.000

Table 7: Correlation matrix between the interest rates on deposits and the 3-months Euribor: pre-crisis period (2003-2006)

relationship between interest rates on deposits across the seven european countries considered during the pre-crisis period. Concerning the correlation coefficients between interest rates on deposits and the Euribor, peripheral countries (Greece, Ireland and Spain) have the highest coefficient; the Netherlands and Germany are less correlated while France is negatively related.

Table 8 shows that, during the crisis period, all the relationships between interest rates on deposits in different European countries are strongly and positively related to each other. This behavior is consistent with the increase of monetary rates.

In Table 9 one can notice that the situation changes after the financial crisis. Two different groups, characterized by strong inner connections emerge: France and Germany on one side (core countries), and Greece, Ireland, Italy and Spain on the other side (peripheral countries). The Netherlands seem to behave independently from other countries, being positively related only to Italy: this peculiarity is due to its relatively high interest rates on deposits, which may

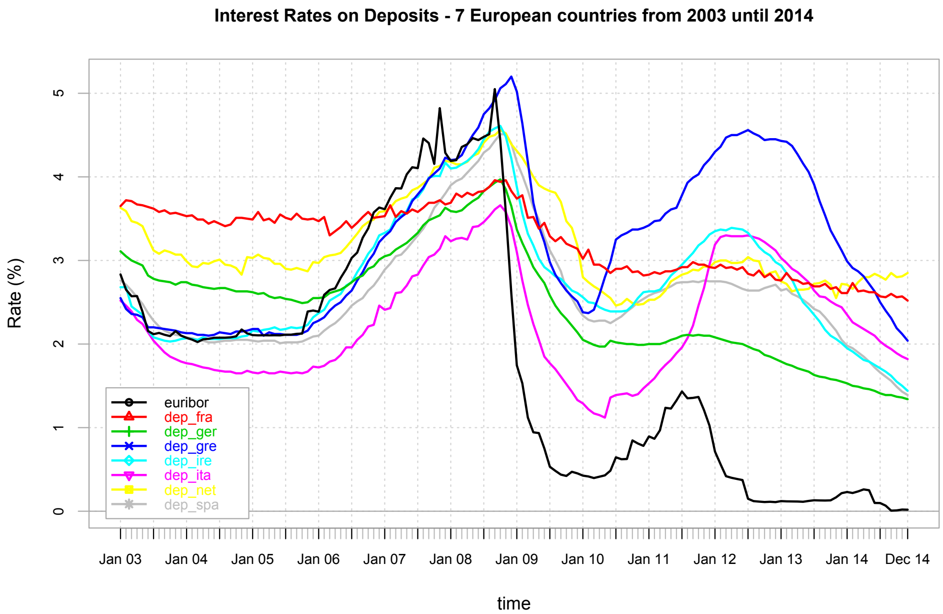

Figure 3: Monthly time evolution of interest rates on deposits and of the 3-months Euribor, from January 2003 until December 2014

depend, as for France, from the peculiarities of its banking system. Last, and coherently with what obtained for the sovereign and the corporate interest rates, France and Germany have the highest and positive correlation coefficients with monetary rates, with the same coefficients negative for Italy.

3.2 Sectorial Spreads

The first step in model estimation consists in deriving the CIR coefficients in (2.2), for the three sectors of each country.

The estimated parameter values obtained for the systematic process S_t (3-months Euribor interest rate), common to all the spread measures, for the three time windows, are reported in Table 10.

From Table 10 note that, during the crisis period, all the parameters (drift and volatility terms) are sensibly higher. In the last period, the drift has returned to the initial values, but the volatility has remained high.

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.968	1.000						
Greece	0.939	0.913	1.000					
Ireland	0.955	0.989	0.909	1.000				
Italy	0.966	0.995	0.925	0.992	1.000			
Netherl.	0.875	0.866	0.943	0.860	0.864	1.000		
Spain	0.889	0.869	0.965	0.865	0.871	0.984	1.000	
Euribor	0.782	0.859	0.600	0.853	0.849	0.511	0.504	1.000

Table 8: Correlation matrix between the interest rates on deposits and the 3-months Euribor: crisis period (2007-2009)

	France	Germany	Greece	Ireland	Italy	Netherl.	Spain	Euribor
France	1.000							
Germany	0.938	1.000						
Greece	0.398	0.503	1.000					
Ireland	0.761	0.830	0.856	1.000				
Italy	-0.080	-0.040	0.773	0.512	1.000			
Netherl.	0.052	0.026	0.363	0.406	0.681	1.000		
Spain	0.754	0.838	0.849	0.933	0.382	0.135	1.000	
Euribor	0.607	0.721	0.165	0.441	-0.306	-0.094	0.526	1.000

Table 9: Correlation matrix between the interest rates on deposits and the 3-months Euribor: post-crisis period (2010-2014)

In Table 11 the estimated parameters of the idiosyncratic processes $y^i_{t,1}$ for the sovereign sector are reported.

Table 11 shows a heterogeneous behavior during the three periods. During the pre-crisis years all countries have the same behavior, with similar drift and volatility terms. During the crisis period, some differences between core countries and peripheral countries start emerging, with the latter having higher drift parameters and higher volatility terms, consistently with the descriptive analysis. The differences in the drift between core countries and peripheral countries amplify in the post-crisis years, with Ireland having the highest volatility parameter due to its recovery and the consequent drop in sovereign interest rates after its crisis of 2011.

Table 12 reports the estimated weight coefficients of the systematic and of the idiosyncratic processes.

From Table 12 note that, during the pre-crisis period, both weights α_1 and β_1 are negative for all countries: the systematic component, therefore, changes faster than the idiosyncratic one. On the con-

	a	v	b
2003-2006	0.014	0.001	0.056
2007-2009	0.899	0.355	0.569
2010-2014	0.006	0.027	0.131

Table 10: Estimated parameters of the systematic process S_t (3-months Euribor), for the pre-crisis, crisis and post-crisis periods

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	$(\theta_1)_1$	0.388	0.390	0.433	0.333	0.392	0.379	0.361
	$(\theta_2)_1$	0.103	0.105	0.109	0.090	0.099	0.101	0.096
	$(\theta_3)_1$	0.077	0.079	0.076	0.077	0.074	0.080	0.076
2007-2009	$(\theta_1)_1$	0.238	0.145	0.760	0.755	0.635	0.231	0.708
	$(\theta_2)_1$	0.062	0.045	0.150	0.157	0.143	0.061	0.169
	$(\theta_3)_1$	0.081	0.093	0.108	0.109	0.074	0.080	0.083
2010-2014	$(\theta_1)_1$	0.448	0.386	0.821	1.000	0.996	0.430	0.990
	$(\theta_2)_1$	0.060	0.089	0.060	0.578	0.018	0.076	0.009
	$(\theta_3)_1$	0.649	0.544	0.899	1.212	1.010	0.626	1.005

Table 11: Estimated parameters of the idiosyncratic processes $y^i_{t,1}$ (interest rates on 10-years maturity government bonds), for the pre-crisis, crisis and post-crisis periods

trary, during the post-crisis years all weights are positive, meaning that the idiosyncratic component changes faster than the systematic one, consistently with the actual situation of almost zero monetary rates. Note also that the weights of the processes change over time, but such changes are homogeneous and similar across the seven considered countries.

Having estimated all necessary parameters, we are now able to estimate the probability of default of each sovereign, $PD^i_{t,1}$, based on the spread between bond interest rates and monetary rates ($Z^i_{t,1}$), calculated with equation (2.11). These probabilities have been derived in each of the three time windows (pre-crisis, crisis and post-crisis periods), and have been merged as in Figure 4.

From Figure 4 it is clear that Greece presents the most critical situation, with the highest PD values. Ireland, instead, presents an anticipated increase in its default probability because of its specific sovereign crisis in 2011, while in the following years it starts performing quite well until reaching very low PD values in 2014. Italy and Spain show similar behaviours of the PD and, finally, France, Germany and the Netherlands behave quite similarly, being characterized

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	α_1	-0.103	-0.115	-0.040	-0.096	-0.043	-0.102	-0.101
	β_1	-0.782	-0.775	-0.759	-0.755	-0.758	-0.772	-0.774
2007-2009	α_1	0.497	0.753	0.253	0.295	0.353	0.496	0.443
	β_1	-0.990	-0.980	-0.980	-0.986	-0.990	-0.991	-0.992
2010-2014	α_1	0.855	0.783	0.973	0.988	0.911	0.824	0.906
	β_1	0.198	0.103	0.245	0.855	0.152	0.141	0.073

Table 12: Weight coefficients of the two components of the general processes $Z^i_{t,1}$ (sovereign spread), for the pre-crisis, crisis and post-crisis periods

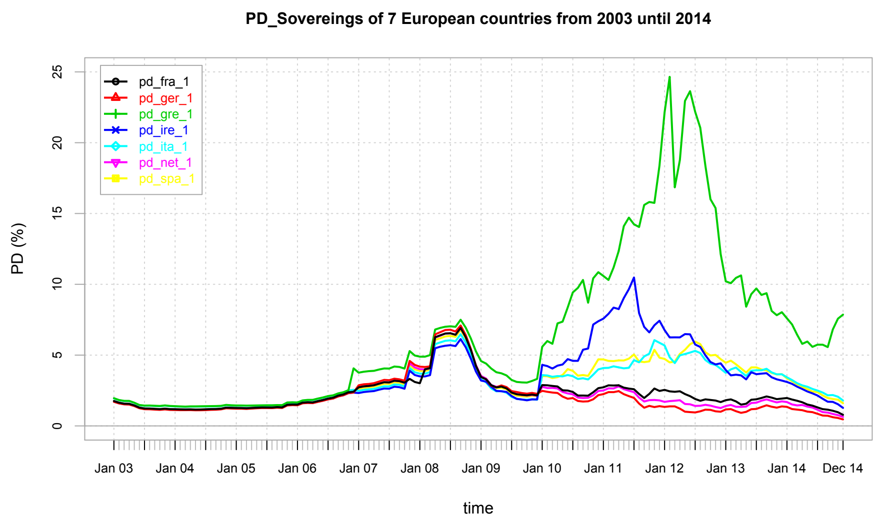

Figure 4: Default probabilities $PD^i_{t,1}$ from 2003 until 2014 - sovereign risk

by the lowest PDs for the entire period.

In Table 13 the estimated parameters of the idiosyncratic processes $y^i_{t,2}$ for the corporate sector are reported.

Table 13 shows that Greece, Ireland, Italy and Spain have the highest volatility parameters during the pre-crisis period. During the crisis period, all the volatility terms are very high, as was the case for the sovereigns. After the crisis, Greece, Italy and Spain present the highest drift terms, consistently with the descriptive statistics.

The weight coefficients of the systematic and the idiosyncratic processes are reported in Table 14.

From Table 14 note that the α_2 weight is low, especially for Greece

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	$(\theta_1)_2$	0.828	0.305	0.006	0.190	0.252	0.366	0.056
	$(\theta_2)_2$	0.191	0.063	0.001	0.041	0.057	0.079	0.015
	$(\theta_3)_2$	0.041	0.012	0.028	0.036	0.030	0.018	0.032
2007-2009	$(\theta_1)_2$	0.001	0.001	0.002	0.001	0.001	0.001	0.001
	$(\theta_2)_2$	0.016	0.019	0.019	0.020	0.017	0.021	0.011
	$(\theta_3)_2$	1.500	1.243	1.518	1.497	1.507	1.483	1.521
2010-2014	$(\theta_1)_2$	0.426	0.447	1.511	0.895	1.350	0.431	1.497
	$(\theta_2)_2$	0.031	0.029	0.001	0.032	0.032	0.035	0.099
	$(\theta_3)_2$	0.306	0.297	1.454	0.306	0.330	0.312	0.644

Table 13: Estimated parameters of the idiosyncratic processes $y^i_{t,2}$ (interest rates on loans to non-financial corporates), for the pre-crisis, crisis and post-crisis periods

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	α_2	0.401	0.720	0.346	0.611	0.486	0.577	0.268
	β_2	-0.091	0.415	-0.550	0.240	0.025	0.170	-0.109
2007-2009	α_2	0.286	0.226	0.050	0.052	0.103	0.276	0.207
	β_2	-0.982	-0.991	-0.968	-0.832	-0.880	-0.984	-0.926
2010-2014	α_2	0.735	0.796	0.868	0.735	0.758	0.730	0.753
	β_2	-0.781	-0.575	-0.609	-0.840	-0.855	-0.902	-0.840

Table 14: Weight coefficients of the two components of the general processes $Z^i_{t,2}$ (corporate spread), for the pre-crisis, crisis and post-crisis periods

and Ireland, during the crisis period, suggesting a prevalence of the systemic effect. In addition, as it occurred for the sovereigns, all weight coefficients change over time but such changes are similar across the different countries.

Having estimated the necessary parameters, we can calculate the default probabilities of the corporate sector for the seven countries, $PD^i_{t,2}$. They are shown in Figure 5.

From Figure 5 it is clear that Spain has the lowest default probability until 2007, due to its very low interest rates on loans to non-financial corporates, while in the last year France and the Netherlands are the two countries with the lowest corporate risk. On the contrary, Greece presents the highest default probabilities, even if they have started decreasing in 2014. Ireland is the most significant case, since it strongly suffered the financial crisis of 2008, with a strong increase of the PD of its corporate system: after that period, however, the corporate sector of Ireland started performing very well.

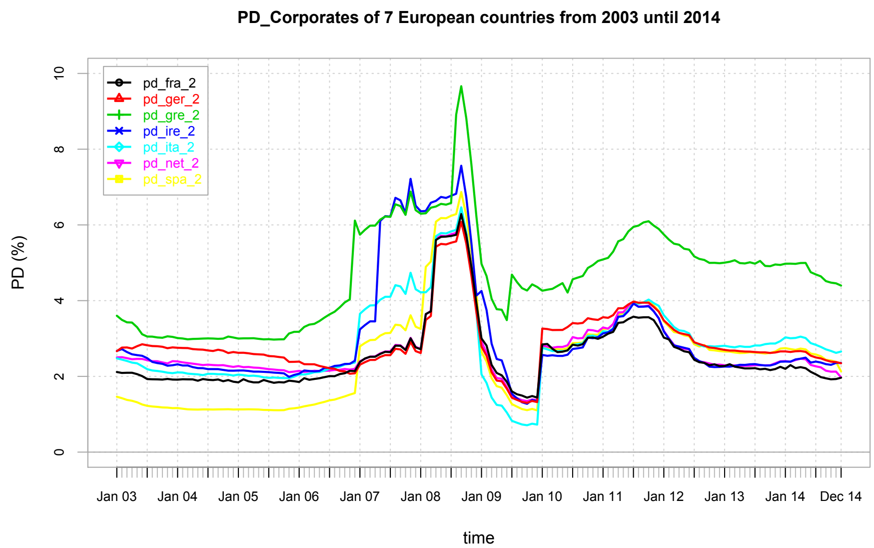

Figure 5: Default probabilities $PD^i_{t,2}$ from 2003 until 2014 - corporate risk

In Table 15 the estimated parameters of the idiosyncratic processes $y^i_{t,3}$ for the bank sector are reported.

Table 15 shows that, during the pre-crisis period, the estimated parameters for the different countries are quite similar to each other: the only one exception consists of France, whose drift term is significantly higher than the others. During the crisis period Greece presents the highest drift and volatility terms, consistently with descriptive statistics. It is also interesting to observe that the volatility parameters have strongly increased in all countries with respect to the previous period. During the post-crisis years a further diversification emerge: core economies, in fact, are characterized by high drift terms and low volatility parameters, while peripheral countries present high volatilities and low drifts.

The weight coefficients of the systematic and the idiosyncratic processes are reported in Table 16.

From Table 16, note that the α_3 coefficient is positive during the three time-windows, lower for France and Germany and higher for peripheral countries.

Having estimated the required parameters, we can calculate the default probabilities of the corporate sector for the seven countries,

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	$(\theta_1)_3$	0.998	0.173	0.012	0.011	0.075	0.500	0.033
	$(\theta_2)_3$	0.285	0.065	0.001	0.001	0.041	0.075	0.015
	$(\theta_3)_3$	0.033	0.021	0.035	0.042	0.039	0.032	0.030
2007-2009	$(\theta_1)_3$	0.522	0.401	1.096	0.023	0.001	0.001	0.033
	$(\theta_2)_3$	0.022	0.012	0.022	0.009	0.011	0.006	0.005
	$(\theta_3)_3$	0.515	0.523	1.480	0.906	0.568	0.620	0.756
2010-2014	$(\theta_1)_3$	1.107	1.050	0.111	0.011	0.253	1.438	0.141
	$(\theta_2)_3$	0.075	0.046	0.004	0.009	0.017	0.094	0.040
	$(\theta_3)_3$	0.019	0.091	0.575	0.236	0.317	0.134	0.121

Table 15: Estimated parameters of the idiosyncratic processes $y^i_{t,3}$ (interest rates on deposits, of both non-financial corporates and households), for the pre-crisis, crisis and post-crisis periods

		France	Germany	Greece	Ireland	Italy	Netherl.	Spain
2003-2006	α_3	0.346	0.712	1.031	0.950	1.318	0.451	1.015
	β_3	-0.981	-0.939	-0.958	-0.895	-0.812	-0.888	-0.871
2007-2009	α_3	0.686	0.859	0.470	0.621	1.167	0.511	0.646
	β_3	-0.995	-0.978	-0.889	-0.963	-0.947	-0.982	-0.936
2010-2014	α_3	0.677	0.538	0.820	0.758	0.655	0.672	0.696
	β_3	-0.953	-0.816	-0.369	-0.334	-0.693	-0.943	-0.553

Table 16: Weight coefficients of the two components of the general processes $Z^i_{t,3}$ (bank spread), for the pre-crisis, crisis and post-crisis periods

$PD^i_{t,3}$. They are shown in Figure 6 where, to avoid computational problems, we have set $S^i_t = \min\left(0; |\frac{\alpha^i_3 y^i_{t,3}}{\beta^i_3}|\right)$.

From Figure 6 it is clear that the banking systems of different countries present similar default probabilities, because of their similar interest rates on deposits. France is characterized by the lowest PD until 2011, followed by Germany after 2011. The Netherlands and Ireland, in spite of the high peak they had in 2008, are now significantly decreasing their bank risk, and Italy and Spain too. Greece presents a strong increase in the bank system default probability not only during the financial distress of 2008, but also in correspondence of the sovereign crisis of 2012.

3.3 Correlated Sectorial Spreads

According to (2.13), we can obtain the correlation matrix between all the $3 \times N + 1$ processes, and for the three different time windows.

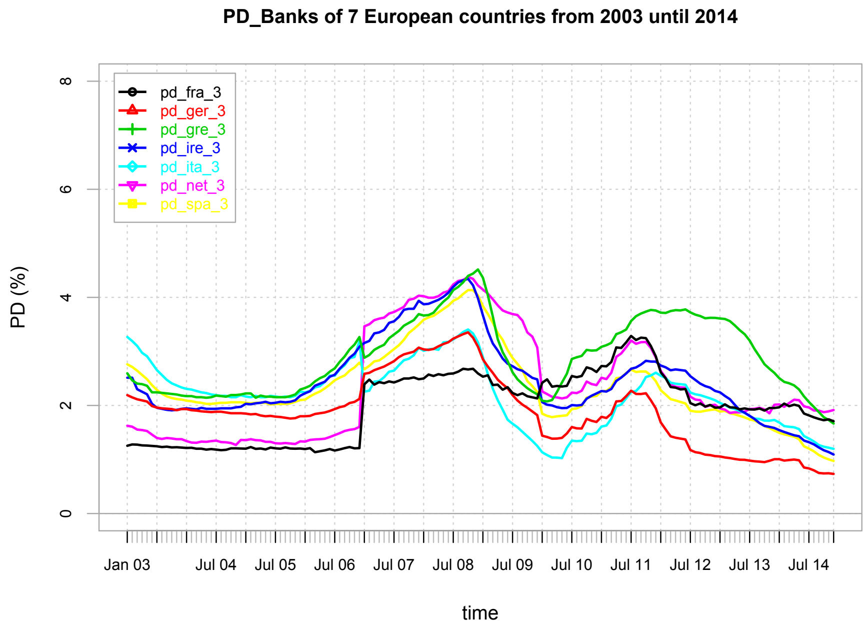

Figure 6: Default probabilities $PD^i_{t,3}$ from 2003 until 2014 - bank risk

These matrices are reported in the Appendix, as Table 18 (pre-crisis period, 2003-2006), Table 19 (crisis period, 2007-2009) and Table 20 (post-crisis period, 2010-2014).

During the pre-crisis period, the idiosyncratic processes $y^i_{t,1}$ and $y^j_{t,2}$ are positively related, as well as $y^j_{t,2}$ with $y^k_{t,3}$. The situation is different between $y^i_{t,1}$ and $y^k_{t,3}$, very low for most pairs of countries.

During the crisis period, all idiosyncratic processes (with the exception of Greece and Ireland) become correlated with each other and also with the systematic Euribor process. This conclusion extends what found in the previous Section to the correlations between countries across different sectors of the economy.

In the post-crisis period, many correlations are still positive, but a large number of negative ones emerges, especially between core and peripheral countries. Thus, once more, the correlation pattern indicates that the financial crisis has lead to a clustering of european countries in two distinct groups (core and peripheral), with Ireland moving from the cluster of peripheral countries into the group of core countries after 2012.

To better understand the clustering behavior, we have calculated the correlations between sovereign spreads ($Z^i_{t,1}$) and bank interest margins ($Z^j_{t,2} - Z^j_{t,3}$). The post-crisis period confirms that the correlation coefficients within the same country are strongly positive in core countries, strongly negative in peripheral countries (Greece, Italy and Spain), and very close to zero in Ireland. This means that, when interest rates on government bonds increase, the banking systems of core countries increase their margins, while the banking systems of peripheral countries reduce them.

We remark that, during the last time window, the correlations $\mathrm{Corr}(Z^i_{t,1}, Z^i_{t,2})$, $\mathrm{Corr}(Z^i_{t,1}, Z^i_{t,3})$ and $\mathrm{Corr}(Z^i_{t,2}, Z^i_{t,3})$ are all positive. This means that, in core countries, the corporate spread measure changes faster than the bank spread measure: more formally, $|\frac{\partial Z^i_{t,2}}{\partial t}| > \frac{\partial Z^i_{t,3}}{\partial t}$ for $i = fra, ger, net$. In peripheral countries, instead, the situation is reversed, with bank spread measure changing faster than corporate spread measures: $|\frac{\partial Z^i_{t,3}}{\partial t}| > \frac{\partial Z^i_{t,2}}{\partial t}$ for $i = gre, ita, spa$.

We now derive, according to the partial correlation model described in (2.17) and (2.18), the network model between all sovereign, corporate and bank spreads. To achieve this aim, it is necessary to calculate the inverse correlation matrix of $Z^i_{t,\{1,2,3\}}$, together with the associated $t - test$.

By considering a significance level of $\alpha = 0.01$, we can thus select the statistically significant partial correlations, and thus derive the partial correlation networks for $Z_{t,\{1,2,3\}}$, for the three different time windows. The results are shown in Figure 7, in which the green lines stand for positive and significant partial correlations, while red lines indicate negative and significant partial correlations; moreover, the thicker the line, the stronger the connection.

From Figure 7 note that the partial correlation pattern has substantially changed over the years: in the pre-crisis period negative correlations prevail; during the crisis period both positive and negative correlations increase in number; finally, in the post-crisis period correlations create two distinct clusters (core and peripheral countries), with positive correlations within each cluster and negative correlations between the two clusters.

In particular, the most significant positive partial correlations are between (a) the sovereign spread in Spain and the sovereign spread in Italy, (b) the bank spread in Italy and the bank spread in Greece,

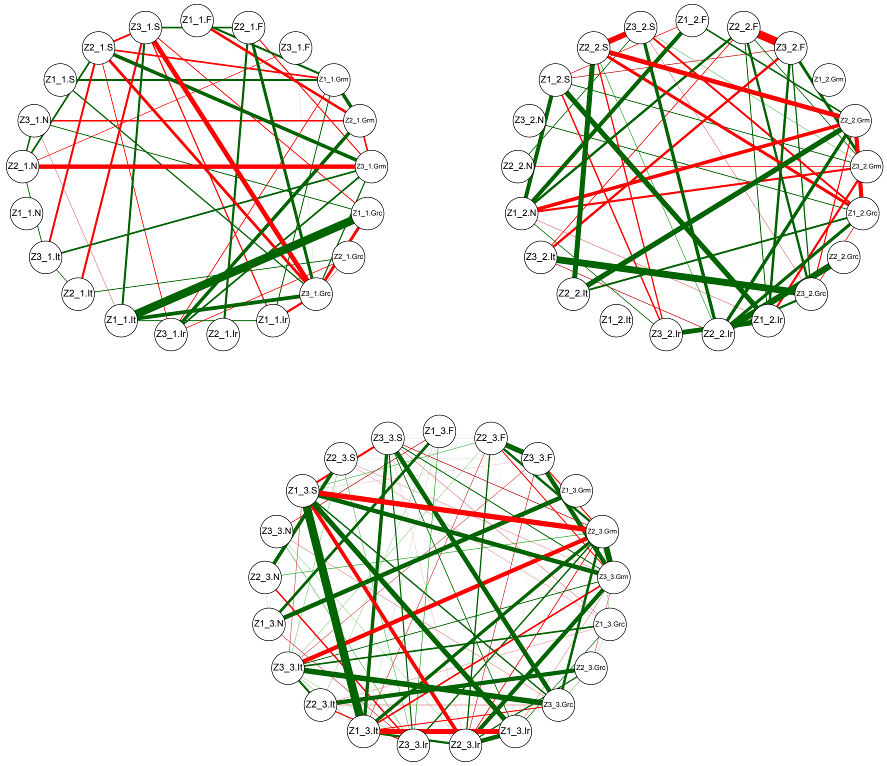

Figure 7: Network graphs for the seven european countries considered in the sample, based on $Z_{t,1}$, $Z_{t,2}$ and $Z_{t,3}$, for the pre-crisis (top left), crisis (top right) and post-crisis (bottom) periods

(c) the bank spread in Spain and the bank spread in Greece. Positive links regard also core countries, especially between (a) corporate and bank spreads in France, (b) corporate and bank spreads in Germany, (c) bank spreads in France and corporate spreads in Germany and (d) sovereign spreads of the Netherlands and Germany. Negative correlations appear, in particular, between (a) sovereign Spanish spreads and corporate German spreads, and (b) corporate German spreads and bank Italian spreads. Finally, note the peculiar case of Ireland, whose sovereign spread is positively related to the one of Spain and negatively related to the Italian one, again as an effect of the peculiar improvement of the Irish economy.

Reading jointly the results obtained through the partial correlation networks with those obtained from marginal correlations, we can improve the interpretation of the results in Figure 7. Before the crisis, countries move together and, therefore, marginal correlations are stronger than partial ones. During crisis periods the number of partial correlation increases, because all the components of the system simultaneously react to stress events. After the crisis, countries divide into different clusters, with strong partial correlations inside each cluster.

Furthermore, Figure 7 indicates that the correlations between spreads referred to the same country are less than the correlations between spreads of different countries. This indicates that the different sectors of the economy, as well as different countries, are strongly interrelated, as the economies, at the european level, are highly integrated.

In order to better understand the systemic risk associated to each country and/or each sector, we have then applied our proposed measure of interconnectedness, calculated as the sum of all the significant partial correlations. For comparison, such a sum has been developed in two ways, one considering signed numbers, the other one considering the absolute value of partial correlations. The results are shown in Table 17, for the three time windows and for the three sectors of the economy.

	2003-2006		2007-2009		2010-2014	
Ec. system	Sign. Sum	Abs. sum	Sign. Sum	Abs. sum	Sign. Sum	Abs. sum
Sovereign	5.352	16.897	5.982	25.776	4.808	22.238
Corporate	-0.570	17.639	5.268	27.732	5.915	23.972
Bank	0.503	21.609	4.782	27.247	5.783	29.435

Table 17: Systemic risk for the three sectors of the economy (sovereign, corporate, bank) and for the three time windows: sum of signed partial correlations vs sum of the absolute values of partial correlations

The absolute value measure in Table 17 shows that the bank sector is the one that carries the highest systemic risk. In addition, while the sovereign and corporate risk peak during the crisis, and decrease afterwards, the bank risk has further increased in the latest period. The signed sum measure shows instead that, while before the crisis the bank sector (and the corporate sector) compensates risks through diversification, this is much less the case in the recent years, as the economies become more interrelated.

To understand how sector-specific risks are spread between countries, Figure 8 shows the time evolution of the contribution of each country to the total degree of connectivity, expressed by the sum of the absolute values of the partial correlations.

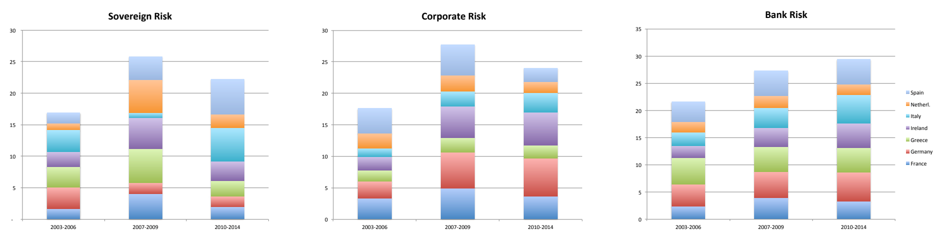

Figure 8: Contribution of each country to systemic risk in the three different economic sectors, and in the three different time windows

From Figure 8 note that, concerning sovereign risk, Greece has increased its relative contribution in the second period, together with France, Ireland and the Netherlands. We can compare this result with the (signed) network proposed in Figure 7, and conclude that: (a) the Greek contribution in the second period mostly comes from negative links with Germany and Spain, (b) the French sovereign, very low connected in the first period, becomes highly and positive related to the Netherlands in the crisis period, (c) the Ireland sovereign, very low connected in the first period, becomes highly and positively connected to Spain and Greece during the crisis period and, finally, (d) the Netherlands sovereign, very low connected in the first period, becomes highly and positive connected to France and Spain in the crisis years. During the post-crisis years, the most important contributions are due to interconnections between peripheral countries (Spain, Italy, and Greece), with core countries contributing very little to sovereign risk.

Concerning corporate risk, all the countries increase their correlations during the crisis period, with the greatest contribution coming from Germany and Ireland. Comparing this result with the network proposed in Figure 7, we can conclude that the contribution of the German corporate sector mostly comes from negative relationships, precisely with the Spanish corporate sector, the Dutch sovereign sector and the German banking sector.

Concerning bank risk, all the countries have almost equally increased their contribution to systemic risk along time, with no substantial differences between them. This result can be explained by the stronger integration of the european bank sector with respect to the corporate and sovereign sectors.

4 Conclusions

In this work we have proposed a new systemic risk measurement model, based on stochastic processes and correlation network models. The model has been applied to the economies of the European monetary union. For each country we have considered three spread measures (country spread, corporate spread, bank spread), and we have modeled each of them as a linear combination of two stochastic processes: a country-specific idiosyncratic component and a common systematic factor. We have introduced a correlation structure between all countries, within each sector (independent spreads model) and also between sectors (correlated spreads model) thus deriving a statistical representation of the transmission mechanism of systemic risk through the Eurozone, that correctly takes into account interdependence effects.

From an applied viewpoint, the results obtained with the independent spreads model indicate that, before the crisis, all interest spreads of different countries showed a similar behavior, with the corporate and bank sector positively correlated with the monetary rate, especially for the peripheral countries. The estimated drift, volatility and default probability were moderate, and the systematic component prevailed. During the crisis all correlations, drifts, volatilities and default probabilities have increased, in line with the monetary rates. After the crisis, we observe a change in regime, with two evident clusters, characterised by a high within correlation and a low between correlation: core economies (France, Germany, the Netherlands and, recently, Ireland), with low drift, volatility and default probability, with the corporate and bank sector still positively correlated with the monetary rates; peripheral economies (Greece, Italy, Spain), with a high drift, volatility, and only sovereign bonds correlated with monetary rates. In other words, after the crisis the sovereign risk of peripheral countries seems to distort the "natural" relationships between the three sectors of the economy, that is present in all countries before the crisis but only in the core economies afterwards.

The results obtained with the correlated spreads model and, in particular, with the resulting partial correlation network, confirm the above findings. Before the crisis, countries move together and, therefore, marginal correlations are stronger than partial ones. During crisis periods the number of partial correlations increases, because all the components of the system simultaneously react to stress events. After the crisis, countries divide into different clusters, with strong partial correlations within clusters and very weak or negative ones between clusters.

At the overall European level, the bank sector is the one that carries the highest systemic risk. While the sovereign and corporate risk peak during the crisis, but decrease afterwards, the bank risk has further increased in the latest period. At the country level, our results show that peripheral european countries, differently from core ones, have had a limited benefit from the drop of monetary rates that has followed the financial crisis.

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A Tables

		France			Germany			Greece			Ireland			Italy			Netherl.			Spain			Eur.
		y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	
Fra	y_1	1.00																					
	y_2	0.49	1.00																				
	y_3	0.12	0.65	1.00																			
Ger	y_1	1.00	0.49	0.11	1.00																		
	y_2	0.64	0.82	0.74	0.62	1.00																	
	y_3	0.52	0.93	0.59	0.53	0.82	1.00																
Gre	y_1	0.98	0.50	0.05	0.99	0.58	0.54	1.00															
	y_2	0.25	0.75	0.29	0.27	0.40	0.85	0.35	1.00														
	y_3	0.03	0.47	0.00	0.06	0.02	0.58	0.14	0.91	1.00													
Ire	y_1	1.00	0.52	0.15	1.00	0.67	0.56	0.98	0.28	0.05	1.00												
	y_2	0.39	0.91	0.56	0.40	0.71	0.97	0.44	0.91	0.69	0.43	1.00											
	y_3	-0.10	0.42	-0.01	-0.08	-0.05	0.53	0.03	0.89	0.97	-0.08	0.66	1.00										
Ita	y_1	0.99	0.48	0.03	0.99	0.57	0.53	1.00	0.33	0.14	0.99	0.42	0.01	1.00									
	y_2	0.39	0.90	0.53	0.40	0.70	0.97	0.44	0.92	0.70	0.43	0.99	0.67	0.42	1.00								
	y_3	0.30	0.87	0.58	0.31	0.67	0.94	0.35	0.90	0.69	0.34	0.99	0.68	0.32	0.98	1.00							
Net	y_1	1.00	0.49	0.12	1.00	0.63	0.52	0.98	0.25	0.04	0.99	0.39	-0.10	0.98	0.39	0.30	1.00						
	y_2	0.60	0.91	0.69	0.60	0.94	0.96	0.59	0.68	0.36	0.64	0.90	0.29	0.58	0.89	0.86	0.60	1.00					
	y_3	0.26	0.84	0.51	0.27	0.59	0.91	0.31	0.93	0.76	0.29	0.96	0.75	0.29	0.96	0.96	0.26	0.80	1.00				
Spa	y_1	1.00	0.49	0.12	1.00	0.64	0.52	0.98	0.25	0.03	1.00	0.39	-0.10	0.99	0.39	0.30	1.00	0.61	0.26	1.00			
	y_2	0.29	0.83	0.44	0.31	0.56	0.92	0.37	0.97	0.82	0.33	0.97	0.80	0.34	0.98	0.97	0.29	0.80	0.97	0.29	1.00		
	y_3	0.21	0.79	0.41	0.22	0.48	0.88	0.29	0.97	0.84	0.24	0.95	0.84	0.26	0.96	0.96	0.21	0.74	0.96	0.21	0.99	1.00	
Eur	S_t	-0.01	0.45	-0.02	0.02	-0.02	0.54	0.11	0.90	0.98	0.01	0.67	0.99	0.10	0.68	0.68	-0.01	0.31	0.75	-0.01	0.80	0.84	1.00

Table 18: Correlation coefficients between the idiosyncratic processes ($y^i_{t,\{1,2,3\}}$) and the systematic process (S_t): pre-crisis period (2003-2006)

		France			Germany			Greece			Ireland			Italy			Netherl.			Spain			Eur.
		y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	
Fra	y_1	1.00																					
	y_2	0.74	1.00																				
	y_3	0.58	0.94	1.00																			
Ger	y_1	0.97	0.69	0.48	1.00																		
	y_2	0.79	0.99	0.90	0.76	1.00																	
	y_3	0.69	0.97	0.97	0.60	0.95	1.00																
Gre	y_1	-0.34	-0.26	-0.01	-0.53	-0.33	-0.10	1.00															
	y_2	0.78	0.99	0.92	0.73	0.99	0.96	-0.26	1.00														
	y_3	0.40	0.83	0.94	0.26	0.77	0.91	0.24	0.80	1.00													
Ire	y_1	-0.46	-0.54	-0.31	-0.61	-0.61	-0.38	0.85	-0.55	-0.09	1.00												
	y_2	0.80	0.98	0.88	0.78	1.00	0.94	-0.34	0.99	0.75	-0.63	1.00											
	y_3	0.68	0.96	0.95	0.59	0.94	0.99	-0.11	0.95	0.91	-0.41	0.93	1.00										
Ita	y_1	0.74	0.68	0.75	0.57	0.68	0.77	0.27	0.71	0.75	0.08	0.67	0.76	1.00									
	y_2	0.75	0.99	0.93	0.70	0.99	0.97	-0.26	0.99	0.83	-0.56	0.99	0.97	0.70	1.00								
	y_3	0.66	0.97	0.97	0.57	0.95	0.99	-0.09	0.96	0.93	-0.39	0.94	0.99	0.75	0.97	1.00							
Net	y_1	0.99	0.75	0.62	0.95	0.80	0.72	-0.27	0.80	0.47	-0.39	0.81	0.70	0.79	0.77	0.69	1.00						
	y_2	0.81	0.98	0.87	0.79	1.00	0.93	-0.38	0.99	0.73	-0.65	1.00	0.93	0.65	0.99	0.93	0.81	1.00					
	y_3	0.38	0.74	0.88	0.22	0.68	0.87	0.22	0.71	0.94	0.03	0.65	0.86	0.75	0.74	0.86	0.44	0.64	1.00				
Spa	y_1	0.91	0.71	0.67	0.81	0.74	0.74	0.02	0.75	0.58	-0.14	0.74	0.72	0.92	0.73	0.70	0.95	0.73	0.56	1.00			
	y_2	0.67	0.97	0.97	0.59	0.95	1.00	-0.10	0.96	0.92	-0.40	0.94	1.00	0.76	0.97	1.00	0.70	0.93	0.86	0.72	1.00		
	y_3	0.34	0.74	0.89	0.18	0.68	0.87	0.28	0.71	0.96	0.03	0.65	0.86	0.73	0.74	0.87	0.40	0.64	0.98	0.54	0.87	1.00	
Eur	S_t	0.86	0.93	0.78	0.87	0.97	0.86	-0.48	0.95	0.60	-0.71	0.97	0.85	0.59	0.94	0.85	0.85	0.98	0.51	0.74	0.86	0.50	1.00

Table 19: Correlation coefficients between the idiosyncratic processes ($y^i_{t,\{1,2,3\}}$) and the systematic process (S_t): crisis period (2007-2009)

		France			Germany			Greece			Ireland			Italy			Netherl.			Spain			Eur.
		y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3	
Fra	y_1	1.00																					
	y_2	0.88	1.00																				
	y_3	0.81	0.93	1.00																			
Ger	y_1	0.91	0.71	0.65	1.00																		
	y_2	0.89	0.96	0.88	0.77	1.00																	
	y_3	0.84	0.96	0.94	0.67	0.95	1.00																
Gre	y_1	0.28	0.55	0.49	-0.07	0.45	0.59	1.00															
	y_2	-0.02	0.16	0.04	-0.35	0.05	0.13	0.68	1.00														
	y_3	0.20	0.41	0.40	-0.11	0.30	0.50	0.78	0.74	1.00													
Ire	y_1	0.80	0.86	0.69	0.65	0.88	0.84	0.56	0.34	0.45	1.00												
	y_2	0.45	0.62	0.40	0.23	0.63	0.52	0.54	0.53	0.25	0.77	1.00											
	y_3	0.55	0.77	0.76	0.24	0.67	0.83	0.86	0.56	0.86	0.69	0.47	1.00										
Ita	y_1	0.59	0.73	0.67	0.24	0.65	0.76	0.83	0.65	0.80	0.71	0.57	0.92	1.00									
	y_2	-0.29	-0.12	-0.23	-0.60	-0.25	-0.20	0.48	0.87	0.43	-0.02	0.38	0.25	0.37	1.00								
	y_3	-0.30	-0.09	-0.08	-0.61	-0.25	-0.04	0.65	0.81	0.77	-0.06	0.05	0.51	0.50	0.79	1.00							
Net	y_1	0.95	0.76	0.70	0.99	0.81	0.72	0.03	-0.25	-0.02	0.71	0.30	0.32	0.35	-0.51	-0.53	1.00						
	y_2	0.85	0.92	0.78	0.70	0.96	0.87	0.45	0.19	0.26	0.91	0.78	0.60	0.64	-0.08	-0.23	0.75	1.00					
	y_3	-0.20	0.07	0.05	-0.48	-0.08	0.03	0.62	0.62	0.36	0.03	0.38	0.41	0.40	0.76	0.68	-0.41	0.00	1.00				
Spa	y_1	0.56	0.68	0.65	0.28	0.60	0.76	0.79	0.55	0.86	0.71	0.38	0.93	0.92	0.18	0.49	0.38	0.54	0.26	1.00			
	y_2	0.34	0.54	0.42	-0.02	0.44	0.50	0.79	0.87	0.71	0.61	0.72	0.76	0.83	0.71	0.62	0.10	0.55	0.64	0.70	1.00		
	y_3	0.65	0.78	0.75	0.41	0.70	0.84	0.69	0.50	0.85	0.75	0.41	0.93	0.88	0.10	0.38	0.49	0.63	0.13	0.93	0.67	1.00	
Eur	S_t	0.75	0.80	0.61	0.66	0.86	0.72	0.31	0.20	0.16	0.90	0.83	0.44	0.52	-0.07	-0.31	0.70	0.94	-0.09	0.43	0.48	0.53	1.00

Table 20: Correlation coefficients between the idiosyncratic processes ($y^i_{t,\{1,2,3\}}$) and the systematic process (S_t): post-crisis period (2010-2014)