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Abstract

We examine the role of *per capita* income in closed and open economy models of monopolistic competition based on non-homothetic directly additive preferences *à la* Dixit-Stiglitz, as in Krugman (1979). In a closed economy with free entry income is always neutral on markups and firm size. In a two-country trade model without transport costs, markups are higher in the country with higher income if the elasticity of substitution is decreasing in consumption. Pricing to market also emerges with transport costs.

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There is a consistent evidence that markups are higher in richer countries: see e.g. Alexandria and Kaboski (2011) and Simonovska (2013). The classic motivation for this form of *pricing to market* (Krugman, 1986) is that richer countries have a less elastic demand, so as to require higher prices under imperfect competition. However, the workhorse model of the new trade theory, the monopolistic competition model of Dixit and Stiglitz (D-S, 1977) *cum* CES preferences, implies constant markups and an endogenous number of firms that is proportional to both the number of consumers and their *per capita* income. Therefore, it cannot account for the variability of markups in both a closed and an open economy (see Krugman, 1980). Krugman (1979) has used the general version of the D-S model with directly additive (non-homothetic) preferences as a source of variable markups, but its implications for the impact of income differences between countries have been usually neglected in the literature,[2] which has focused on the role of the number of consumers, the so-called "market size" (see Zhelobodko *et al.*, 2012).

In this note we first formalize a result which is rarely noticed in the literature: even the general closed-economy D-S model with non-homothetic preferences generates, under free entry, a neutrality of the market structure with respect to income. Namely, markups and firm size are unaffected by changes in consumer income/expenditure. For this reason, alternative models of monopolistic competition are needed to explain the variability of markups over the business cycle.[3] For instance, to break the neutrality of income, Bertoletti and Etro (2013) suggest to adopt preferences represented by additively separable *indirect* utility functions: together with those directly additive, they belong to the class of preferences satisfying Generalized Additive Separability (Pollak, 1972), for which the demand of each good depends on its own price and on a price index.

In a trade model *à la* Krugman (1979), however, allowing firms to price discriminate between countries restores a role for pricing to market. We extend the Krugman model to different income levels between countries to show that, without transport costs, markups are higher in the country with higher per capita income if the elasticity of substitution is decreasing in consumption, generating pricing to market. More involved forms of pricing to market emerge in the presence of transport costs.

Other papers have recently dealt with multi-country trade models and D-S preferences. Behrens and Murata (2012) analyze (negative) exponential preferences (see Bertoletti, 2006) but assuming away the possibility of price discrimination. Bertoletti and Epifani (2012) and Zhelobodko *et al.* (2012) consider the general case but with identical countries. Finally, Simonovska (2013) presents a multicountry trade model with heterogeneous firms and obtains prices increasing in income for a special example of the D-S preferences but, as far as we know, ours is the first characterization of the general results for closed and open economies.

The article is organized as follows. In Section 1 we present the closed econ-

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[2]Krugman (1979) assumed countries with equal income/productivity.
[3]Notice that the neutrality of income may hold in other setups as well: see e.g. Tarasov (2013).

omy model and the neutrality result. In Section 2 we analyze the two country model. In Section 3 we conclude.

1 The Closed-economy Model

Consider a closed economy with L identical agents consuming a mass of n goods under the following symmetric and additively separable direct utility function:

$$U = \int_0^n u(x_j)\, dj, \tag{1}$$

where x_j is the consumption of variety j and we assume $u(0) = 0$, $u'(x) > 0 > u''(x)$ for any $x > 0$. Each consumer maximizes utility under a budget constraint $E = \int_0^n p_j x_j dj$, where $E > 0$ is the income of each agent to be spent in a continuum of differentiated varieties, and $p_j > 0$ is the price of variety j. Using the wage as the *numeraire*, E can be interpreted as the labor endowment of each agent (in efficiency units). The inverse demand function can be derived as $p_j = u'(x_j)/\lambda$, where $\lambda = \int_j u'(x_j) x_j/E$ is the marginal utility of income.

With marginal cost c and fixed cost F the profit of each firm i is:

$$\pi_i = \left[\frac{u'(x_j)E}{\int_j u'(x_j)x_j} - c \right] x_j L - F. \tag{2}$$

Defining $r(x) = u(x)x$, it is assumed that $r'(x) = u''(x)x + u'(x) > 0 > r''(x)$ to ensure positive and decreasing marginal revenue. For a given number of firms n, the first order condition for profit maximization in a symmetric equilibrium can be written as:

$$\frac{p-c}{p} = \frac{1}{\theta\left(\frac{E}{pn}\right)}, \tag{3}$$

where we use the equilibrium budget constraint condition $x = E/pn$ and define the inverse demand elasticity $\theta(x) = -u'(x)/u''(x)x$. Notice that θ is larger than unity under our assumptions and measures the elasticity of substitution between any two varieties in the case of a common individual consumption level x: see Bertoletti and Epifani (2012).[4] Notice that the optimal markup depends on E through the properties of θ. This derives from non-homotheticity: if for instance $\theta' < 0$, when (real) income increases demand becomes less elastic and each firm increases its own markup.

However, the number of firms generates a countervailing effect to the one exerted by income. The latter effect becomes crucial when the number of firms is endogenous so as to reduce the symmetric profits $\pi = (E/nx - c)\, xL - F$ to zero. In particular, the implications of assuming free entry can be summarized with the following equilibrium system:

$$\frac{p^e - c}{p^e} = \frac{1}{\theta(x^e)}, \quad x^e = F\frac{\theta(x^e) - 1}{cL} \quad \text{and} \quad n^e = \frac{EL}{F\theta(x^e)}. \tag{4}$$

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[4] According to the terminology of Zhelobodko *et al.* (2012), $1/\theta(x)$ is the "relative love for variety".

Here both the equilibrium price and the quantity consumed of each good do depend on the population L, which in turn affects non-linearly the number of firms: the exact impact depends on the sign of $\theta'(x)$. Notice that the first two relations determine price and firm size independently from income E. This neutrality result can be summarized as follows:

Proposition 1. *Under directly additive preferences, a closed economy with monopolistic competition and endogenous entry generates prices and firm size independent from per capita income, and a number of firms increasing linearly with it.*

As in the standard D-S model *cum* CES preferences, free entry eliminates any impact of income on prices, and markups cannot be affected by changes in consumer spending over the business cycle. This happens because the direct impact of income on the demand elasticity is neutralized by the opposite impact exerted by the endogenous increase in the number of firms. Namely, a richer economy expands the set of consumed varieties without affecting the equilibrium elasticity of demand.

As a first example, consider the negative exponential utility $u(x) = 1 - \exp(-ax)$ with $a > 0$ (Bertoletti, 2006). Since $\theta(x) = 1/ax$, the equilibrium price can be derived as:

$$p^e = \frac{c}{1 - \frac{2}{\Psi}\left(\sqrt{1+\Psi} - 1\right)}, \tag{5}$$

where $\Psi \equiv 4cL/aF$. The consumption of each consumer is:

$$x^e = \frac{F\left(\sqrt{1+\Psi} - 1\right)}{2cL} \tag{6}$$

and both price and firm size are independent from income. Finally, the number of firms is:

$$n^e = \frac{aE\left(\sqrt{1+\Psi} - 1\right)}{2c} \tag{7}$$

As a second example, consider the subutility $u(x) = (x+b)^{1-1/\vartheta} - b^{1-1/\vartheta}$ with $b > 0$ and $\vartheta > 1$. Notice that this corresponds to a symmetric version of the so-called Stone-Geary utility function (see Klein and Rubin, 1947-48, Geary, 1949-50 and Stone, 1954). Since $\theta(x) = \vartheta(1+b/x)$, we obtain the following price:

$$p^e = \frac{\vartheta c\left(b + \delta_1 + \sqrt{\delta_1^2 + \delta_2}\right)}{\vartheta b + (\vartheta - 1)\left[\delta_1 + \sqrt{\delta_1^2 + \delta_2}\right]},$$

where $\delta_1 = (\vartheta - 1)F/(2cL)$ and $\delta_2 = \vartheta bF/(cL)$, with consumption per capita:

$$x^e = \delta_1 + \sqrt{\delta_1^2 + \delta_2},$$

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where income is neutral. The number of firms is:

$$n^e = \frac{E\left(\delta_3 + \sqrt{\delta_3^2 + \delta_4}\right)}{b + \delta_1 + \sqrt{\delta_1^2 + \delta_2}},$$

where $\delta_3 = (\vartheta - 1)/(2c\vartheta)$ and $\delta_4 = bL/(cF\vartheta)$. A simple case emerges for $\vartheta \to 1$ (that implies $\delta_1 = 0 = \delta_3$ and $\delta_2 = bF/(cL) = (F/L)^2 \delta_4$), which corresponds to the particular example $u(x) = \ln(x + b) - \ln b$ examined by Simonovska (2013). In such a case we obtain:

$$p^e = c + \sqrt{\frac{cF}{bL}}, \quad x^e = \sqrt{\frac{bF}{cL}} \quad \text{and} \quad n^e = \frac{EL}{F + \sqrt{bcLF}}. \tag{8}$$

These examples exhibit an elasticity of substitution decreasing with the size of consumption, therefore the price decreases and the number of firms increases less than proportionally with respect to the number of consumers (see Zhelobodko *et al.*, 2012, and Bertoletti and Epifani, 2012).

2 The Two-country Trade Model

Following Krugman (1979, 1980) we now consider trade between two countries sharing the same non-homothetic preferences (1) and the same technology, as embedded into the costs c and F, which are given in labor units, but possibly with different numbers of consumers L and L^*. Differently from Krugman, we allow income to differ across countries.

In particular, we assume that the labor endowments of consumers in the Home and Foreign countries are respectively e and e^*,[5] so that income levels are $E = we$ and $E^* = w^*e^*$. Accordingly, the marginal and fixed costs in the domestic and foreign countries are respectively wc and wF and w^*c and w^*F. Let us assume that to export each firm bears an "iceberg" cost $d \geq 1$, and, as standard, let us rule out the possibility of parallel imports aimed at arbitraging away price differentials (i.e., international markets are segmented). Consider the profit of a firm i, based in the Home country, which has to choose its domestic sales x_i and exports x_{zi}:

$$\pi_i = \left[\frac{u'(x_i) x_i L}{\lambda} - wcx_i L\right] + \left[\frac{u'(x_{zi}) x_{zi} L^*}{\lambda^*} - wcdx_{zi}L^*\right] - wF. \tag{9}$$

A symmetric expression holds for a Foreign firm j, choosing x_j^* and x_{zj}^*, while in a symmetric equilibrium (across firms based in the same country) we have:

$$\lambda = \frac{nu'(x) x + n^* u'(x_z^*) x_z^*}{E} \quad \text{and} \quad \lambda^* = \frac{nu'(x_z) x_z + n^* u'(x^*) x^*}{E^*}.$$

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[5] Of course, one may interpret this as a difference in labor productivity.

Given the optimal markup $m(x) = \theta(x) / [\theta(x) - 1]$ (for which it holds that $m'x + m > 1$), we obtain the following equilibrium pricing rules:

$$p = m(x)wc, \qquad p_z = m(x_z)wdc, \tag{10}$$
$$p^* = m(x^*)w^*c, \quad p_z^* = m(x_z^*)w^*dc, \tag{11}$$

where $m' > 0$ iff $\theta' < 0$. The endogenous entry condition for the firms of the Home country reads as:

$$[m(x) - 1]\, cxL + [m(x_z) - 1]\, cdx_z L^* = F, \tag{12}$$

and a corresponding one holds for the firms of the Foreign country:

$$[m(x^*) - 1]\, cx^*L^* + [m(x_z^*) - 1]\, cdx_z^*L = F. \tag{13}$$

We close the model with the budget constraints:

$$E = nm(x)wcx + n^*m(x_z^*)w^*dcx_z^*, \tag{14}$$

$$E^* = n^*m(x^*)w^*cx^* + nm(x_z)wdcx_z, \tag{15}$$

and the resource constraints:

$$eL = n\left[c\left(xL + dx_z L^*\right) + F\right], \tag{16}$$
$$e^*L^* = n^*\left[c\left(x^*L^* + x_z^*L\right) + F\right], \tag{17}$$

and normalize the home wage to $w = 1$.

2.1 Costless trade

Let us consider the case of costless trade ($d = 1$). Each firm (independent from the country where it is based) faces the same demand conditions, and this induces wage equalization $w = w^* = 1$ (otherwise the zero-profit condition of free entry could not be satisfied in both countries). This implies that all firms sell the same quantity at the same price within the same country, i.e., $x = x_z^*$ with $p = p_z^*$, and $x_z = x^*$ with $p_z = p^*$. Moreover, we have $xL + x_z L^* = x^*L^* + x_z^*L$, that is the firm size is the same across countries. Notice that the budget constraints (14) and (15) imply $E = (n + n^*)\, m(x)xc$ and $E^* = (n + n^*)\, m(x^*)x^*c$, therefore we have:

$$\frac{E}{E^*} = \frac{m(x)x}{m(x^*)x^*}. \tag{18}$$

Since $m(x)\, x$ is an increasing function, when $E > E^*$ (18) implies $x > x^*$. This requires that $p = p_z^* > p_z = p^*$ if (everywhere) $\theta' < 0$, which is a case of pricing to market where each firm sells its own variety at a higher price in the country with higher per capita income, in line with the well known empirical evidence in trade.

To complete the analysis of the equilibrium, notice that, given the same production across all firms, the resource constraints (16) and (17) imply:

$$\frac{n}{n^*} = \frac{EL}{E^*L^*},\tag{19}$$

i.e., the distribution of firms is equal to the distribution of total income. The quantities x and x^* can be derived by the following conditions (where we use the zero profit conditions and the budget constraints):

$$[m(x) - 1]\, xL + [m(x^*) - 1]\, x^*L^* = F/c,\tag{20}$$

$$Em(x^*)x^* - E^*m(x)x = 0.\tag{21}$$

Finally, using again the budget constraints, we obtain the total number of firms:[6]

$$n + n^* = \frac{EL + E^*L^*}{c\,(xL + x^*L^*) + F}.\tag{22}$$

Summing up, we have:

Proposition 2. *Under directly additive preferences, a two-country economy with costless trade, monopolistic competition and endogenous entry generates higher prices in the country with higher per capita income if the elasticity of substitution is decreasing.*

Accordingly, pricing to market emerges naturally in the Krugman (1979) model as long as price discrimination is possible. The reason relies on the fact that while all consumers spread their consumption over all varieties, richer consumers have more of each of them: if their demand is less elastic the country profit-maximizing price is higher. Accordingly, firms from the country with poorer consumers sell their goods at a higher price abroad compared to the domestic price whenever $\theta' < 0$. Of course, the opposite result occurs in the (arguably less realistic) case where $\theta' > 0$ (then goods would be sold at a lower price in the high-income country whose demand is more elastic).

We can finally look at the impact of income and population on consumption per capita in the two countries. Total differentiation of the system (20)-(21) shows that:

$$\begin{bmatrix} \frac{\partial x}{\partial L} < 0 & \frac{\partial x}{\partial E} > 0 \\ \frac{\partial x^*}{\partial L} < 0 & \frac{\partial x^*}{\partial E} < 0 \end{bmatrix}.$$

Without trade costs, a rise of population (no matters in which country) always reduces the consumption size of any variety, decreasing the corresponding markup if $\theta' < 0$ and generating gains from variety.[7] On the contrary, an increase of domestic income increases the domestic consumption of all varieties

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[6]For example, in the CES case we have:

$$x = \frac{E(\theta - 1)F}{[EL + L^*E^*]\,c} = \frac{E}{E^*}x^* \quad \text{and} \quad n + n^* = \frac{EL + E^*L^*}{\theta F}.$$

[7]A decrease in either x or x^* must, by the budget constraints, increase the total number of firms $n + n^*$.

(also of the imported ones), while decreasing the foreign consumption levels. Accordingly, if $\theta' < 0$ markups raise at home (for both domestic and foreign sales) and decrease abroad. Notice that an income increase is prosper-thy-neighbor: foreign consumers gain (even if less than the domestic consumers) from the increase in variety provision and the price decrease.

2.2 Costly trade

Finally, we can investigate what happens when there are transport costs ($d > 1$). The balanced trade condition requires:

$$nm(x_z)x_z L^* = n^* m(x_z^*)x_z^* w^* L.$$

It is easy to extend a classic result by Krugman (1980) to our set-up and verify that under CES preferences and costly trade we have $w \gtrless w^*$ if and only if $eL \gtrless e^* L^*$. However, markups remain the same and each firm applies the same price (net of transport costs) for any destination country.

The analysis of pricing to market becomes more involved when we depart from CES preferences and allow for non-homotheticity. In general, with $m' \neq 0$, wage equalitazion will not take place unless countries are identical (the case examined by Krugman, 1979, and Bertoletti and Epifani, 2012). To gain more insights, let us consider the case of two countries with the same population $L = L^*$. Then, it is intuitive that $w^* \gtreqless w$ if and only if $e^* \gtreqless e$ (wages must be higher in the more productive economy "to compensate" for the larger consumption of imports in that country if exporting prices were the same). Now, let us focus on the case of decreasing elasticity of substitution, that is $\theta' < 0$, and compare the markup of domestic goods sold abroad and at home. If the two countries have identical productivity ($e = e^*$), trade costs imply $p_z > p$ and $x_z < x$, therefore the markup on exports is smaller than the markup on domestic sales. It is only when foreign productivity and income are significantly larger than their domestic counterparts that the above result can be reversed: when a richer foreign agent consumes more than a domestic one in spite of the transport costs ($x_z > x$), then the markup on foreign sales is higher than the markup on domestic sales. Opposite results emerge under increasing elasticity of substitution ($\theta' > 0$), but in all cases firms target different markets with different prices.

3 Conclusions

We have studied monopolistic competition with non-homothetic preferences *à la* Krugman (1979). In a two-country economy without transport costs, markups are higher in the country with higher per capita income if the elasticity of substitution is decreasing in the level of consumption, generating a sort of *pricing to market*. It would be interesting to study the impact of income differences in an extended two-sector model.

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