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The role of cognitively biased imitators in a small scale agent-based financial market

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Abstract

We analyze the consequences of the presence of imitators in a financial market populated by boundedly rational speculators. We consider imitators that only look at the recent success of the available trading rules. We show that the introduction of this kind of imitators makes the results more complicated but even more realistic. In particular, under some specific circumstances, imitators may stabilize an otherwise unstable market or, at the opposite, make unstable an otherwise stable scenario.

Keywords: Cognitive biases; Bounded rationality; Financial markets; Agent-based models.

1 Introduction

Over the last few years a lot of attention has been raised on the psychology of financial markets. This is probably a consequence of the failure of the traditional approach to the study of financial markets, which is essentially based on the assumption of perfectly rational agents, cornerstone of the so-called Efficient markets hypothesis (see Fama 1965a,b). These theories dramatically failed in anticipating and explaining how financial bubbles like the dot.com bubble or the US real estate bubble originate, grow larger and then burst (see Shiller, 2005). The consequences of the explosions of such bubbles can be huge and nowadays we know that they can also influence real economy, triggering deep recessions. Bubbles and crashes are not the only stylized facts of financial markets that the mainstream approach is unable to explain in a convincing manner. The list of other prominent features of stock markets includes excess volatility, fat tails of returns' distribution together with their virtual unpredictability. We also mention volatility clustering and long memory effects among the facts to be explained.

Deviations from the perfectly rational behavior have been founded and analyzed since many years before the current economic crisis[1]. A

[1] See for instance the famous Allais paradox (Allais, 1953).

systematic study and classification of the irrationalities that plague humans' decision making started with the works of Kahneman and Tversky in the seventies (see Tversky and Kahneman (1974) for a list of the most common biases). They and their scholars proved that people who make decisions follow simple rules of thumb (called *heuristics*) that many times represent an easy way to make a good decision, but may also lead to systematic deviations (called *biases*) from what a perfectly rational agent should do. The good thing is that, given their regularity, these biases can in some sense be foreseen.

Financial traders (professional or not) decide whether to buy or sell and asset following simple heuristics, too. These rules of thumb can be subdivided into two main categories: fundamental and chartists trading rules, followed by fundamentalists and chartists (or technicians), respectively (see Frankel and Froot, 1986 and Menkhoff and Taylor, 2007 for empirical validations and Hommes, 2011 for a review of laboratory experiments). The former are convinced that, even in the short period, prices will come back to their fundamental values, so they buy undervalued assets and sell overvalued ones. They can be considered as traders looking at the long period. Chartists (or technicians) look at the time series of prices to find a clue for understanding the (near) future price movements.

An interesting strand of research consists in studying small-scale heterogeneous agent-based financial market models (HAMs henceforth) with behavioral assumptions. The pioneering work in this field is due to Day & Huang (1990) and after that the interactions between heterogeneous market participants have been developed in many directions (see Chiarella, Dieci and He, 2009; Hommes and Wagener, 2009: Lux, 2009; Westerhoff, 2009 for recent surveys). The strong point of HAMs lies in the connection between the behavioral assumptions that are supported by empirical and experimental evidence, and the small scale of the dynamical systems that explain asset price movements and the underlying mechanisms that cause them. This permits to analytically study the most of these models, making understandable the endogenous causes of particular price movements. HAMs lead to irregular endogenous price dynamics even in their deterministic version, through the nonlinearities that are introduced in the trading rules and/or in the switching mechanism. The emergence of chaotic dynamics permits to replicate stylized facts like bubbles and crashes and excess volatility. These results reduce the role played by stochastic variables that can still be useful to replicate some quantitative aspects of real time series, but are not necessary for a qualitative explanation of the most of these facts.

HAMs can also be used for taking into consideration the imitative strategies that a (sub)group of traders can adopt to obtain better performances. In this sense the works of Lux (1995, 1998) and Lux and Marchesi (1999, 2000) deserve to be mentioned. In their works there are fundamentalists and chartists. These latters use a combination of trend following and imitative strategies and can be optimistic or pessimistic. They decide which subgroup to imitate looking at what the majority is doing[2] and at expected and realized excess profits of the available strategies. A similar mechanism is used in Franke and Westerhoff (2011). Further HAMs with imitation are Bischi et al. (2006) and Foroni & Agliari (2008).

Our work is also related to the strand of literature in which financial markets are viewed as evolutionary adaptive systems, populated by boundedly rational interacting agents (see Brock & Hommes 1997, 1998, Chiarella and He, 2000, 2002, Farmer, 2002, among the others). In these models agents are allowed to switch among the different trading strategies, trying to learn the best one. The authors of these papers are interested in the final outcome of this evolutionary competition and remarkable results emerge when the fractions of agents adopting each strategy, continuously vary over time, never reaching a fixed final value.

In our model there are some agents that keep fixed their strategies, no matter what happens in the markets. They are overconfident and can be affected by the so-called *confirmation bias* (see Barber and Odean, 2002, 2006). Roughly speaking, they favor information that support their strategies and give less importance to the others. This bias explains, for instance, why even beliefs that have been heavily discredited survive in the mind of some people (Kunda, 1999). On the other hand, we also consider a fraction of traders that are not so self-confident and at each time period reconsider their strategies myopically looking at the performance of the alternatives. We consider the amount of agents adopting each strategy as a consequence of the imitative behavior and not as a cause. In this sense we do not model an *herding behavior* (in which traders follow the crowd). Our approach is even more simple because it does not require for imitators to know the number of traders that use each available strategy. They only look at their neighborhood where they can find examples of traders adopting the various strategies. When they look at the outcome of their strategies, they decide who among them should be imitated. In some context this behavior can be a good one. Especially when a best strategy really exists. Financial markets is not one of these cases because we cannot state, for instance, that the fundamentalists approach is always better than the chartists

[2]In this case we talk about *herding* rather than *imitation,* that we use in a more generic meaning.

one or the opposite, and in this case, as we will see, the role of imitators can by quite important and drastically influence price dynamics.

The paper is structured as follows. In Section 2 we build a typical HAM with fundamentalists and chartists that keep fixed their strategies. In Section 3 we introduce a third group of traders that imitate the behavioral rule of the others by using a simple rule of thumb. The consequences of the introduction of imitators in the market are studied in Section 4. Section 5 concludes the paper.

2 The benchmark model

In this Section we build a typical HAM describing a financial market where only one asset is exchanged. The market is populated by two kinds of speculators: fundamentalists and chartists.

In the spirit of Day and Huang (1990), a market maker adjusts the log of the asset price (P) according to this rule:

$$P_{t+1} = P_t + a\left(D_t^f + D_t^c\right) \tag{1}$$

where D_t^f and D_t^c represent the orders placed at time t by fundamentalists and chartists, respectively. The positive parameter a measures the intensity of the adjustment.

Fundamentalists are assumed to believe in the reversion of the asset price towards its (exogenously given) fundamental value F. As a consequence, they buy the asset when its price is below the fundamental value and sell it when it is overvalued. Their behavioral rule is the following:

$$D_t^f = f(F - P_t) \tag{2}$$

where $f > 0$ is a reaction parameter.

At the opposite, chartists optimistically interpret the signal given by a price above the fundamental. So they buy the overvalued asset because they think that the positive trend will go on, at least in the short period. Nevertheless, we introduce a nonlinearity in their trading rule (an arctangent) that permits us to take into consideration a certain degree of prudence when the difference between actual price and fundamental value becomes extremely large. Orders placed by chartists are given by:

$$D_t^c = c \cdot atn(P_t - F) \tag{3}$$

where c is a positive reaction parameter[3].

[3]Note that we would obtain the same qualitative results that are explained in the following by using $(P_t - P_{t-1})$ instead of $(P_t - F)$, so considering some sort of positive feedback investors (see De Long et al. 1990). We prefer to avoid this characterization of chartists because it would increase the dimensionality of the dynamical system explaining the price dynamics.

The price adjustment rule (1) combined with the trading rules (2) and (3), gives us a dynamic model explaining the movements of the asset price as a function of the previous period price:

$$P_{t+1} = P_t + a\left[f(F - P_t) + c \cdot atn(P_t - F)\right] \tag{4}$$

We can use the auxiliary variable $x_t = P_t - F$ representing deviation from the fundamental value, to obtain the map:

$$x_{t+1} = f(x_t) = x_t + a\left[c \cdot atn(x_t) - f x_t\right] \tag{5}$$

where "$'$" denotes the unit-time advancement operator.

2.1 Steady states and local stability

One of the fixed points of the map (5) corresponds to a price equal to the fundamental value: $x^* = 0$ (we call it *fundamental fixed point* in what follows). Nevertheless, the price may not converge to it if the it is not locally stable. To check the local stability of x^* we must use the first derivative of $f(x_t)$, i.e.:

$$f'(x_t) = 1 + a\left(\frac{c}{1 + x_t^2} - f\right) \tag{6}$$

and evaluate it at the fundamental fixed point value:

$$f'(0) = 1 + a(c - f) \tag{7}$$

The fixed point is locally stable if the value of the derivative is lower than 1 in absolute value. We can easily obtain the local stability condition in terms of the chartists' reactivity coefficient c:

$$-\frac{2}{a} + f < c < f \tag{8}$$

Condition (8) has a straight interpretation: the asset price converges to the fundamental value provided that fundamentalists are more reactive (or trade more aggressively) than chartists, but not too much[4].

The fulfillment of condition (8) only ensures the *local* stability of the fundamental fixed point. In other words, we know that starting with an initial value of the price that is close enough to the fundamental value, then the price will converge to it. But how close the price should be

[4] This second case may appear less easy to interpret but the explanation is straight: when fundamentalists strongly dominate the market the price oscillations are huge and overvalued and undervalued prices alternate. From the mathematical point of view this is a so-called flip (or period-doubling) bifurcation.

to the fundamental value? Note that this is a quite relevant question because another way of formulating the same question is the following: if a shock hits the market, are we sure that the price will come back to the fundamental value? The larger is the interval made up by initial conditions leading to the fundamental value, the more robust is the system that only requires some settlement periods for reabsorbing shocks (see Fig.1a).

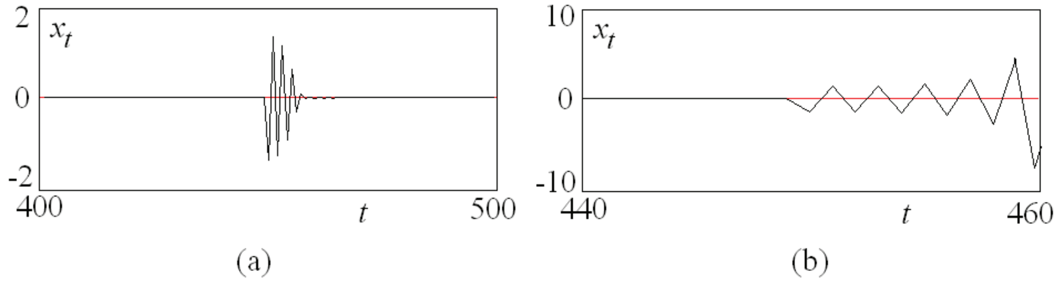


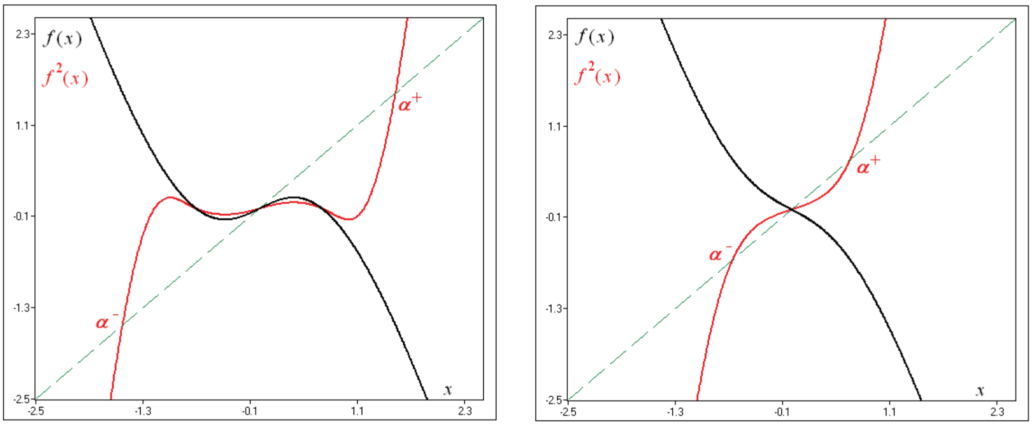
Figure 1. Timeplots obtained by using the parameters' values: $a = 3$, $c = 0.8$ and $f = 1.2$. At time period $t = 450$ an additive shock is introduced. The shock hitting the system in panel b is larger than the one of panel a.

The set of initial conditions leading to a fixed point (x^*) is usually called its *basin of attraction* $(\beta(x^*))$ in the dynamical systems' literature. In our case the basin of attraction of the fundamental value is given by:

$$\beta(x^*) =]\alpha^-, \alpha^+[\tag{9}$$

where α^- and α^+ are the points of an (unstable) cycle of period 2 that we can find by looking at the second iterate $f^2(x)$ (see Fig.2).

By keeping fixed the value of f and by varying the value of c inside the range of local stability of the fundamental fixed point (8), we find that the larger is c the larger is $\beta(x^*)$. This means that when fundamentalists are much more reactive than chartists, even if condition (8) is not violated, only initial conditions quite close to the fundamental value lead to it.



(a) (b)

Figure 2: first and second iterates of the map $f(x)$. In (a) we have that $a = 3$, $f = 1.64$ and $c = 1.5$. The fixed points of the second iterate (the red curve) bound the basin of attraction of the fixed point. This set is smaller when c is reduced to 1.1 as in (b).

In other terms, even starting from a scenario in which price is equal to the fundamental value, a small shock may have heavy consequences on the price dynamics. And what happens to initial conditions outside $\beta(x^*)$? In these cases the strong dominance of fundamentalists combined with a sufficiently large mispricing drives fundamentalists to overreact to the market signal and the price diverges alternating high and low values, each time more distant from the fundamental one (Fig. 1b).

Let us focus on what happens when chartists are more aggressive than fundamentalists (i.e. $c > f$) by looking at the following bifurcation diagram, obtained by keeping fixed $a = 3$ and $f = 1.2$ and varying the values of c between 0.5 and 4:

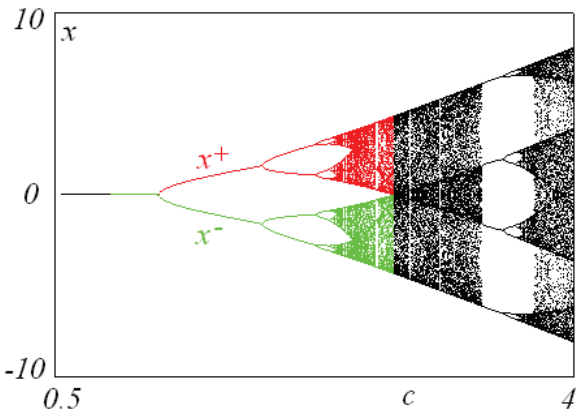


Figure 3. Bifurcation diagram.The red diagram is obtained by using an initial condition $x_0 = 0.1$ while the green one is obtained starting from

$x_0 = -0.1$. The black portion of the diagram is not influenced by the initial condition chosen.

Until the value of the chartists' reactivity is lower than the fundamentalists' one, the asset price converges to the locally stable fundamental fixed point. At $c = f$ a *pitchfork bifurcation* occurs and two further steady states are created. For values of c not excessively higher than f the fundamental state $x^* = 0$ is locally unstable and price converges to one of the other two steady states, depending on the initial condition. Note that one steady state (x^+) corresponds to an overvalued price while the other one (x^-) to a situation where the price is undervalued. By further increasing the reactivity of chartists, the two coexisting fixed points also become locally unstable via a simultaneous flip (or *period doubling*) bifurcation, giving rise to coexisting cycles of period 2, 4, 8 and so on. After the typical cascade of period-doubling bifurcations, two coexistent chaotic attractors arise. When the system converges to one of them, the price erratically moves in it, keep remaining in the bull or in the bear region. In the rightmost part of the diagram, for high values of c, the two attractors become one (a so-called homoclinic bifurcation of x^* occurs) and the price oscillate between the bull and the bear regions in an almost unpredictably way, no matter the initial condition. In figure 4 we have a typical timeplot obtained by using $c = 4.528$:

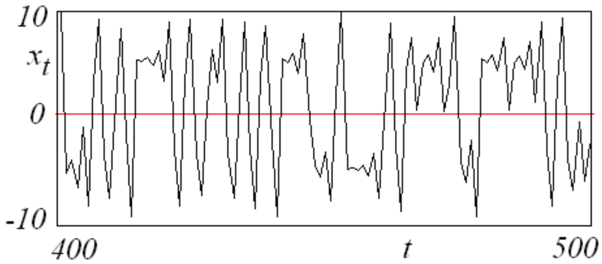


Figure 4: One hundred consecutive values of the price when the attractor is chaotic and covers both bull and bear regions.

This is the most interesting scenario. In fact the dynamics represented in Figure 4 are hardly distinguishable from those obtained by using a more sophisticated stochastic model. This simple deterministic model is able to qualitatively replicate some important facts of the financial markets like bubbles and crashes and excess volatility. In this scenario, periods in which the price get closed to the fundamental value, alternate with periods in which it moves away from it. This means that in some periods the fundamentalists trading rule seems to be better than the one used by chartists, but this situation never persists forever. It is (almost) impossible to identify a rule that would permit to systematically *beat the market*.

The following table summarizes the main features of the benchmark model:

Parameters'values	Stability of fund. fixed point	Effects of a price shock
$-\frac{2}{a}+f < c < f$	- locally stable - not globally stable	- small $\Rightarrow$ reabsorbed - big $\Rightarrow$ divergence
$c > f$	- locally unstable - attractors are periodic or chaotic, i.e. price constantly fluctuates possibility of realistic bull and bear dynamics	- reabsorbed

To this benchmark model we will add in the next Section a third group of traders, the imitators, and we will analyze the consequences of their presence.

3 The model with imitators

If we want to consider a more realistic asset market, we must take into consideration that it is not only populated by traders that do not never call into question their beliefs about the future price's movements. There also exist traders that change their behavioral rules. Among them there is someone that looks at the beliefs of the other traders, trying to learn the best strategy. This means to check who has been right between chartists and fundamentalists, that is if it meaningful to believe that the price will suddenly approach the fundamental value or not. We know that a rational behavior consists in waiting a long time and comparing a long series of daily return, in order to be relatively sure that the results are caused by the relative values of the trading strategies and not by chance. This is a consequence of the so called *law of great numbers* that permits to the best strategy (if there is one) to asymptotically emerge. Unfortunately, there exists a huge amount of experimental evidence and data that people believe instead in the so-called *law of small numbers*[5]. People believing in the "false" law of small numbers, according to definition of Shefrin (2001) "attribute negative serial correlation to an identical and independently distributed stochastic process". In other words, a sample, especially if small, is erroneously considered highly representative of the population. For our purposes this means that after some periods in which the asset price has moved closer and closer to the fundamental value, imitators start thinking that the fundamentalists' belief

[5] See Tversky and Kahneman (1971). To be precise, this false belief is a consequence of the so-called *representativeness heuristic* (Tversky and Kahneman, 1974) according to which people evaluate the probability of whether A originated from process B by the degree to which they resamble each other.

is the best one and decide to behave like them. The opposite when the price has moved away from the fundamental value.

Besides the fact that this false law may lead to erroneous evaluation even if a best strategy really exists, we will show that it may have heavy consequences when a best strategy does not exist, like in the case of the chaotic motion of price showed in the previous section.

Considering how people are influenced by the actions of the others is not new in the literature. Banerjee (1992) and Bikhchandani et al. (1992) show that in markets with asymmetric information where decisions are taken sequentially, it is possible to create information cascades that sometimes lead to the herding of wrong behaviors. Orléan (1995) removes the hypothesis of sequential decisions in a Bayesian setting. These are all examples of *rational* herding, in the sense that from the point of view of the single decision maker, following the signal given by the actions of the others can be the right thing to do, even if it can lead to undesirable consequences at the aggregate level. In HAMs literature of financial markets, tho most important works on imitation are probably those of Lux (1995, 1998) and Lux and Marchesi (1999, 2000) where agents are influenced by the actual price trend and also by the opinion of the majority.

Our model is different from those predecessors in two ways. First of all, we take into consideration an imitative behavior that is not necessarily an herding behavior. In fact, imitators do not want to *follow the crowd,* or the majority, simply because they are unable to survey the opinion of all the traders. Moreover, when we talk about money, it is possible that it is not so relevant to belong to the majority if the majority is going to lose money. We build a model in which there is no majority between the fundamentalists and the chartists behavioral rules before the decision of imitators. The behavior of imitators create a majority that in this sense is a consequence and not a cause of the imitation[6]. The second difference with respect of the existing literature is the simplicity of the decisional mechanism adopted by imitators. As we have seen, a huge amount of evidence exists in support of the idea that people follow simple rule of thumbs for making decisions. We try to keep their behavioral rule as simple as possible, obtaining the same results, from a qualitative point of view, of more complicated models.

3.1 The complete model

Let us now introduce a third kind of traders to the benchmark model analyzed in the previous section. As we have seen, we call them *imitators*

[6] The pivotal role of imitators in our model resembles the role of undecided voters in elections when the other parties are not able to get majority without them.

because they, at each time period, decide which of the other two groups to imitate. In order to take into consideration the relative number of imitators, we also explicitly introduce three parameters, representing the amount of traders belonging to each group: n_c, n_f and n_i denoting the number of chartists, fundamentalists and imitators, respectively.

The market maker rule takes now the following form:

$$P_{t+1} = P_t + a\left(n_f D_t^f + n_c D_t^c + n_i D_t^i\right) \tag{10}$$

where D_t^i denotes the imitators'orders.

Considering that we are interesting in the role played by imitators, we can normalize the values of n_f and n_c to 1. This means that fundamentalists are as many as chartists. A value of n_i=0.5 implies that imitators are a half with respect to chartists or fundamentalists (and a fifth of the total number of traders), while a value of $n_i = 2$ implies that the number of imitators is twice the number of fundamentalists or chartists (and a half of the total number of traders), and so on. As a consequence, imitators cannot choose looking at what the majority is doing because the number of fundamentalists and chartists is the same. They will create a majority with their decision. This majority can be temporary because they could change their minds in the future.

The behavioral rules of chartists and fundamentalists are the same of the benchmark model, expressed in equations (2) and (3), respectively.

We need to specify now the behavioral rule of imitators. We assume that they use a very simple heuristic: at each time t they look at the current price P_t and at its previous value P_{t-1}. If P_t is closer than P_{t-1} to the fundamental value, than they conclude that the fundamentalists' strategy has been successful and they imitate fundamentalists in time $t+1$. The opposite is true if the distance between the price and the fundamental is grown up in the last period[7].

Remembering that we have already introduced an auxiliary (x_t) that measure the distance between the current price and the fundamental value, the behavioral rule of imitators is the following:

$$D_t^i = \begin{cases} ix_t & \text{if} \quad |x_t| > |x_{t-1}| \\ -ix_t & \text{if} \quad |x_t| < |x_{t-1}| \end{cases} \tag{11}$$

where the positive parameter i measures the reactivity of imitators

[7]This kind of modeling could also represent the net effect of the decisions of subgroups of imitators that look backward and that differ for the number of past periods they consider. The net effect of each new data is that some group will switch towards the belief they support.

to the market signal[8].

By combining behavioral rules (2), (3) and (11) with the market maker rule (10), we obtain the dynamical system regulating how the asset price evolves:

$$x_{t+1} = \begin{cases} x_t + a\left[c \cdot atn(x_t) - (f - n_i i)x_t\right] & \text{if} \quad |x_t| > |x_{t-1}| \\ x_t + a\left[c \cdot atn(x_t) - (f + n_i i)x_t\right] & \text{if} \quad |x_t| < |x_{t-1}| \end{cases} \tag{12}$$

that is a system of second-order difference equations. By introducing the auxiliary variable $y_t = x_{t+1}$ we have the equivalent system of first-order difference equations:

$$T: \begin{cases} x_{t+1} = \begin{cases} x_t + a\left[c \cdot atn(x_t) - (f - n_i i)x_t\right] & \text{if} \quad |x_t| > |y_t| \\ x_t + a\left[c \cdot atn(x_t) - (f + n_i i)x_t\right] & \text{if} \quad |x_t| < |y_t| \end{cases} \\ y_{t+1} = x_t \end{cases} \tag{13}$$

Before going on with the analysis of the consequences of the introduction of imitators in the market, let us subdivide the phase plane (x_t, y_t) into two sub-regions, according to the behavior of imitators. We have this situation:

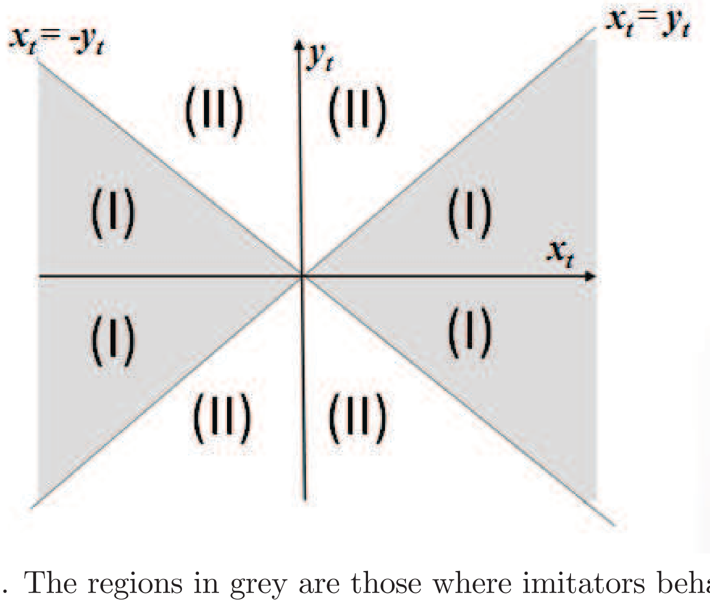


Figure 5. The regions in grey are those where imitators behave like fundamentalists. In the white regions they interpret the market signals as chartists do.

[8] We use in both cases a linear trading rule for imitators in order to avoid introducing a further nonlinearity in the model. By using the arcotangent function when imitators behave like chartists we would obtain the same results we will show in the rest of the paper.

The grey regions (I) of the phase plane represent situations in which the last value of the price is more distant from the fundamental value with respect to the previous price value, so imitators will follow chartists. In the white areas (II), the opposite is true so imitators interpret the market signal as they were fundamentalists, so they are optimistic when price is low (and buy the asset) while they are pessimistic when the price is high (and sell the asset).

We can look at the dynamical system (13) as the combination of two subsystems:

$$\begin{aligned} F_{(I)} &: \begin{cases} x_{t+1} = x_t + a\left[c \cdot atn(x_t) - (f - n_i i)x_t\right] \\ y_{t+1} = x_t \end{cases} \\ F_{(II)} &: \begin{cases} x_{t+1} = x_t + a\left[c \cdot atn(x_t) - (f + n_i i)x_t\right] \\ y_{t+1} = x_t \end{cases} \end{aligned} \tag{14}$$

where system $F_{(I)}$ governs the dynamics in the region (I) of the phase plane while system $F_{(II)}$ is active in the other subregion of the phase plane. Obviously, the dynamics may switch from on region to the other.

In the next Section we analyze what happens to the local stability properties of the fundamental steady state as a consequence of the introduction of imitators.

4 Imitators and local stability of the fundamental steady state

We have seen in the benchmark model that the fundamental fixed point becomes locally unstable via pitchfork bifurcation when chartists are more aggressive than fundamentalists. How does the introduction of imitators affect this result?

In this case the fundamental fixed point belongs to the border that separates region I and II. The stability conditions in the two regions become:

$$SC_{(I)} : f > c + in_i \;\text{ and }\; SC_{(II)} : f + in_i > c, \tag{15}$$

respectively.

As a consequence, we can distinguish among four scenarios:

a) $f > c + in_i > c$: fundamentalists are much more reactive than chartists and imitators are a few and/or have a low reactivity (*strong fundamentalists dominance scenario*);

b) $c+in_i > f > c$: fundamentalists are more aggressive than chartists but if imitators behave like chartists they can be predominant (*weak fundamentalists dominance scenario*);

c) $f + in_i > c > f$: chartists dominates the market but if imitators and fundamentalists behave similarly they are more reactive than chartists (*weak chartists dominance scenario*);

d) $c > f + in_i > f$: chartists are much more reactive than fundamentalists and imitators are a few and/or have a low reactivity (*strong chartists dominance scenario*).

The local stability properties of the fundamental fixed point in each scenario are summarized in the following table.

	Region I	*Region II*
sfds	stable	stable
wfds	unstable	stable
wcds	stable	unstable
scds	unstable	unstable

Table 2: Scenarios and local stability of the fundamental fixed point

In what follows we separately analyze these four scenarios.

4.1 The strong fundamentalists dominance scenario

The first case we consider is also the easiest to analyze. In fact, in both regions (I) and (II) the fundamental steady state is locally stable. The stability conditions (15) are both fulfilled. As a consequence only the initial condition can belong to the region (I) of the phase plane because fundamentalists are more reactive than chartists and imitators. So the price becomes closer to its fundamental value (that is the system passes to and remains in region (II)) and imitators immediately start behaving like fundamentalists. Only a shock that moves the price in the opposite direction with respect to its fundamental value can make imitators change their minds and make the system come back to region (I), but only for the length of the shock, because the price will start again to approach the fundamental value as soon as the shock is finished.

We must also note that there are two main differences with respect to the corresponding benchmark case (i.e. with $n_i = 0$). First of all, in this case the speed of convergence is faster that the one that we would see without imitators. This is obvious because imitators in this scenario can be considered as additional fundamentalists, so we would obtain the same effect by increasing the value of f in the benchmark model. The second difference lies in the size of the fundamental fixed point's basin of attraction. With imitators behaving as fundamentalists this basin is reduced, and in fact we have seen in the benchmark model that the larger is the value of f the smaller is the set of initial price values leading to the fundamental fixed point. This means that some shocks that were reabsorbed in the benchmark model could not be reabsorbed now.

4.2 The weak fundamentalists dominance scenario

This case is more interesting than the previous one, because imitators can make the difference in the dynamics of the asset price.

Let us start by considering an initial condition belonging to the subregion (II) of the phase plane. In particular it belongs to the basin of attraction of the fundamental fixed point. This means that the current price is closest to the fundamental value than its previous period value. In this case imitators decide to believe, as fundamentalists do, that the price will converge towards its fundamental value and they behave accordingly. Considering the stability condition for the subregion (II) (see eq. 15) holds and price moves closer to the fundamental value. In other words the system stays in the subregion (II) of the phase plane, and so on for the next periods. But what would happen as a consequence of a shock that move the asset price away from the fundamental value? The orbit, after the shock, moves from the subregion (II) to the subregion (I) where the stability condition $SC_{(I)}$ is violated, in fact $c + in_i > f$. The shock makes the imitators change their mind and they start following the chartists' trading rule, giving them the additional influence that permits to obtain complicated price dynamics. We have seen in the benchmark model that when chartists are more reactive than fundamentalists, periods in which the asset price goes away from the fundamental value alternate with periods in which it moves closer it. So, we should expect that sooner or later imitators will behave again as fundamentalists, bringing back stability to the system. That is true provided that when fundamentalists prevail they system is in the basin of attraction of the fundamental fixed point. Otherwise, the overreaction of fundamentalists lead the price to move away from the fundamental value and imitators to switch to the chartists' rule. This is confirmed by the following timeplot:

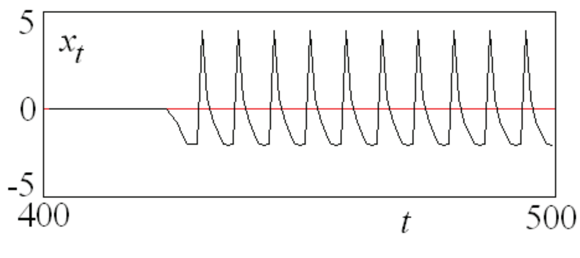


Figure 6: timeplot obtained by keeping fixed $a = 3$; $c = 1.1$; $f = 1.2$, $i_m = 0.6$ and $n_i = 0.9$. Theshock is additive and introduced at iteration 425.

where we introduce a (positive) additive shock to the price after which price movements are characterized by fluctuations.

The same happens starting directly with an initial condition in the subregion (II) of the phase plane but not belonging to the basin of attraction of the fundamental fixed point. In a case like the one represented in Fig. 6 a paradoxical situation seems to occur. In fact, a price value below the fundamental fixed point is followed by an overvalued price, whose deviation from the fundamental fixed point is increased. According to the imitators' behavioral rule, they decide to behave like chartists, even if the fundamentalists' strategy has been successful. Our hypothesis is that imitators look at the beliefs of the other groups and not at their gains. Fundamentalists take their decisions thinking that the price will converge to the fundamental fixed point, so their successful, in this case, is not considered a signal of the accuracy of their belief.

The overreaction of fundamentalists is more probable when the number of imitators is not negligible. The is confirmed by the following bifurcation diagram:

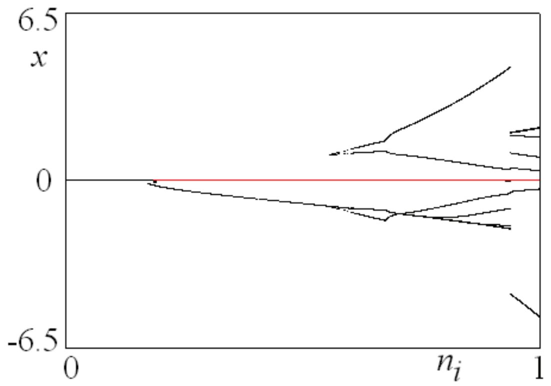


Figure 7: Bifurcation diagram obtained keeping fixed $a = 3$; $c = 1.1$; $f = 1.2$ and $i_m = 0.6$. The relative number of imitators varies between 0 and 1.

We can see that if the number of imitators is high enough, the fundamental steady state could not be reached and the complexity of the dynamics increases with the number of imitators.

So imitators play a key role in this scenario and if we think at more frequent shocks we can easily imagine how complicated the dynamics may appear.

4.3 The weak chartists' dominance scenario

This case is specular to the previous one and share with it the importance of the initial condition. Starting from Region II, that is from a value of the asset price closer to its fundamental value than the former period price (or immediately after a shock that moved the price towards its fundamental value), the system may start to converge to the fundamental value, despite the fact that without imitators we would see convergence to an attractor different from the fundamental steady state. The following bifurcation diagram, obtained by using an initial condition in Region II can clarify this situation:

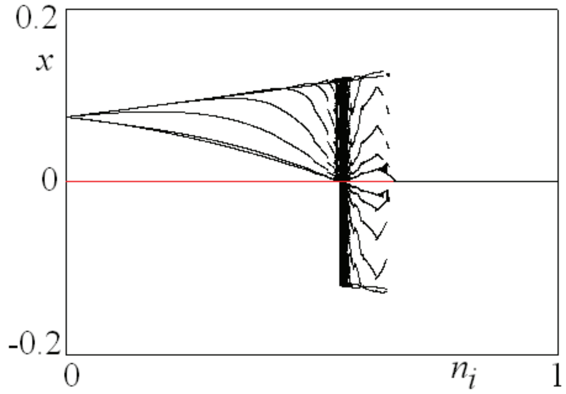


Figure 8: Bifurcation diagram obtained by keeping fixed $a = 3$, $f = 1.2$, $c = 1.4$, $i = 0.3$. The share of imitators n_i varies between 0 and 1.

As can be seen, until imitators are too few to compete with chartists, they contribute in originating dynamics that do not convergence to the fundamental value. The initial condition (or the shock) becomes even more relevant when imitators are numerous enough the make the price change direction when they opt for imitating the fundamentalists' trading rule. Stabilizing forces are now dominating with respect to the destabilizing role played by chartists and price starts moving towards its fundamental value, as can be seen in the right part of Figure 8, the one representing this third scenario. This dominance of stabilizing forces is strong because even if a new price shock happens, moving away the price from the fundamental value, we know that sooner or later the price will move again towards the fundamental value and imitators definitely behave like fundamentalists. Unless the system would not be outside the basin of attraction of the fundamental fixed point (as we have seen in the previous subcase)

The role of imitators is extremely important here because their are pivotal: when they believe that price will go close to the fundamental value, it actually does.

The left part of figure 8 is also representative of what would happen starting from an initial condition in region (I). Imitators follow the trading rule of chartists and the more they are the more complicated the price dynamics appears. We delve into this point analyzing the last scenario.

4.4 The strong chartists' dominance scenario

In this case, the asset price does not never converge to its fundamental value. Stability conditions (15) are both violated. Chartists dominate the market so strongly that even a shock that moves the price towards the fundamental value has not any long period consequence. After the first period in which imitators follow the fundamentalists' trading rule, the price starts again the move towards an attractor that is periodic or chaotic. We know that in this case imitators often change their minds and alternate behavioral rules. Let us consider the case in which, even without imitators, the attractor is chaotic and bull and bear dynamics occur, i.e. when chartists strongly dominate the market (as in the case represented in Fig.4). Under these circumstances, values of the asset price close the fundamental value alternate with values far away from it in an almost unpredictable way. In such a case it can be natural to expect that by introducing imitators the price's variance will increase. That is fluctuations should be amplified. This is only partially confirmed by simulations. Let us look at the bifurcation diagram represented in Fig. 9:

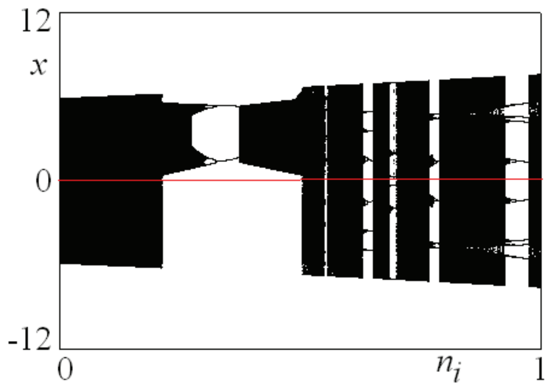


Figure 9: Bifurcation diagram obtained by keeping fixed $a = 3$, $f = 1.2$, $c = 3.3$, $i = 0.3$. The share of imitators n_i varies between 0 and 1.

We can see that for certain values of the relative number of imitators n_i the chaotic attractor reduces its size, sometimes even becoming periodic. This result is quite interesting and can be seen as a possible base model for reproducing some stylized facts of financial markets like volatility clustering. In our simple model, in fact, we consider the model's parameters as exogenously given. In real markets we expect that they can vary over time. It is not strange to consider the number of imitators one of them. If this is so, the changing number of imitators can drastically modify the boundaries of the price movements as we have just seen. So, a general tendency for an increasing of the price variations as imitators increases is present but cannot be considered a general rule. This interesting scenario deserves, in our opinion, to be deepen investigated. We plan to do that in future research.

5 Conclusions

The discovery of deviations from the rational behavior described in the microeconomics textbooks, has a long history. In the *heuristics-and-biases* program, Kahneman and his collaborators find a lot of deviations that are systematic and in some sense predictable. Among the various strands of the theoretical economic literature that try to incorporate such behavioral assumptions, there is one in which the mathematics of dynamical systems meets behavioral economics. The aim of the researchers working in this field consists in building small scale models representing markets populated by heterogeneous and boundedly rational agents. The small scale of the models usually permits their analytical study that, together with the use of numerical simulations, makes possible to understand the causes of the emergence of some phenomena. These models are usually not so sophisticated to be immediately calibrated. Hardly the simulated time series could quantitatively be compared with real ones[9]. The main aim of these model consists in qualitatively replicating some of the most puzzling stylized facts. At a later stage, these model can be the base for the building of more complicated and complete models that also quantitatively replicate some market phenomena. One of the main results of this strand of literature is that in order to qualitatively reproduce a lot of stylized facts, the introduction of some well thought behavioral assumptions in a completely deterministic setting may be enough.

The most of these models tries to explain some features of financial markets. In this model we have analyzed the role of cognitively biased imitators in a single asset market. Imitators trade together with fundamentalists and chartists and at each time period they decide which of the two available trading rules is the best for the next trading session. This decision is based on the last performance of the available strategies. Our results permit to enlight the role of imitators in destabilizing or in stabilizing the market. In particular, some if the scenarios we obtain seems to be suitable for being used as a skeleton for replicating important facts of financial markets like long memory effects and volatility clustering.

[9] Even if there are also some recent and successful attempts to do that (see Franke and Westerhoff, 2012 and Tramontana and Westerhoff, 2013).

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