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Monetary-Policy Tradeoff in Overlapping Generations DSGE Models

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Abstract

We evaluate optimal monetary policy in the standard sticky-price DSGE model in which overlapping-generations households à la Blanchard-Yaari replace the infinitely lived representative household. With nominal rigidities, monetary policy faces a new trade-off between inflation and intergenerational inequalities. This trade-off is related to the inability of future generations to trade on the financial market in order to hedge against shocks that appear during the birth period. If the fiscal authority hedges households against those shocks and then equalizes consumption among households, non-Ricardian behaviors disappear and the monetary policy trade-off is the same as in a fully Ricardian economy. Otherwise optimal monetary policy should be less proactive and let appear a much larger inflation surge.

1

1 Introduction

The birth of new agents in an overlapping-generations (OLG) model is a way to break the Ricardian equivalence: government bonds are, at least partially, net wealth for current households (Barro, 1974) as future generations bear a share of taxes the government will levy to repay bonds. More generally, non-Ricardian behavior is, in overlapping-generation models, related to intergenerational redistribution, whether the government issues bonds or not (Weiss, 1980; Weil, 1991).

The last decade has seen a revival of OLG models in the macroeconomic literature. These models are mainly used to analyze the medium and long-run effects of fiscal policy on output, the private saving rate and, in open-economy models, the current account (Cardia, 1991; Borgy & Hairault, 2001; Caballero et al., 2008), or more recently, precautionary foreign reserves (Carroll & Jeanne, 2009). Bénassy (2005) derives the condition for price determinacy in a non-Ricardian model with flexible price and cash-in advance, whereas Piergallini (2006) shows that the Taylor principle is not valid in a non-Ricardian model with endogenous labor supply and sticky prices. The first medium-scale (multi-country, with capital, real and nominal rigidities) macroeconomic model, GIMF, introduced by the IMF (Kumhof & Laxton, 2007) has dragged a collection of policy-oriented papers both at the IMF (Freedman et al., 2010a,b) and in central banks (Almeida et al., 2011) and represents today the state-of-the-art in macro models with an OLG framework. The main parameters driving the non-Ricardian feature of the model, the birth rate, have been estimated by Castelnuovo & Nisticò (2010), based on the effect of the stock market on the macroeconomic dynamic. However, no normative analysis of monetary policy has been developed in an OLG framework.

This paper develops the discrete-time Yaari (1965)-Blanchard (1985) perpetual-youth OLG model in a DSGE framework. The calculation of an exact aggregation of cohort-specific variables (consumption, labor supply and welfare) requires some restriction in the utility function and the ability of current households to trade a complete set of contingent claims. In addition, like in Yaari (1965), life insurance pays a premium on the financial wealth against the ownership of the corresponding assets at the death of the household.

Thanks to the complete set of contingent claims, the relative consumption of current cohorts is fixed by history. By contrast, the newborns did not have the opportunity to trade financial assets in order to hedge against shocks occurring the current period (Gordon & Varian, 1988; Ball & Mankiw, 2007). Therefore, the evolution of wealth distribution comes down to a single term: the consumption of the young (newborns, newcomers) relative to the aggregate consumption in the economy. This term enters the inter-temporal Euler equation (or, equivalently, in the definition of the stochastic discount factor) and the social welfare. The other components of the model are identical to the infinitely-lived household model. The relative consumption of the young is equal to the ratio of the total wealth of the young to the total wealth of the average household; its variations are therefore driven by the relative variation

of the components of total wealth: human wealth (current and future labor income), financial wealth (accumulated by households overtime) and the present value of future taxes.

Compared with the Ricardian model, price stickiness implies a new monetary-policy trade-off related to its re-distributive effects among cohorts. An analogy with the basic new-Keynesian framework enlightens the point : (a) if the competitive equilibrium (the steady-state) differs from the efficient equilibrium, monetary policy faces a trade-off between inflation and output, leading to a strictly positive inflation if monetary policy is discretionary; (b) if the gap between the natural equilibrium (absent nominal rigidities) and the efficient equilibrium is not constant, the optimal monetary policy does not coincide with strict inflation targeting. Those two results can be transposed in the overlapping-generations case: (a) if the competitive equilibrium (even if efficient) does not coincide with the social-welfare optimization, monetary policy may be biased in favor of a positive or negative inflation in the discretion case; (b) if the natural equilibrium (even if efficient) induces variations of intergenerational inequalities, the optimal monetary policy does not coincide with strict inflation targeting.

The interaction between monetary policy and redistribution issues has been intensively discussed in the literature. A money-supply increase modifies real wages (the distribution of income between labor and capital), the real interest rate (the distribution of income between creditors and debtors) or the relative wealth of money holders. This interaction is also at the core of the credit channel (Bernanke & Gertler, 1989; Bernanke et al., 1999): due to credit frictions, investment is constrained by the net wealth of agents that have the opportunity to invest (entrepreneurs) and monetary policy is able to modify the relative wealth of this class. In all these perspectives, the redistribution of wealth affects the macroeconomic dynamic but is not *per se* an objective of monetary policy. The model developed in this article emphasizes the impact of redistribution shocks on welfare and monetary policy as a way to accommodate these shocks. Closer to our approach but in a very different context (two types of households, where one type is not able to trade on the financial market), Anand & Prasad (2010) evaluates optimal simple monetary rules.

The evaluation of monetary policy hinges on the calculation of the social welfare, i.e. the aggregation of cohort-specific welfare. We restrict the analysis to the case where the social welfare is (i) time independent and (ii) recursive or time consistent. Under those two restrictions, at the optimum, the consumption level of a cohort depends geometrically on its age and the growth rate is constant over time. Given this social welfare function, the distribution of wealth is an explicit objective of monetary policy and interacts with its effects on the macroeconomic dynamic.

This article is very close to Nisticò (2011) that also evaluate optimal monetary policy base on the second order Taylor expansion of social welfare in a non-Ricardian framework. He shows that productivity shocks may generate a tradeoff between inflation and redistribution. His results comes from the mechanism that implement a socially-optimal steady-state (equitable

consumption across generations): a fixed financial endowment for newcomers. In our model, we also implement an equitable steady-state, but the mechanism is different (labor endowment is decreasing with age) and generates no tradeoff for productivity shocks. We show that, for a low elasticity of labor supply, monetary policy has sizable effects on the level of inequalities between workers (young households) and owners of financial assets (old households). This motivates a less reactive monetary policy when the economy faces a mark-up shock. We then extend the analysis in an economy with sticky wages and credit shocks. Contrary to Bernanke & Gertler (2001), the OLG framework implies that central banks should respond to asset prices, in addition to the effect of their movements on inflation.

2 The model

Comparing to Ricardian models, overlapping generations introduce intergenerational redistribution issues. Fiscal policy is the natural instrument to deal with redistribution. Apart from any discussion of feasibility, we first wonder how fiscal policy is able to both levy a certain amount of tax and achieve a given level of redistribution among households. In our framework, net taxes aim to subsidy labor in order to offset the firm's mark-up on labor supply at the steady-state. This section presents the model for any possible taxation scheme (i.e. how to share the burden across cohorts for a given aggregate taxation). Then, in order to find a closed-form solution of the aggregate consumption function, we specify a particular "neutral" taxation scheme, i.e. without any re-distributive gaol, and solve the model completely in order to assess the ability of monetary policy to accommodate shocks that redistribute wealth among households.

2.1 Households

The size of the population is constant, but young households replace dying ones at a rate $1-\theta$. Thus, the economy is populated by heterogeneous households, related to the date of birth, and we assume perfect risk-sharing among them such that the pricing kernel for assets and the program of firms and unions (price and wage setting) will be properly defined. Let ξ_t^{t+1} the relevant description of the state of nature at $t+1$. It counts for the realization of macroeconomic shocks (there is no idiosyncratic shock households need to hedge against). $\mathcal{F}_t^{t+1}$ denotes the real stochastic discount factor, *i.e.* the price at date t of an asset that delivers 1 unit of the consumption good at date $t+1$ when the state of nature ξ_t^{t+1} is realized, normalized by the probability of this event.

2.1.1 Households at the cohort level

The representative household of the cohort born at date a and still living at date t purchases goods $C_{a,t}$, supplies labor $L_{a,t}$ and decides the portfolio for the complete set of contingent claims (the decision at t concerns the amount of real unit of consumption $B_{a,t+1}$ the household receives — or pays if negative — at $t+1$, depending on the realized state of nature) and the portfolio $Z_{a,t}(i)$ of equities ($i \in \mathcal{I}$, the set of existing equities) which real price at the end of the current period is $EP_t(i)$ and real dividend is $D_t(i)$. The inter-temporal utility of the household is given by

$$\mathcal{W}_{a,t} \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k U_{t+k-a}(C_{a,t+k}, L_{a,t+k}) \qquad (1)$$

where $0 < \beta < 1$ is the discount factor and $0 < \theta < 1$ the probability to stay in life the following period. For general utility functions, the first order conditions at the cohort level are easy to derive, but aggregation among cohorts is almost intractable but with some restrictions on the class of utility functions. The first class considers utility functions leading to a labor supply linear in consumption and a consumption function linear in total wealth. Utility then writes $U(C, L) = (1-\kappa)\log(C) + \kappa\log(\bar{L} - L)$ ($\kappa > 0$ is the utility of leisure and $\bar{L}$ is labor endowment). The second class considers utility functions leading to a labor supply which is independent on the level of consumption and a consumption function which is linear in total wealth at the cohort level. Utility then writes $U(C, L) = \log(C - V[L])$. Each utility function has its own advantages and restrictions, but the main conclusions of the paper remains in both.[1]

In the rest of the paper, we assume the first class utility function and we introduce an age-dependent time endowment. This new feature may capture retirement: time endowment decreases with age. By definition, young households have no financial assets (they born without any asset) whereas older one has accumulated some. The higher time endowment of young households allows to compensate for financial inequalities. So, at the steady-state, all households share the same level of total wealth (financial and human) and the same level of consumption. For convenience, we assume a geometric progression of time endowment ($\bar{L}_j = \gamma^j(1-\theta\gamma)/(1-\theta)\bar{L}$) and the parameter γ will be chosen in order to avoid inequalities at the steady-state. The smaller γ, the larger the human wealth of young households comparing to older ones. Of course, when macroeconomic shocks occurs, the composition of wealth (mostly human or mostly financial) will matter.

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[1]The first class is the more natural extension of Blanchard (1985) with endogenous labor supply have already been intensively used both in a continuous-time or a discrete-time framework. It allows a decreasing labor endowment with age and so the introduction of a pay-as-you-go pension system. The second class have been introduced by Ascari & Rankin (2007) and avoids the negative labor supply for very old (and rich) households that appears in the first class if the steady-state natural interest rate exceeds $1/\beta$.

The budget constraint $\Omega_{a,t}$ denotes the financial wealth at the beginning of the period t of the representative household born at period a (we assume $\Omega_{t,t} = 0$). The budget constraint of an household writes

$$P_t C_{a,t} + \mathbb{E}_t(P_{t+1} \mathcal{F}_t^{t+1} B_{a,t+1}) + P_t \int_{i \in \mathcal{I}} EP_t(i) Z_{a,t}(i) \mathrm{d}i = P_t \Omega_{a,t} + \tau W_t L_{a,t} - P_t \mathcal{T}_{a,t}, \quad (2)$$

where P_t is the consumption price level, W_t the level of wage, τ the labor subsidy, and $\mathcal{T}_{a,t}$ the real lump-sum tax of cohort a. As in Woodford (2003), the labor subsidy compensates for the inefficiency created by the price mark-up and allows to reproduce the optimal labor supply at the steady-state. Aggregate taxes cover each period the cost of subsidies and the way the tax burden is shared across cohorts is discussed below.

Households hedge against life duration: alive households receive a premium on their financial wealth in exchange of the corresponding assets it in case of death. For $a < t$, this translates into

$$\Omega_{a,t} = \frac{1}{\theta} B_{a,t} + \frac{1}{\theta} \int (EP_t(i) + D_t(i)) Z_{a,t-1}(i) \mathrm{d}i. \quad (3)$$

First order conditions   First order condition on claims $(B_{a,t+1})$, equities $(Z_{a,t})$ and labor supply $(L_{a,t})$ and the definition of the risk-free one-period nominal interest rate write

$$\mathcal{F}_t^{t+1} = \beta \frac{C_{a,t}}{C_{a,t+1}} \quad (4) \qquad EP_t(i) = \mathbb{E}_t \left\{ \frac{P_{t+1}}{P_t} \mathcal{F}_t^{t+1} (EP_{t+1}(i) + D_{t+1}(i)) \right\} \quad (6)$$

$$R_t \mathbb{E}_t \left\{ \frac{P_{t+1}}{P_t} \mathcal{F}_t^{t+1} \right\} = 1 \quad (5) \qquad L_{a,t} = \bar{L}_{t-a} - \frac{\kappa}{1-\kappa} \frac{P_t C_{a,t}}{\tau W_t} \quad (7)$$

Consumption at the cohort level   The derivation of the consumption function is based on the forward iteration of the budget constraint (2) and the transversality condition. Finding a closed-form is difficult, as current and future labor income depends on the labor supply of the household and the level of taxes, which also may be time and cohort dependent. The analysis of the effect of taxes and subsidies is first conduces without a closed-form solution. Then, Section 2.3 derives the closed-form solution in the particular case we consider. With complete financial markets, future buys and sells of equities are not relevant to compute the consumption of a household: they are fully hedged by contingent claims. The iteration writes

$$\mathbb{E}_t \sum_{k \geq 0} \theta^k \mathcal{F}_t^{t+k} C_{a,t+k} = \Omega_{a,t} + \underbrace{\mathbb{E}_t \sum_{k \geq 0} \theta^k \mathcal{F}_t^{t+k} \frac{\tau W_{t+k}}{P_{t+k}} L_{a,t+k}}_{HW_{a,t}} - \underbrace{\mathbb{E}_t \sum_{k \geq 0} \theta^k \mathcal{F}_t^{t+k} \mathcal{T}_{a,t+k}}_{TB_{a,t}}.$$

$HW_{a,t}$ denotes the human wealth at t of the representative household born at a (the expected present value of future labor income) and $TB_{a,t}$ the tax burden (the expected present value of future taxes). The first order condition on claims (4) allows to simplify the left-hand side

of the equation. The consumption function writes

$$C_{a,t} = (1 - \beta\theta)\left(\Omega_{a,t} + HW_{a,t} - TB_{a,t}\right). \tag{8}$$

2.1.2 Aggregation

Labor Thanks to the affine relation (7) between consumption and labor at the cohort level, the aggregate labor supply is straightforward.

$$L_t = \bar{L} - \frac{\kappa}{1 - \kappa}\frac{P_t C_t}{\tau W_t} \tag{9}$$

Consumption At the cohort level, the relation between consumption and total wealth is linear and so the aggregation is immediate. On aggregate, net supply of contingent claims is zero and aggregate financial wealth reduces to the value of equities (see the program of firms and the definition of dividends below). The aggregate consumption function then writes

$$C_t = (1 - \beta\theta)\left[EW_t + HW_t - TB_t\right] \tag{10}$$

where aggregate human wealth, aggregate tax burden and aggregate equity wealth are given by

$$HW_t \equiv (1 - \theta)\mathbb{E}_t \sum_{k=0}^{\infty} \mathcal{F}_t^{t+k} \sum_{a \leq t} \theta^{t+k-a}\frac{\tau W_{t+k}}{P_{t+k}} L_{a,t+k}$$

$$TB_t \equiv (1 - \theta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_t^{t+k} \sum_{a \leq t} \theta^{t+k-a} TB_{a,t+k} \right\}$$

$$EW_t \equiv \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_t^{t+k} D_{t+k} \right\} \tag{11}$$

The Aggregate consumption function, Equation (10) and the consumption of the young, Equation (8) with $a = t$ allows to define the relative consumption of the young (or relative consumption)

$$\chi_t \equiv C_{t,t}/C_t. \tag{12}$$

2.2 Non-Ricardian features

2.2.1 The fundamental equation of OLG models

The pricing kernel $\mathcal{F}_t^{t+1}$ enters in the firms' optimization program, and forms, together with Equation (5), the aggregate IS equation. To calculate the pricing kernel, aggregate consumption is decomposed in two terms: the consumption of the young and the consumption of older

households

$$C_{t+1} = (1-\theta)C_{t+1,t+1} + (1-\theta)\sum_{a \leq t} \theta^{t+1-a}C_{a,t+1} \tag{13}$$

The definition of aggregate consumption at t leads to

$$\frac{C_{t+1}}{C_t} = (1-\theta)\frac{C_{t+1,t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t} + \theta\frac{(1-\theta)\sum_{a \leq t}\theta^{t-a}C_{a,t+1}}{(1-\theta)\sum_{a \leq t}\theta^{t-a}C_{a,t}}$$

The first order condition on claims, Equation (4), allows to simplify the previous expression

$$(1 - (1-\theta)\chi_{t+1})\mathcal{F}_t^{t+1} = \theta\beta\frac{C_t}{C_{t+1}}. \tag{14}$$

This fundamental equation of OLG models, which gives the relation between aggregate consumption behavior and the pricing kernel, is the generalization of the definition of the pricing kernel in the representative-agent model ($\mathcal{F}_t^{t+1} = \beta\frac{C_t}{C_{t+1}}$).[2] It emphasizes the role of inter-generational inequalities, measured by the relative consumption of the young, on the macroeconomic dynamic. Aggregate consumption in $t+1$ is the sum of consumption of households already alive in t, which determines the pricing kernel between t and $t+1$, and newborns at $t+1$. If one expect a well-endowed cohort tomorrow (the relative consumption of the young is expected higher than 1), the consumption growth rate of already alive households is smaller that the aggregate consumption growth rate and so $\mathcal{F}_t^{t+1} > \beta\frac{C_t}{C_{t+1}}$. In the opposite case ($\chi_{t+1} < 1$), the consumption growth rate of already alive households is higher than the aggregate consumption and $\mathcal{F}_t^{t+1} < \beta\frac{C_t}{C_{t+1}}$. The composition effect (i.e. entry and exit of households) on the aggregate Euler equation has been noticed in Attanasio & Weber (1993, 1995). The authors emphasizes the bias in the estimation of inter-temporal elasticity of substitution when looking at aggregate data and ignoring composition effects.

The evaluation of the relative consumption of the young requires the consumption function of the young (see Equation (8) with $a = t$) and the aggregate consumption function (see Equation (10)). But whatever the expression of these two quantities which depends on the specification of the model, the relation defined by Equation (14) still remains.

2.2.2 The relative consumption of the young and the taxation scheme

The young differ from the aggregate as they have a different human wealth, tax burden and no financial wealth at all. The relative consumption of the young writes

$$\chi_t = \frac{HW_{t,t} - TB_{t,t}}{EW_t + HW_t - TB_t}. \tag{15}$$

The aggregate IS equation (Equations (5), (14) and (15)) allows to measure the effect of non-

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[2]For an inter-temporal elasticity of substitution equals to 1.

distorsive fiscal policy on aggregate consumption. For example, the introduction of a pension system that taxes young households and redistributes to elder ones raises consumption in the short run and increases the natural interest rate in the long run. More generally, any given bounded sequence of χ is implementable through a convenient tax scheme, through the level of redistribution between the young (newborns) and older households.

Proposition 1. *Let a bounded sequence of aggregate taxation $(\mathcal{T}_t)_{t \geq 0}$ and a bounded sequence of the relative consumption of the young $(\chi_t)_{t \geq 0}$. There exists a taxation scheme (the burden sharing of taxes among cohorts) such that the market equilibrium implements $(\mathcal{T}_t)_{t \geq 0}$ and $(\chi_t)_{t \geq 0}$.*

The proof is given in Appendix A. This proposition implies that an omniscient government is able to reproduce any possible dynamic of aggregate consumption for a given monetary policy. In order to shift the IS curve at date t, the government has to commit to increase, at date $t+1$, transfers to households that are already born at date t (i.e. to decrease χ_{t+1}). The very same argument is developed in Bénassy (2007), Chapter 8, in an OLG model with two generations. The author emphasizes that, absent information advantage of the government, welfare improving activist policies can be implemented. Departing from this perspective, we will assume here that *fiscal* policy is neutral (will not modify the aggregate IS equation through redistributive taxation) and focus on the ability of *monetary* policy to partially offset inefficient variations of intergenerational inequalities.

Definition 1. *A taxation scheme is neutral if, for all periods and states of nature, the lump-sum tax exactly compensate for the additional income given by the labor subsidy*

$$\mathcal{T}_{a,t} = (\tau - 1)W_t L_{a,t}$$

If the tax is set according to the average labor supply of the cohort (instead the hours supply of a particular household), the tax does not modify the hours supplied by households.

2.3 A closed-form consumption function

We derive and exact form for the aggregate consumption function and the relative consumption of the young when the taxation scheme is neutral.

The cohort level The taxation scheme is assumed to be neutral such that $\tau_t W_t L_{a,t} - P_t \mathcal{T}_{a,t} = W_t L_{a,t}$. This relation, together with Equation (7), allows to simplify the budget constraint at the cohort level (2) by eliminating the endogenous labor supply. This writes

$$\left(1 + \frac{\kappa}{1-\kappa}\frac{1}{1+\mu}\right)P_t C_{a,t} + \mathbb{E}_t(P_{t+1}\mathcal{F}_t^{t+1}B_{a,t+1}) + P_t \int_{i \in \mathcal{I}} EP_t(i)Z_{a,t}(i)\mathrm{d}i = W_t \bar{L}_{t-a} + P_t \Omega_{a,t}.$$

Let $w_t = W_t/P_t$ the real wage, the forward iteration of the budget constraint writes

$$\left(1 + \frac{\kappa}{1-\kappa}\frac{1}{1+\mu}\right)\mathbb{E}_t\left\{\sum_{k\geq 0}\theta^k \mathcal{F}_t^{t+k} C_{a,t+k}\right\} = \Omega_{a,t} + \mathbb{E}_t\left\{\sum_{k\geq 0}\theta^k \mathcal{F}_t^{t+k} w_{t+k}\bar{L}_{t+k-a}\right\}.$$

We re-define the human wealth at t of the representative household born at a by the present value of future labor income, as if all its disposable time were worked, i.e.

$$HW_{a,t} \equiv w_t\bar{L}_{t-a} + \theta\mathbb{E}_t\left\{\mathcal{F}_t^{t+1} HW_{a,t+1}^s\right\}. \tag{16}$$

We then introduce σ, the propensity to consume wealth, which is independent of the cohort: $\sigma \equiv (1 - \beta\theta)/\left(1 + \frac{\kappa}{1-\kappa}\frac{1}{1+\mu}\right)$. σ is smaller than $1 - \beta\theta$, as wealth allows to increase both consumption and leisure. The consumption function writes

$$C_{a,t} = \sigma\left(\Omega_{a,t} + HW_{a,t}\right). \tag{17}$$

Aggregation The only term on which one has to pay attention is aggregate human wealth. It resumes to the actual value of net of tax future labor income of households living today. This leads to use $\theta\gamma\mathcal{F}_t^{t+1}$ instead of $\mathcal{F}_t^{t+1}$ in the actualization.

$$HW_t \equiv \mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\gamma)^k \mathcal{F}_t^{t+k} w_{t+k}\right\}\bar{L} \tag{18}$$

the aggregate consumption writes

$$C_t = \sigma\left[EW_t + HW_t\right] \tag{19}$$

with aggregate human wealth given by

$$HW_t \equiv \mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\gamma)^k \mathcal{F}_t^{t+k} w_{t+k}\right\}\bar{L} \tag{20}$$

The closed-form solution for the consumption of the young and aggregate consumption allows to evaluate relative consumption of the young

$$\chi_t = \frac{HW_{t,t}}{EW_t + HW_t} = \frac{1-\theta\gamma}{1-\theta}\frac{HW_t}{EW_t + HW_t}.$$

The same variable is evaluated as a function of the value of equities as a share of GDP, $Q_t \equiv \frac{EW_t}{C_t}$, a measure of the wealth gap between the young and the average consumer.

$$\chi_t = \frac{1-\theta\gamma}{1-\theta}\left(1 - \sigma Q_t\right). \tag{21}$$

The previous expression together with the fundamental equation of overlapping-generation model (14) leads to the expression of the real stochastic discount factor

$$\mathcal{F}_t^{t+1} = \frac{\theta\beta}{\theta\gamma + (1-\theta\gamma)\sigma Q_{t+1}} \frac{C_t}{C_{t+1}}. \tag{22}$$

The definition of financial wealth, Equation (11), and the value of the stochastic discount factor allows to calculate the dynamic of Q_t

$$Q_t = \frac{D_t}{C_t} + \mathbb{E}_t \left\{ \frac{\theta\beta}{\theta\gamma + (1-\theta\gamma)\sigma Q_{t+1}} Q_{t+1} \right\}. \tag{23}$$

The previous equation means that the dynamics of Q_t only depends on the share of dividends in the GDP (which here reduces to consumption). This results comes from the unit inter-temporal elasticity of substitution.

2.4 Welfare

2.4.1 Welfare at the cohort level

The welfare of an household born at date a and still alive at date t is given by Equation (1). We eliminate labor supply with Equation (7). Up to a constant, the welfare is given by

$$\forall a \leq t, \quad \mathcal{W}_{a,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\log(C_{a,t+k}) - \kappa \log\left(\frac{\tau W_{t+k}}{P_{t+k}}\right) \right]. \tag{24}$$

To take into account future generations in the social welfare, one has to define the present value of their welfare. We then generalize Equation (24) for $a > t$ (see below for a discussion of the welfare of future generations)

$$\forall a > t, \quad \mathcal{W}_{a,t} = \beta^{a-t}\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\log(C_{a,a+k}) - \kappa \log\left(\frac{\tau W_{a+k}}{P_{a+k}}\right) \right]. \tag{25}$$

2.4.2 Aggregate welfare

There are heterogeneous agents in the model. The way they are weighed in the social welfare function is an important issue. The first formulation of social welfare (or social planner's objective) in a perpetual-youth OLG model is given by Calvo & Obstfeld (1988). Translated in our framework, the two authors assume that welfare at t of an household that will born at $a > t$ is given by

$$\mathbb{E}_t \sum_{k=a-t}^{\infty} (\beta\theta)^{t+k-a} \left[\log(C_{a,t+k}) - \kappa \log\left(\frac{\tau W_{t+k}}{P_{t+k}}\right) \right],$$

instead of our Equation (25). To preserve the time-consistency of the social planner's objective (the arbitrage between the welfare of two different cohorts is not put into question the next period), they are logically forced to assume that utility of alive households is discounted "*back to their birthdates*" and so writes

$$\mathcal{W}_{a,t} = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^{t+k-a}\theta^k \left[\log(C_{a,t+k}) - \kappa \log\left(\frac{\tau W_{t+k}}{P_{t+k}}\right) \right],$$

instead of our Equation (24), which "*may seem unnatural*" (see Calvo & Obstfeld (1988), p. 414).

For cosmetic purpose, we formulate differently the problem. Let assume that, in the social welfare, the instantaneous utility of future generations is discounted from the current date t instead their birthdate. In this case, the utility of alive household is also discounted from the current date t which seems natural.

Like Calvo and Obstfeld, we also assume a time-independent social welfare, i.e. implying that the arbitrage between cohort born at a and born at $a+s$ does not depends on a. Under time-consistency and time-independency, there exist $\beta < \zeta < 1/\theta$ such that the social welfare is given by

$$\mathcal{W}_t \equiv (1-\theta)\sum_{a\leq t}(\zeta\theta)^{t-a}\mathrm{W}_{a,t} + (1-\theta)\sum_{a>t}(\zeta)^{t-a}\mathrm{W}_{a,t} \tag{26}$$

Let $g(\chi) = (1-\theta)\log(\chi) + \theta\zeta\log\left(\chi + \frac{1-\chi}{\theta}\right)$, one has

Proposition 2.

$$\mathcal{W}_t = (1-\kappa)\log(C_t) + \kappa\log(\bar{L} - L_t) + \frac{g(\chi_t)}{1-\beta\theta} + (\beta/\zeta)\mathbb{E}_t\{\mathrm{W}_{t+1}\} \tag{27}$$

See Appendix B for the proof. The social planner's objective is the sum of two terms. The first one collects aggregate consumption and aggregate labor supply and is common with the representative infinitely-lived household model. The second one is related to the distribution of consumption across cohorts. In the rest of the paper, $\zeta = 1$, i.e. all generations are treated equally in the social welfare.

Efficiency (that reduces to the equalization of the marginal rate of substitution and the technical rate of transformation) does not imply that social welfare is maximum. The social welfare given by Equation (26) implies that, at the optimum, households share the same level of consumption. Subsequently, consumption is constant during the life-cycle at the steady-state, *i.e.* the natural interest rate is equal to $1/\beta$. In the basic monopolistic-competition DSGE model, the first-best is realized through a compensatory taxation/subsidy scheme (a subsidy on labor for instance) to equalize marginal rate of substitution and marginal rate of transformation. In the overlapping-generation model, the first best is achieved through an

age-dependent labor subsidy (or, equivalently, a labor subsidy and a pay-as-you-go retirement system) to equalize consumption among households.

When the natural equilibrium and the first-best do not coincide, a tradeoff appears for monetary policy, leading to inflation-bias under discretion. The very same appears in the OLG framework: when consumption increases with age (the real interest rate is higher than $1/\beta$) monetary policy will try to favor the young against the older.

2.5   Firms and price setting

The production function is common across firms and linear in labor input ($Y_t(i) = A_t L_t(i)$) thus, firms share the same marginal cost. Dividends (capital income) consists in the price mark-up over the marginal production cost ($D_t(i) = (P_t(i) - W_t/A_t)Y_t(i)$). To generate redistributive shocks from labor to capital, we will assume that the desired mark-up μ_t varies around it steady-state value μ. The consumption bundle writes

$$C_t = \left(\int_{i=0}^{1} c_{i,j}(t)^{\frac{1}{1+\mu_t}} \, di \right)^{1+\mu_t}$$

Prices are sticky *à la* Calvo and firms that have the ability to set freely their price at t will all choose the same optimal price P_t^{opt}, due to their homogeneity. They maximize their profit expectancy until next free price setting. Let $p_t = \frac{P_t^{opt}}{P_t}$, the program writes

$$\max_{p_t} \mathbb{E}_t \sum_{s=0}^{\infty} \mathcal{F}_t^{t+s} \nu^s \left(p_t - \frac{MC_{t+s}}{P_t} \right) \left(p_t \frac{P_t}{P_{t+s}} \right)^{-\frac{1+\mu_{t+s}}{\mu_{t+s}}} \frac{P_t}{P_{t+s}} C_{t+s} \qquad (28)$$

The log-linearization of the price-setting decision of firms leads to the following New-Keynesian Phillips curve (see Galí (2008), Chap. 3)

$$\pi_t = \frac{(1-\nu\mathcal{F})(1-\nu)}{\nu} (\mu\hat{\mu}_t + \hat{mc}) + \mathcal{F} \, \mathbb{E}_t \{\pi_{t+1}\}$$

3   The steady-state and log-linearization

This section derives the steady-state of the model, the natural equilibrium and the log-linearization of the flexible wage model. This log-linearization will allow to find the possible values of parameters of a simple rule ensuring determinacy.

Figure 1: Asset demand, asset supply and the steady-state
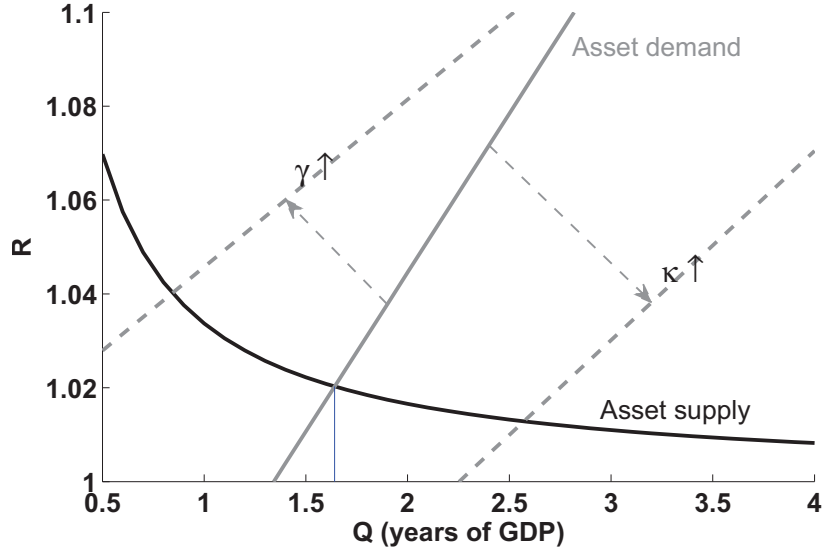
3.1 The steady-state

Technically efficient steady-state. Efficiency implies that no inflation at the steady-state ($\pi = 1$). The free parameters are β, θ, γ, κ, $\bar{L}$, μ and A. The government decides the rate of labor subsidies that achieves optimal labor supply, assuming a neutral taxation scheme across cohorts. Let $\mathcal{F}$ the steady-state value of the real stochastic discount factor (see below), the steady-state is given by

$$C = (1-\kappa)A\bar{L} \qquad \mathrm{mrs} = A \qquad MC = \frac{1}{1+\mu} \qquad R = \frac{1}{\mathcal{F}} \qquad Q = \frac{\mu}{1+\mu}\frac{1}{1-\mathcal{F}}$$

The steady-value of $\mathcal{F}$ is determined by asset supply and asset demand curves in the long run. Asset supply is given by $Q = \frac{\mu}{1+\mu}\frac{1}{1-\mathcal{F}}$. Asset demand is given by the value of the relative consumption of the young, Equation (21), together with the fundamental equation of OLG models, Equation (14). The two curves appears in Figure 1. An increase of labor endowment ($\gamma \uparrow$) of the old reduces savings, as falling labor income when households get older do not need to be compensate through higher financial income.

At the steady-state, $\mathcal{F}$ is a root of the following polynomial

$$P(\mathcal{F}) = \theta\gamma\mathcal{F}^2 - [\theta(\gamma + \beta) + (1 - \theta\beta)(1 - \theta\gamma)\varphi]\,\mathcal{F} + \theta\beta.$$

with $\varphi = \frac{\mu(1-\kappa)}{1+\mu(1-\kappa)}$. It can be shown that this degree 2 polynomial equation has 2 real and strictly positive roots. One exactly is strictly smaller than 1 and has an economic interpretation, it also verifies $\theta\beta < \mathcal{F} < \beta/\gamma$. The other one is strictly larger than one and is excluded.[3]

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[3]It is sufficient to check that $P(\theta\beta) > 0$, $P(1) < 0$ and $P(\beta/\gamma) < 0$.

Table 1: Parameters of the model

Description	Name	Value
Psychologic discount factor	β	0.99
Probability to stay alive	θ	0.94
Consumption-leisure preference parameter	κ	0.1
Natural price mark-up	μ	0.15
Propensity to consume wealth	σ	$(1-\beta\theta)/\left(1 + \frac{\kappa}{1-\kappa}\frac{1}{1+\mu}\right)$
Growth rate of labor endowment	γ	Eq (30)
Stochastic discount factor at the steady-state	$\mathcal{F}$	Eq (29)
Non-Ricardian reduced parameter	$\mathcal{G}$	$\mathcal{F}\gamma/\beta$

The economic root is given by

$$\mathcal{F} = \frac{[\theta(\gamma+\beta) + (1+\theta\beta)(1+\theta\gamma)\varphi] - \sqrt{[\theta(\gamma+\beta) + (1-\theta\beta)(1-\theta\gamma)\varphi]^2 - 4\theta^2\gamma\beta}}{2\theta\gamma} \quad (29)$$

Socially efficient steady-state. We restrict the analysis on the case where $\mathcal{F} = \beta$, *i.e.* where the steady-state also maximizes the social welfare with equal weighting. We therefore restrict the value of γ such that the human wealth of a new born is equal to the average total wealth in the economy. This implies

$$\gamma = \frac{\theta - \frac{1-\theta\beta}{1-\beta}\varphi}{1 - \frac{1-\theta\beta}{1-\beta}\varphi} \quad (30)$$

3.2 The natural equilibrium

We assume two shocks in the economy. The mark-up shock is additive and normalized such that a 1 percent shock moves the share of dividends in value added by one percent. The technology shock is multiplicative.

$$\mu_t = \mu + \epsilon_t^\mu/(1+\mu) \quad (31) \qquad\qquad A_t = A(1+\epsilon_t^A) \quad (32)$$

The share of dividends in value added is given by $1-MC$ and marginal costs depends on technology and the labor supply. This translates into

$$1 = R_t\mathbb{E}_t\left\{\frac{\theta\beta}{\theta\gamma + (1-\theta\gamma)\sigma Q_{t+1}}\frac{C_t}{C_{t+1}}\frac{1}{\pi_{t+1}}\right\}, \quad (33)$$

$$Q_t = \left(1 - \frac{\kappa}{1-\kappa}\frac{C_t}{A_t\bar{L}-C_t}\frac{1}{1+\mu}\right) + \mathbb{E}_t\left\{\frac{\theta\beta}{\theta\gamma + (1-\theta\gamma)\sigma Q_{t+1}}Q_{t+1}\right\}. \quad (34)$$

Let $\mathcal{G} = \mathcal{F}\gamma/\beta < 1$, the flexible price (natural) dynamics and its log-linearization around the

steady-state is given by[4]

$$C_t^n = \frac{A_t \bar{L}}{1 + \frac{1+\mu_t}{1+\mu}\frac{\kappa}{1-\kappa}} \qquad\qquad \hat{c}_t^n = \hat{a}_t - \kappa\mu\hat{\mu}_t \tag{35}$$

$$Q_t^n = \frac{\mu_t}{1+\mu_t} + \mathbb{E}_t\left\{\frac{\theta\beta}{\theta\gamma + \sigma(1-\theta\gamma)Q_{t+1}^n}Q_{t+1}^n\right\} \qquad \hat{q}_t^n = (1-\mathcal{F})\hat{\mu}_t + \mathcal{F}\mathcal{G}\mathbb{E}_t\{\hat{q}_{t+1}^n\} \tag{36}$$

$$1 = R_t^n\mathbb{E}_t\left\{\frac{C_t^n}{C_{t+1}^n}\frac{\theta\beta}{\theta\gamma + \sigma(1-\theta\gamma)Q_{t+1}^n}\right\} \qquad \hat{r}_t^n = \hat{c}_{t+1}^n - \hat{c}_t^n + (1-\mathcal{G})\mathbb{E}_t\left\{\hat{q}_{t+1}^n\right\} \tag{37}$$

An important result is the dependency of the natural real interest rate on the ratio of equity price to GDP (Q_t) which is the signature of non-Ricardian models. If this ratio is expected to be higher than the steady-state, the natural real interest rate increases. The reason is the following. If equity prices are higher tomorrow, the relative consumption of the young is expected to be lower or, equivalently, the relative consumption of alive households is expected to be higher. This implies that the evolution of alive households consumption is higher than the evolution of aggregate consumption, and so the natural real interest rate.

The mark-up shock modifies the share of total compensation in GDP and so the ratio equity price to GDP, whereas the productivity shock affects in the same proportion GDP and equity prices. The monetary will face no trade-off in case of productivity shock (stabilizing inflation also stabilizes the intergenerational inequalities) but the trade-off inflation-inequalities may appears in case of mark-up shocks.

3.3 The Phillips curve and deviations from the natural equilibrium

The deviation of each variable from the natural equilibrium (denotes with a tilde) allows to evaluate the dynamic property of the model. One has

$$\tilde{q}_t = -\frac{1-\mathcal{F}}{\mu\kappa}\tilde{c}_t + \mathcal{F}\mathcal{G}\mathbb{E}_t\left\{\tilde{q}_{t+1}\right\} \tag{AP}$$

$$\tilde{c}_t = \mathbb{E}_t\left\{\tilde{c}_{t+1}\right\} - (i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - \hat{r}_t^n) + (1-\mathcal{G})\mathbb{E}_t\left\{\tilde{q}_{t+1}\right\} \tag{IS}$$

$$\pi_t = \frac{(1-\nu\mathcal{F})(1-\nu)}{\kappa\nu}\tilde{c}_t + \mathcal{F}\mathbb{E}_t\left\{\pi_{t+1}\right\} \tag{NKPC}$$

In the Ricardian model, $\mathcal{G} = 1$ and the stock-market price does not influence aggregate consumption. In the non-Ricardian model, monetary policy affects consumption (through the IS) and so both inflation and the asset price (asset price (AP) equation and the Phillips curve (NKPC)).

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[4]For convenience, we define $\hat{\mu}_t$ as the log-deviation of $\mu_t/(1+\mu_t)$

4 Monetary policy

4.1 The monetary-policy trade-off

Productivity shock In Ricardian models, a productivity shock respects the divine coincidence: the welfare-relevant output gap does not vary. The divine coincidence can be extended in OLG models as productivity shocks do not modifies the equity-price to GDP ratio, as appearing in Equation (36). When a productivity shock occurs, Equity prices and production share the same dynamic: no socially inefficient variations in the level of inequalities appear. To illustrate this point, we consider the equations of financial wealth and human wealth to GDP ratio, as derived from Equations (11) and (20)

$$Q_t = (1 - MC_t) + \mathbb{E}_t \left\{ \mathcal{F}_t^{t+1} \frac{C_{t+1}}{C_t} Q_{t+1} \right\}, \tag{38}$$

$$\frac{HW_t}{C_t} = \left(MC_t + \frac{\kappa}{1-\kappa} \frac{1}{\tau} \right) + \theta\gamma \mathbb{E}_t \left\{ \mathcal{F}_t^{t+1} \frac{C_{t+1}}{C_t} \frac{HW_{t+1}}{C_{t+1}} \right\}. \tag{39}$$

The unitary inter-temporal elasticity of substitution leads to the fundamental equation of OLG models, Equation (14), that links $\mathcal{F}_t^{t+1} \frac{C_{t+1}}{C_t}$ with the level of inequalities at $t+1$. When productivity increases, compensations and dividends are affected by the same proportion (according to the Cobb-Douglas production function). Even if future compensations are more discounted than future dividends (the human wealth should increases less than financial wealth) this effect is exactly compensate through the evolution of the stochastic discount factor. Thus, in OLG models, the divine coincidence for productivity shocks are related to two assumptions: (i) productivity shocks are neutral on factors shares and (ii) the inter-temporal elasticity of substitution is equal to 1.

Mark-up shock On the contrary, the mark-up shock induces a variation of the welfare-relevant output gap. First, the optimal mark-up of firms varies and so the natural equilibrium is no more technically efficient (the marginal rate of substitution and the productivity do not share the same dynamic). In addition, in the OLG framework, this shock induces a redistribution between workers (the young) and owners (older households). Welfare is reduced due to rising inequalities. In addition to the traditional inflation-output gap trade-off, a new trade-off appears between inflation and intergenerational inequalities. To compare the two trade-offs, the monetary authority will choose either the "Ricardian" welfare criterion (absent the term with inequalities in Equation (27)) or the "non-Ricardian" one (including the term with inequalities). The Ricardian welfare makes the monetary-policy trade-off equivalent to the Ricardian model, and the efficient frontier is the same. In addition to the cost of inflation and the output-gap, time-varying inequalities enters in the non-Ricardian welfare and the efficient frontier moves.

We first derive conditions for the determinacy of the dynamical system (Blanchard-Kahn conditions), then we evaluate the effect of a monetary shock on the economy and particularly on the level of inequalities. Then we assess th optimal simple rule and optimal monetary policy we the economy is affected by cost-push shocks.

4.2 Simple rules and determinacy in OLG models

The OLG model modifies the standard Ricardian framework through the introduction of inter-generational inequalities and the main driver of intergenerational inequalities is asset prices evolution. In a Ricardian model, asset prices does not matter but some financial frictions exists (Bernanke et al., 1999). In the OLG model, asset prices matter both for the macroeconomic dynamic and welfare because it induces variation of intergenerational inequalities, whatever financial frictions are present or not. As far as some real time information on this new variable of the model (inequalities) is available to monetary authorities, the policy rule may consider this variable. It seems difficult to collect monthly data on inequalities and to disentangle the intergenerational dimension from others. But data on asset prices exists. We thus consider a monetary policy rule such that the nominal interest rate react to inflation, the output-gap and asset prices that are encompassed by the variable Q.

$$\log\left(\frac{R_t}{R^{ss}}\right) = \phi_\pi \log\left(\frac{\pi_t}{\pi^{ss}}\right) + \phi_c \log\left(\frac{C_t}{C^{ss}}\right) + \phi_q \log\left(\frac{Q_t}{Q^{ss}}\right) \tag{40}$$

The main condition that insures determinacy of the dynamic and inflation anchoring turns into (see Appendix C for the complete set of conditions)

$$[\phi_\pi - 1] + \frac{(1-\mathcal{F})\nu\kappa}{(1-\nu\mathcal{F})(1-\nu)}\phi_c - \frac{(1-\mathcal{F})^2\nu}{(1-\nu\mathcal{F})(1-\nu)(1-\mathcal{F}\mathcal{G})\mu}[\phi_q - (1-\mathcal{G})] > 0. \tag{41}$$

To our knowledge, this condition have not already been derived in the general case. Airaudo et al. (2007) derives a similar relation in a special case with a policy rule that includes forward variables. Nisticò (2012) has already studied numerically the determinacy condition and emphasizes the positive slope of the determinacy frontier in the plane (ϕ_q, ϕ_π). Piergallini (2006) had noticed that, absent reaction on the output-gap and on equity prices ($\phi_c = 0$ and $\phi_q = 0$), the Taylor principle does not hold and determinacy occurs even if $\phi_\pi = 1$.[5]

4.3 How monetary policy affects intergenerational inequalities?

As far as monetary policy is able to modify the relative consumption of the young, it can attenuate or exacerbate the welfare loss due to rising inequalities. An interest-rate drop

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[5]The model developed by the author is different: firm's shares are not traded and money is the only asset households accumulate along the life-cycle. As a consequence, consumption is independent from the stock market.

reduces margins (the share of capital income in value added). The financial wealth to GDP ratio, Equation (38), and the human wealth to GDP ratio, Equation (39), shows that the ratio of human wealth to financial wealth increases and so a monetary expansion reallocates wealth from the old to the young.

In our purely forward model, the dynamic evolution of variables following a shock reduces to the impact. The effect of monetary policy can thus be analyzed through the sacrifice ratio, i.e. the ratio between the consumption surge and the inflation surge when the monetary shock occurs. However, in addition to consumption and inflation, the relative consumption of the young also enters the social welfare. The sacrifice ratio now has two components : the aggregate consumption to inflation ratio and the relative consumption to inflation ratio.

The elasticity of labor supply is the main parameter driving the two components of the sacrifice ratio as depicted in Figure 5, panel b. The largest κ, the highest the elasticity of labor supply and the largest consumption decreases following a restrictive monetary shock. The inequalities component of the sacrifice ratio varies the other way around: low values of κ are related to a large redistribution of consumption from the young to the old following a restrictive monetary shock. For our baseline calibration ($\kappa = 0.1$), the sacrifice ratio is around 2 for consumption (1% decrease in inflation leads to a 2% decrease in aggregate consumption) and around 6 for the relative consumption of the young (1% decrease in inflation leads to a 6% decrease in the relative consumption of the young). This illustrates the potentially high effect of monetary policy on redistribution.

Following a mark-up shock, both aggregate consumption and relative consumption goes down (see Figure 5, panel a) in the natural equilibrium, creating variations of the welfare-relevant output-gap and inequalities. A monetary-policy shock is able to increase simultaneously aggregate consumption and relative consumption and thus to reduce the welfare cost induces by the mark-up shock, at the cost of a higher inflation. The two trade-offs push monetary in the same direction: inducing inflation to mitigate the mark-up shock.

4.4   Optimal simple rules and optimal monetary policy

Depending on the value of κ, monetary policy turns more or less effective in accommodating re-distributive shocks. If κ is low (low the elasticity of labor supply to real wage), a monetary expansion induces a large wage increase and so a large reallocation of wealth from the old to the young. In our simulations, we restrict to the case where the natural interest rate is equal to $1/\beta$ at the steady-state; the parameter γ, the growth rate of time endowment, is adjusted in accordance. For a low elasticity of labor supply (low κ), monetary policy should react actively to inequalities.

We first evaluate the optimal simple rule when the mark-up shock has no persistence. As the model is purely forward, the dynamic reduces to the contemporaneous reaction of macroe-
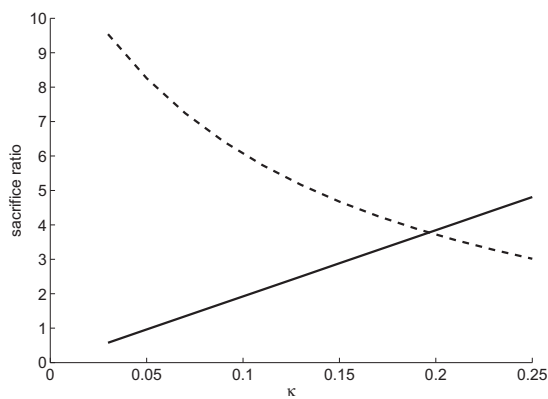
Figure 2: The monetary policy trade-off induces by mark-up shocks

(a) Mark-up shock, natural equilibrium
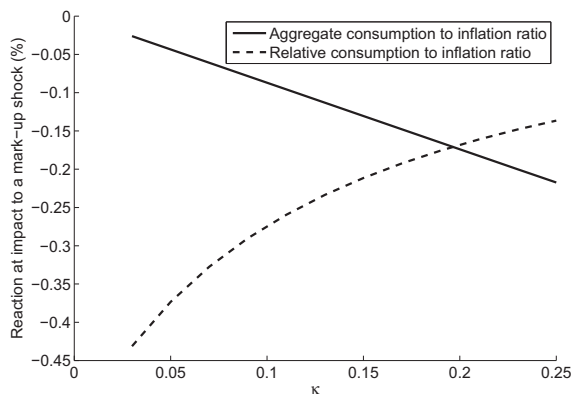
For each value of κ, the figure plots the contemporaneous reaction of aggregate consumption and the relative consumption of the young following a mark-up shock when prices are flexible (natural equilibrium).

(b) Monetary shock

For each value of κ, the figure plots the contemporaneous reaction of the aggregate consumption to inflation ratio and the relative consumption of the young to inflation ratio following a monetary shock when prices are rigid.

Figure 3: Optimal simple rule to a mark-up shock
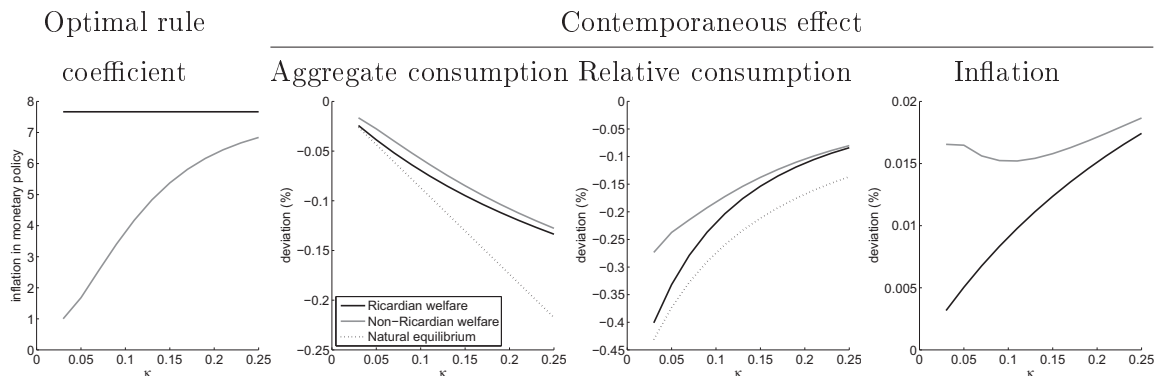
For each value of κ, the first panel plots the optimal value of ϕ_π in the monetary policy rule, and panels 2 to 4 plot the contemporaneous reaction of aggregate consumption, the relative consumption of the young ans inflation to a mark-up shock when monetary policy is optimal.

conomic variables. Furthermore, the optimal simple rule and optimal monetary policy are the same. Figure 3, left panel, depicts the value of the optimal reaction of the policy rule to contemporaneous inflation when the central bank considers the Ricardian welfare (absent inequalities in the social welfare) or the non-Ricardian welfare. For low values of κ, the optimal simple rule is much less reactive when taking into account inequalities than in the correspondent Ricardian model. This induces a higher inflation rate following a mark-up shock. As a consequence, the output-gap and inequalities are more stabilized.

To check the robustness of the result, Figure 4 represents the impulse response function following a positive mark-up shock when the shock is autocorrelated: $\epsilon_t^\mu = \rho^\mu \epsilon_{t-1}^\mu + \nu_t^\mu$, with $\rho^\mu = 0.8$ in Equation (31). Four different cases are considered:

1. Optimal monetary policy with the Ricardian welfare criterion. A mark-up shock induces a trade-off between inflation and the output-gap (consumption in this context). Monetary policy avoid a surge in inflation in the short run through an increase in the nominal real rate, triggering a consumption drop (black, plain lines). The equity price to GDP ratio increases denoting a redistribution of wealth from workers to owners.

2. Optimal monetary policy with the non-Ricardian welfare criterion. The same mark-up shock is accommodated differently when monetary policy takes into account inequalities into account. Any increase in the interest rate lowers the real wage and benefit the owners. The interest rate increase will be lower than in the previous case, inflation increases by more (grey, plain lines).

3. Optimal simple rule with the Ricardian welfare. We assume that the economy is hit by mark-up shocks only. The optimal simple rule is given by $\phi_\pi = 21.5$, $\phi_c = \phi_q = 0$. Inflation is kept low, consumption deceases by large and equity price benefits from monetary policy in the short run (black, doted lines).

21

Figure 4: IRF to a mark-up shock

consumption   inflation   nominal interest rate   Equity price

—— Optimal monetary policy with the Ricardian welfare
—— Optimal monetary policy with the non-Ricardian welfare
– – Optimal simple rule with the Ricardian welfare
– – Optimal simple rule with the non-Ricardian welfare
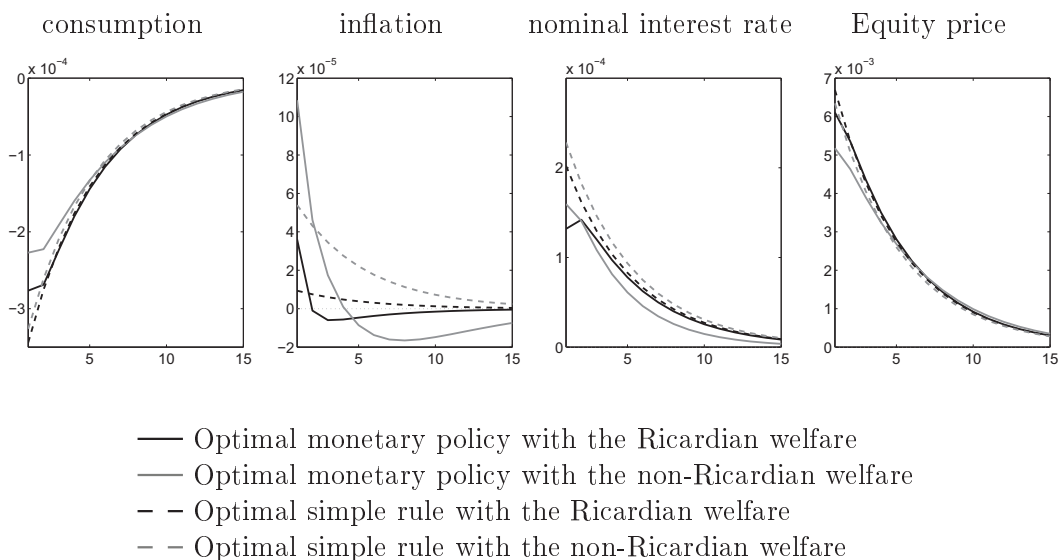
4. Optimal simple rule with the non-Ricardian welfare. The optimal simple rule is given by $\phi_\pi = 4.2$ $\phi_c = \phi_q = 0$. Inflation is 6 times the level of the previous rule, whereas both consumption and the equity price to GDP ratio deviations are limited (grey, dotted lines).

When monetary policy fully consider its impact on intergenerational redistribution, the inflation-output gap trade-off is shift in the favor of the latter. A monetary expansion both favors aggregate consumption (the IS equation) and the relative consumption of the young (the real wage increases). At the natural equilibrium, a mark-up shock also shifts aggregate consumption and the relative consumption of the young in the same direction, monetary policy enjoys a double dividend by letting inflation rise.

4.5 Sticky wages, re-distributive shocks and monetary policy

The design of monetary policy in OLG models departs from the Ricardian as far as (i) shocks lead to variations of inequalities (the relative consumption of the young) at the natural equilibrium and (ii) monetary policy re-distribute wealth between young and older households.

Our simple model encompasses two different shocks: A productivity shock (that has no impact on redistribution) and a mark-up shock (that drives aggregate consumption and relative consumption in the same direction at the natural equilibrium). Based on this analysis, other type of shocks may be considered. A candidate for a shock with redistribution issues is a "credit shock". Assume that firms faces credit frictions *à la* Bernanke et al. (1999). If dividends are more discounted than implied by the interest rate, a positive credit shock (more external funds are at the disposal of firms) allows firms to pay more dividends today:

Figure 5: Sacrifice ratio for different values of price and wage rigidities

Aggregate consumption to inflation ratio   Relative consumption to inflation ratio
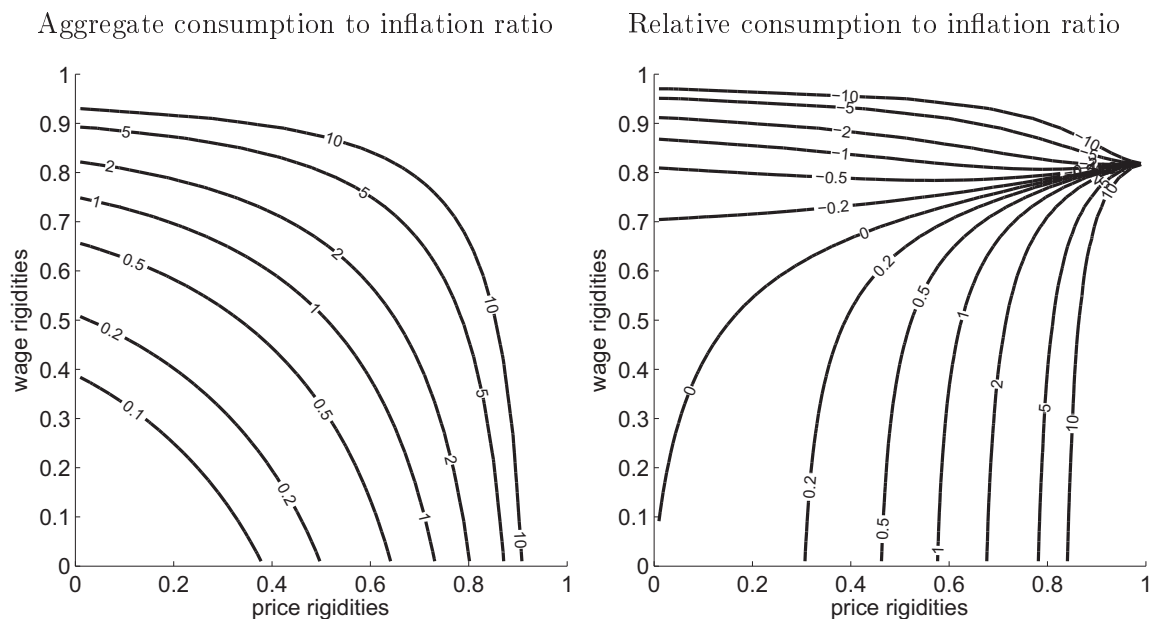
leverage of firms increases. The stock market goes up, redistributing wealth from the young to the old. Such a shock decreases the relative consumption at the natural equilibrium. In the same vein, the housing bubble of the 2000's (related to the ability of households to borrow) has redistributed wealth from households that owned some real estate before the bubble to households that borrowed to buy a house during the bubble or households that missed the rally. Given the credit constraints that household also faces, the housing bubble shock may have induced both an aggregate consumption increase and a relative consumption decrease (the two variables goes in opposite direction).

The way monetary policy should accommodate these shocks depends on the effect of monetary policy on the relative consumption of the young. Absent wage rigidities, an expansionary monetary-policy reallocates income from the old to the young (the relative consumption increases) and raises aggregate consumption. For sizable wage rigidities, an expansionary monetary-policy reallocates income from the young to the old (the relative consumption drops) whereas aggregate consumption surges. Depending on the relative size of price and wage rigidities, monetary policy reallocates wealth in one way or the other. Figure 5 plots the two components of the sacrifice ratio for different values of price rigidities and wage rigidities, and for $\kappa = 0.1$. The sacrifice ratio for aggregate consumption is always positive and increases both with wage and price rigidities. The sacrifice ratio for relative consumption is either positive or negative, depending on wage and price rigidities, and turns negative when wage rigidities increase.

The way monetary policy should react to different shocks depending on the relative size of wage and price rigidities is sum up in Table 2. Absent efficient and reactive re-distributive

23

Table 2: How should monetary policy reacts?

	Low wage rigidities	High wage rigidities
Productivity shock	Inflation targeting in both RIC and OLG cases	Inflation targeting in both RIC and OLG cases
Mark-up shock	OLG monetary policy is less reactive to inflation than RIC monetary policy	OLG monetary policy is more reactive to inflation than RIC monetary policy
Firms' credit shock	Ric monetary policy is no reactive at all, OLG monetary policy eases	Ric monetary policy is no reactive at all, OLG monetary policy is restrictive
Households' credit shock	OLG monetary policy is less restrictive than RIC monetary policy	OLG monetary policy is more restrictive than RIC monetary policy

RIC and OLG monetary policies stand for monetary policy that do not take into account inequalities in the social welfare (like in a Ricardian model) and monetary policy that fully takes into account inequalities between cohorts.

fiscal policy, taking into account the impact of the stock market (or the house price) on owners/non owners (i.e. the old / the young) inequalities should be in the mandate of a central bank.

5 Conclusion

The analysis of monetary policy in an overlapping generation model allows to introduce in an elegant manner some heterogeneity between households. Heterogeneity resumes into a single term, the relative consumption of the young, that enters both in the macroeconomic dynamic and the social welfare criterion. As the main difference between generations is the origin of income (compensation for young households, financial income old ones), the main driver of intergenerational heterogeneities is asset prices. In our framework, monetary policy should consider asset prices not only because it may drive consumption (the wealth effect) and so inflation in the medium run, but also because asset price variations modify the social welfare.

Absent wage rigidities, a monetary policy expansion redistributes wealth from the old to the young: aggregate consumption and the relative consumption of the young shift in the same direction. A positive mark-up shock reduces both consumption and the relative consumption of the young at the natural equilibrium, such that the trade-off of monetary policy between inflation and the output-gap is augmented by a new trade-off between inflation and inequalities. The optimal monetary policy is then biased in favor of the output-gap: monetary policy should be less active.

In our analysis, wages are flexible such that a positive output-gap is related to a reduced mark-up of firms. An expansive monetary policy redistributes wealth from the old to the young.

Sticky wages may reverse the result and an expansive monetary policy shock redistributes, at least in the short run, wealth from the young to the old.

Monetary policy is challenged by redistribution when a shock induces a variation of the relative consumption of the young in the natural equilibrium. Apart from a mark-up shock, a shock on the ability of firms or households to find external funds also raises re-distributive issues. If the discount rate of future income is larger than the interest rate, a positive "credit shock" redistributes wealth from the young to the old. A quantitative analysis is still needed to evaluate the relative role of fiscal, macro-prudential and monetary policy to accommodate the short to medium run impact of the price of assets on intergenerational inequalities. This issue is particularly important in a monetary union where real estate prices of the different countries are not correlated. The role of fiscal policy should be enhanced as the single monetary policy can not fit all.

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A   Proof of Proposition 1

We assume a taxation scheme of the form $\mathcal{T}_{t,t} = \mathcal{T}_t^y$ and $\mathcal{T}_{a,t} = \mathcal{T}_t^o$ for $a < t$ (specific taxation for the young). The value of the relative consumption of the young writes

$$\chi_t = \frac{HW_{t,t} - \mathbb{E}_t \left\{ \sum_{s=1}^{\infty} \theta^s \mathcal{F}_t^{t+s} \mathcal{T}_{t+s}^o \right\} - \mathcal{T}_t^y}{EW_t + HW_t - \mathbb{E}_t \left\{ \sum_{s=1}^{\infty} \theta^s \mathcal{F}_t^{t+s} \mathcal{T}_{t+s}^o \right\} - \mathcal{T}_t}.$$

Let $(\tilde{\chi}_t)_{t \geq 0}$ the desired sequence of the relative consumption of the young (assuming $0 < \underline{\chi} < \tilde{\chi}_t < \overline{\chi} < \infty$), it is sufficient to set

$$\mathcal{T}_t^y = \mathcal{T}_t + HW_{t,t} - \tilde{\chi}_t(EW_t + HW_t) + (\tilde{\chi}_t - 1)\mathbb{E}_t \left\{ \sum_{s=1}^{\infty} \theta^s \mathcal{F}_t^{t+s} \mathcal{T}_{t+s}^o \right\}.$$

The dynamic properties of this kind of taxation scheme (Blanchard-Kahn conditions) has to be analyzed jointly with monetary policy rules.

B   Derivation of aggregate welfare

Aggregate welfare, Equation (26), can be rewrite

$$\mathrm{W}_t = \mathbb{E}_t \left\{ \sum_{s \geq 0} \beta^s \left[\log(C_{t+s}) - \kappa \log \left(\frac{\tau W_{t+s}}{P_{t+s}} \right) \right] + \sum_{s \geq 0} \beta^s \left[(1-\theta) \sum_{a \leq t+s} \theta^{t+s-a} \log \left(\frac{C_{a,t+s}}{C_{t+s}} \right) \right] \right\}.$$
$$(42)$$

The first sum exhibit only aggregate variables and is equal the expression of welfare in the infinitely-lived households model. The second sum exhibits the term $\mathcal{I}_t = (1-\theta) \sum_{a \leq t} \theta^{t-a} \log \left(C_{a,t}/C_t \right)$ which is specific to olg models. This last term is always non positive and is null if and only if cohorts share the same level of consumption: it accounts for inequalities among households. To compute it, we isolate the first term (relative consumption of the young) and we use the perfect risk-sharing among alive households hypothesis, Equation (4).

$$\mathcal{I}_t = (1-\theta)\sum_{a\leq t}\theta^{t-a}\log\left(\frac{C_{a,t}}{C_t}\right)$$

$$= (1-\theta)\log\left(\frac{C_{t,t}}{C_t}\right) + (1-\theta)\sum_{a\leq t-1}\theta^{t-a}\log\left(\frac{C_{a,t}}{C_t}\right)$$

$$= (1-\theta)\log\left(\chi_t\right) + (1-\theta)\sum_{a\leq t-1}\theta^{t-a}\log\left(\frac{\beta}{\mathcal{F}_{t-1}^t}\frac{C_{t-1}}{C_t}\frac{C_{a,t-1}}{C_{t-1}}\right)$$

$$= (1-\theta)\log\left(\chi_t\right) + \theta\log\left(\chi_t + \frac{1-\chi_t}{\theta}\right) + \theta\mathcal{I}_{t-1}$$

$$= \sum_{t'\leq t}\theta^{t-t'}\left[(1-\theta)\log\left(\chi_{t'}\right) + \theta\log\left(\chi_{t'} + \frac{1-\chi_{t'}}{\theta}\right)\right] \tag{43}$$

C   Determinacy

C.1   Contemporaneous rules

We assume a policy rule of the form $i_t = \phi_\pi \pi_t + \phi_c \hat{c}_t + \phi_q \hat{q}_t$. The recursive dynamic of the sticky price model reduces to (we skip any natural variable that do not interact with BK conditions)

$$\begin{bmatrix} 1+\phi_c & \phi_\pi & \phi_q \\ -\frac{(1-\nu\mathcal{F})(1-\nu)}{\kappa\nu} & 1 & 0 \\ \frac{1-\mathcal{F}}{\kappa\mu} & 0 & 1 \end{bmatrix}\begin{pmatrix} \tilde{c}_t \\ \pi_t \\ \tilde{q}_t \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1-\mathcal{G} \\ 0 & \mathcal{F} & 0 \\ 0 & 0 & \mathcal{F}\mathcal{G} \end{bmatrix}\mathbb{E}_t\begin{pmatrix} \tilde{c}_{t+1} \\ \pi_{t+1} \\ \tilde{q}_{t+1} \end{pmatrix},$$

or, equivalently, $AX_t = BX_{t+1}$.

The characteristic polynomial   We look for the set of parameters $(\phi_\pi, \phi_c, \phi_q)$ such that the model meets the Blanchard-Kahn conditions. With three forward variables, every root of the characteristic polynomial $P(\lambda) = \det(A - \lambda B)$ has to lie out the unit circle. To simplify the exposition, we adopt some new notations:

$$\epsilon = \frac{(1-\nu\mathcal{F})(1-\nu)}{\kappa\nu}, \qquad x = \frac{1}{\kappa\mu}.$$

The determinant writes

$$P(\lambda) = \begin{vmatrix} \phi_c + (1-\lambda) & \phi_\pi - \lambda & \phi_q - (1-\mathcal{G})\lambda \\ -\epsilon & 1 - \mathcal{F}\lambda & 0 \\ (1-\mathcal{F})x & 0 & 1-\mathcal{F}\mathcal{G}\lambda \end{vmatrix}.$$

29

Table C.3: Coefficient of polynomial P

	$\epsilon\phi_\pi$	ϵ	ϕ_c	1	$(1-\mathcal{F})x\phi_q$	$(1-\mathcal{F})x(1-\mathcal{G})$
$a_0 =$	1	0	1	1	-1	0
$a_1 =$	$-\mathcal{F}\mathcal{G}$	-1	$-\mathcal{F}(1+\mathcal{G})$	$-(1+\mathcal{F}+\mathcal{F}\mathcal{G})$	$\mathcal{F}$	1
$a_2 =$	0	$\mathcal{F}\mathcal{G}$	$\mathcal{F}^2\mathcal{G}$	$\mathcal{F}(1+\mathcal{G}+\mathcal{F}\mathcal{G})$	0	$-\mathcal{F}$
$a_3 =$	0	0	0	$-\mathcal{F}^2\mathcal{G}$	0	0

Table C.4: Coefficient of polynomial Q

	$\epsilon\phi_\pi$	ϵ	ϕ_c	1	$(1-\mathcal{F})x\phi_q$	$(1-\mathcal{F})x(1-\mathcal{G})$
$b_0 =$	$1-\mathcal{F}\mathcal{G}$	$-(1-\mathcal{F}\mathcal{G})$	$(1-\mathcal{F})(1-\mathcal{F}\mathcal{G})$	0	$-(1-\mathcal{F})$	$1-\mathcal{F}$
$b_1 =$	$3-\mathcal{F}\mathcal{G}$	$-(1+\mathcal{F}\mathcal{G})$	$4-(1+\mathcal{F})(1+\mathcal{F}\mathcal{G})$	$2(1-\mathcal{F})(1-\mathcal{F}\mathcal{G})$	$-(3-\mathcal{F})$	$1+\mathcal{F}$
$b_2 =$	$3+\mathcal{F}\mathcal{G}$	$1-\mathcal{F}\mathcal{G}$	$4-(1-\mathcal{F})(1-\mathcal{F}\mathcal{G})$	$4(1-\mathcal{F}^2\mathcal{G})$	$-(3+\mathcal{F})$	$-(1-\mathcal{F})$
$b_3 =$	$1+\mathcal{F}\mathcal{G}$	$1+\mathcal{F}\mathcal{G}$	$(1+\mathcal{F})(1+\mathcal{F}\mathcal{G})$	$2(1+\mathcal{F})(1+\mathcal{F}\mathcal{G})$	$-(1+\mathcal{F})$	$-(1+\mathcal{F})$

The first line of the determinant is developed

$$P(\lambda) = [\phi_c+(1-\lambda)]\begin{vmatrix} 1-\mathcal{F}\lambda & 0 \\ 0 & 1-\mathcal{F}\mathcal{G}\lambda \end{vmatrix} - [\phi_\pi-\lambda]\begin{vmatrix} -\epsilon & 0 \\ (1-\mathcal{F})x & 1-\mathcal{F}\mathcal{G}\lambda \end{vmatrix} + [\phi_q-(1-\mathcal{G})\lambda]\begin{vmatrix} -\epsilon & 1-\mathcal{F}\lambda \\ (1-\mathcal{F})x & 0 \end{vmatrix}.$$

The polynomial writes $P(\lambda) = a_0+a_1\lambda+a_2\lambda^2+a_3\lambda^3$ with the coefficients given in Table C.3.

C.2 Conditions for stability

The conditions for the roots of a given polynomial to lie out the unit circle are uneasy to write. We then use the transformation of the complex plane that maps the unit circle with complex numbers with a positive real part ($z \to \frac{1-z}{1+z}$). So we define $Q(X) = P\left(\frac{1-X}{1+X}\right)(1+X)^3$. The roots of P are out the unit circle if and only if the roots of Q have a negative real part. If $P(\lambda) = \sum_{i=0}^{3} a_i\lambda^i$ and $Q(\lambda) = \sum_{i=0}^{3} b_i\lambda^i$, then

$$b_0 = a_0 + a_1 + a_2 + a_3$$
$$b_1 = 3[a_0 - a_3] + [a_1 - a_2]$$
$$b_2 = 3[a_3 + a_0] - [a_2 + a_1]$$
$$b_3 = a_0 - a_1 + a_2 - a_3$$

The coefficient of polynomial Q is given in Table C.4.

The condition that "all the roots of Q have a negative real part" can be expressed as conditions on its coefficients (Hurwitz, 1964).

Proposition 3 (Routh-Hurwitz criterion). *Every root of polynomial Q has a strictly negative real part if and only if one of the two following conditions is satisfied (i) $b_0, b_1, b_2, b_3 > 0$ and $b_2 b_1 > b_0 b_3$; or (ii) $b_0, b_1, b_2, b_3 < 0$ and $b_2 b_1 > b_0 b_3$.*

In our particular case, the Routh-Hurwitz criterion can be simplified in

Proposition 4. *For given ϕ_c and ϕ_q, the dynamic system meets the Blanchard-Kahn condition is and only if one of the two conditions holds (i) $b_0, b_3 > 0$ and ϕ_π is larger than the largest real root of $b_1 b_2 - b_0 b_3 = 0$, or (ii) $b_0, b_3 < 0$ and ϕ_π is smaller than the smallest real root of $b_1 b_2 - b_0 b_3 = 0$.*

The demonstration of this proposition is pure geometry and relies on conic properties. The proposition settles that one can forget conditions $b_1 = 0$ and $b_2 = 0$ to draw the boundary of the determinacy area.

The condition $b_0 > 0$ or $b_0 < 0$

$$b_0 = (1 - \mathcal{F}\mathcal{G})\epsilon[\phi_\pi - 1] + (1 - \mathcal{F}\mathcal{G})(1 - \mathcal{F})\phi_c - (1 - \mathcal{F})^2 x[\phi_q - (1 - \mathcal{G})]. \tag{44}$$

This equation for $b_0 > 0$ is the generalization of the Taylor principle in overlapping-generation model.

The condition $b_3 > 0$ or $b_3 < 0$

$$b_3 = (1 + \mathcal{F}\mathcal{G})\epsilon[\phi_\pi + 1] + (1 + \mathcal{F}\mathcal{G})(1 + \mathcal{F})[\phi_c + 2] - (1 - \mathcal{F}^2)x[\phi_q + (1 - \mathcal{G})]. \tag{45}$$

Figure C.6 plots the determinacy area.

D   Alternative utility function

Piergallini (2004) develops the OLG model with an alternative utility function given by
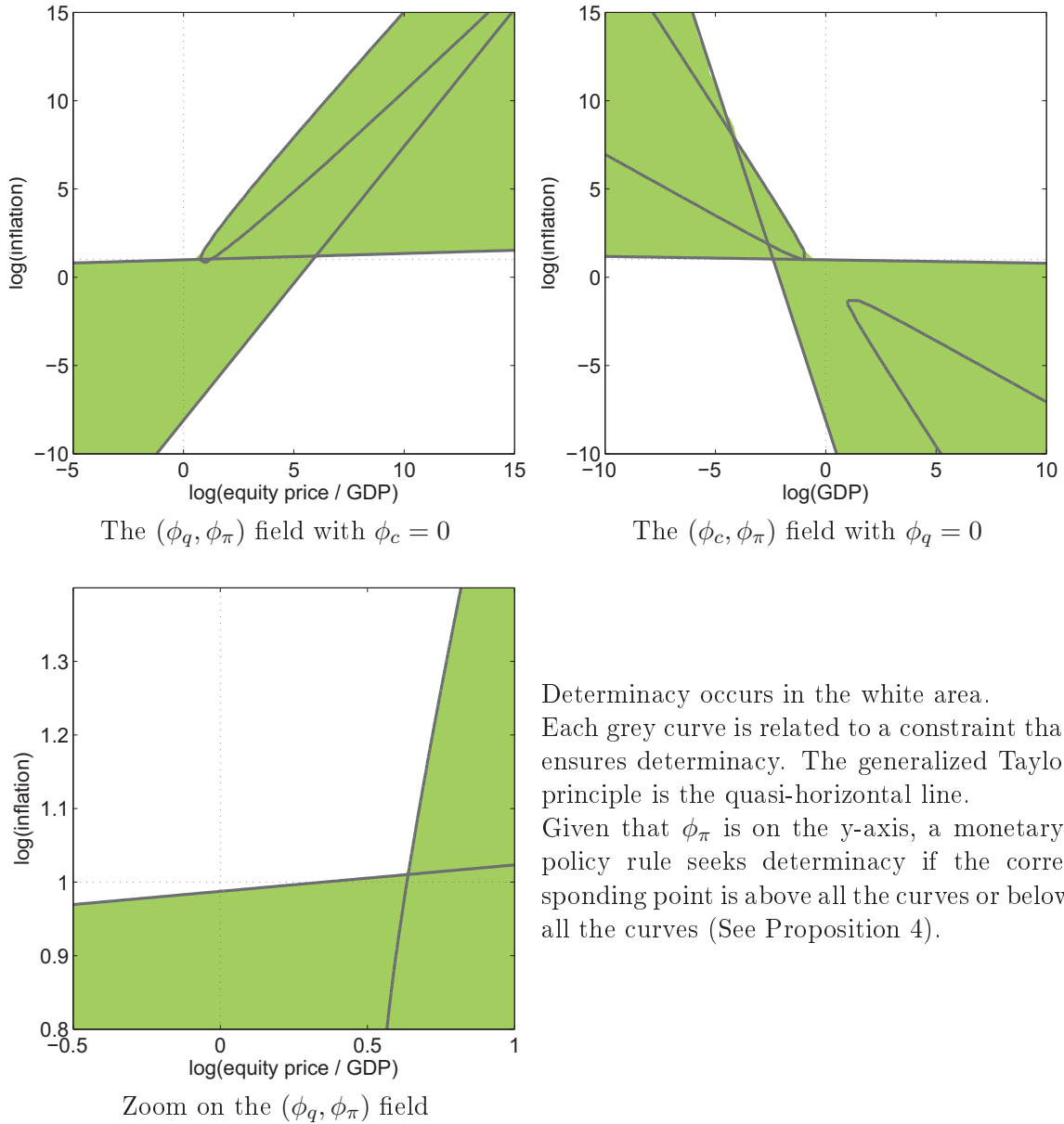
$$U(C_{a,t}, L_{a,t}) = \log(C_{a,t} - V(L_{a,t})), \tag{46}$$

where $V' > 0$ and $V'' > 0$. This utility function have been proposed by Ascari & Rankin (2007) in order to rule out negative labor supply which may arise for older households.

D.1   Aggregate consumption and labor supply

Given the budget constraint (2), this functional form modifies the FOC for contingent claims holding and labor supply

Figure C.6: Determinacy area of the simple rule in the flexible wage model

The (ϕ_q, ϕ_π) field with $\phi_c = 0$

The (ϕ_c, ϕ_π) field with $\phi_q = 0$

Zoom on the (ϕ_q, ϕ_π) field

Determinacy occurs in the white area.
Each grey curve is related to a constraint that ensures determinacy. The generalized Taylor principle is the quasi-horizontal line.
Given that ϕ_π is on the y-axis, a monetary-policy rule seeks determinacy if the corresponding point is above all the curves or below all the curves (See Proposition 4).

$$\mathcal{F}_t^{t+1} = \beta \frac{\tilde{C}_{a,t}}{\tilde{C}_{a,t+1}}, \qquad (47) \qquad\qquad V'(L_{a,t}) = \mathrm{mrs}_t, \qquad (48)$$

with $\tilde{C}_{a,t} = C_{a,t} - V(L_{a,t})$. The labor supply is independent of the cohort ($L_{a,t} = L_t$). The forward iteration of the budget constraint writes

$$\mathbb{E}_t \left\{ \sum_{s \geq 0} \theta^s \mathcal{F}_t^{t+s} \tilde{C}_{a,t+s} \right\} = \Omega_{a,t} + \mathbb{E}_t \left\{ \sum_{s \geq 0} \theta^s \mathcal{F}_t^{t+s} \left(\frac{W_{t+s}}{P_{t+s}} L_{t+s} - V(L_{t+s}) \right) \right\}.$$

The first order condition on contingent claims implies

$$\tilde{C}_{a,t} = (1{-}\beta\theta) \left[\Omega_{a,t} + HW_t \right], \qquad \text{with} \qquad HW_t = \mathbb{E}_t \left\{ \sum_{s \geq 0} \theta^s \mathcal{F}_t^{t+s} \left(\frac{W_{t+s}}{P_{t+s}} L_{t+s} - V(L_{t+s}) \right) \right\}.$$

Aggregation across cohorts writes $\tilde{C}_t = (1{-}\beta\theta) \left[EW_t + HW_t \right]$.

D.2 Aggregate welfare

The social welfare writes

$$\mathrm{W}_t = \mathbb{E}_t \left\{ \sum_{s \geq 0} \beta^s \left[\log(\tilde{C}_{t+s}) + \mathcal{I}_{t+s} \right] \right\} \qquad \text{where} \qquad \mathcal{I}_t = (1{-}\theta) \sum_{a \leq t} \theta^{t-a} \log\left(\frac{\tilde{C}_{a,t}}{\tilde{C}_t} \right).$$

The calculation of $\mathcal{I}_t$ follows Section B when one has replaced χ_t by $\tilde{\chi}_t = \frac{\tilde{C}_{t,t}}{\tilde{C}_t}$ in Equation (14). This leads to

$$\mathcal{I}_t = (1{-}\theta) \log\left(\tilde{\chi}_t \right) + \theta \log\left(\tilde{\chi}_t + \frac{1 - \tilde{\chi}_t}{\theta} \right) + \theta \mathcal{I}_{t-1}$$