ISSN: 2281-1346

Department of Economics and Management

DEM Working Paper Series

Transparency, Expectations Anchoring and the Inflation Target

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 # 22 (11-12)

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http://epmq.unipv.eu/site/home.html

November 2012

Transparency, Expectations Anchoring and the Inflation Target *

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Abstract

This paper proves that a higher inflation target unanchors expectations, as feared by Fed Chairman Bernanke. It does so both asymptotically, because it shrinks the E-stability region when a central bank follows a Taylor rule, and in the transition phase, because it slows down the speed of convergence of expectations. Moreover, the higher the inflation target, the more the policy should respond to inflation and the less to output to guarantee E-stability. Hence, a policy that increases the inflation target and increase the monetary policy response to output would be "reckless". Moreover, we show that transparency is an essential component of the inflation targeting framework and it helps anchoring expectations. However, the importance of being transparent diminishes with the level of the inflation target.

JEL classification: E5.

Keywords: Trend Inflation, Learning, Monetary Policy, Transparency.

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*The authors thank Klaus Adam, Alessandro Gobbi and Seppo Honkapohja for helpful comments. Ascari thanks the MIUR for financial support through the PRIN 09 programme. The usual disclaimer applies.

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1 Introduction

Blanchard et al. (2010) have recently proposed to increase the central bank's inflation target in order to deal with the problem of the zero lower bound on interest rates. In various speeches, Fed Chairman Bernanke contrasted the Blanchard et al. (2010) argument because of the fear that an higher inflation target could unanchor inflation expectations.[1] The New Keynesian literature has convincingly shown that price stability should be the goal of monetary policy even taking into account the perils of hitting the zero lower bound (e.g., Coibion and Gorodnichenko, 2010, and Schmitt-Grohè and Uribe, 2010). However, these papers can not address the Fed Chairman's concern about the possibility that a higher inflation target could unanchor inflation expectations. A natural framework to study such an issue is learning, as suggested by Bernanke himself.[2] In this paper, we therefore consider a New Keynesian macromodel with trend inflation and learning to answer the following research question: would it be more difficult for the central bank to stabilize inflation expectations at higher values of the inflation target? We thus investigate the link between inflation expectations under adaptive learning and the level of the inflation target. We characterize how the set of policy rules that guarantees E-stability of the rational expectations equilibrium (REE) changes with the inflation target. This would allow us to address questions like: if the central bank targets a higher inflation level, does it need to respond more aggressively to inflation to stabilize expectations?

Moreover, another main component of an inflation targeting framework is the communication strategy.[3] We aim to capture this element by distinguishing between trans-

[1] *"In this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence [...]. Inflation expectations would also likely become significantly less stable"*, Chairman Ben S. Bernanke, remarks at the 2010 Jackson Hole Symposium.

[2] *"What is the right conceptual framework for thinking about inflation expectations in the current context? [...] Although variations in the extent to which inflation expectations are anchored are not easily handled in a traditional rational expectations framework, they seem to fit quite naturally into the burgeoning literature on learning in macroeconomics. [...] In a learning context, the concept of anchored expectations is easily formalized"* Fed Chairman Ben S. Bernanke, speech at the NBER Monetary Economics Workshop, July 2007.

[3] *"The second major element of best-practice inflation targeting (in my view) is the communications strategy, the central bank's regular procedures for communicating with the political authorities, the financial markets, and the general public."* Fed Chairman Ben S. Bernanke, speech at the At the Annual

parency and opacity, based on Preston (2006). A central bank is said to be transparent if agents know the policy rule and they use this information in their learning process.[4] If not, it is said to be opaque. Thus, for each level of the inflation target, we study whether the ability to anchor expectations differs under transparency and under opacity. That is, does a central bank that fixes a higher level of inflation target need to be more transparent?

First, we analyze the standard specification of the New Keynesian model, that is, assuming a zero inflation steady state. Economic agents do not have rational expectations but rather form their forecasts by using recursive learning algorithms. This very first part of the paper is similar to Bullard and Mitra (2002), but it adds the distinction between transparency and opacity. Here we are mostly interested in the effects of the communication strategy on the learnability of the REE. More precisely, whether a transparent central bank is better able to anchor inflation expectations, and which features of the economy and of the monetary policy rule affect the different ability of anchoring expectations under transparency and under opacity. The main results of this Section are as follows: (i) transparency helps anchoring expectations, that is, the E-stability region is wider under transparency than under opacity; (ii) a pure inflation targeting central bank needs to be transparent to anchor inflation expectations; (iii) the more flexible are the prices, the more transparency is valuable; (iv) under opacity, a more aggressive response to inflation could destabilize inflation expectations, while a larger response to output tends to stabilize them. Our results, thus, substantiate the claim that transparency is an essential component of the inflation targeting approach

Washington Policy Conference of the National Association of Business Economists, Washington, D.C., March 2003.

[4] *"Why have inflation-targeting central banks emphasized communication, particularly the communication of policy objectives, policy framework, and economic forecasts? [...] a given policy action [...] can have very different effects on the economy, depending (for example) on what the private sector infers from that action about likely future policy actions, about the information that may have induced the policymaker to act, about the policymaker's objectives in taking the action, and so on. [...] Most inflation-targeting central banks have found that effective communication policies are a useful way, in effect, to make the private sector a partner in the policymaking process. To the extent that it can explain its general approach, clarify its plans and objectives, and provide its assessment of the likely evolution of the economy, the central bank should be able to reduce uncertainty, focus and stabilize private-sector expectations".* Fed Chairman Ben S. Bernanke, speech at the At the Annual Washington Policy Conference of the National Association of Business Economists, Washington, D.C., March 2003.

to monetary policy.

We then turn to the analysis of a New Keynesian model with positive steady state inflation with adaptive learning, to tackle the main research question of the paper. The analysis of the zero inflation case proved to be very useful because its analytical results provide very revealing insights for this part of the paper, where only numerical results are possible. We compute, for different values of the inflation target, both the E-stability and the determinacy regions both in case of transparency and in case of no communication by the central bank. The main result of the paper is consistent with the Fed Chairman statement: a higher inflation target tends to destabilize expectations, because it shrinks the E-stability region for a given Taylor rule. Moreover, the higher the inflation target, the more the policy should be hawkish with respect to inflation to stabilize expectations, while it should not respond too much to output. This result completely dismisses arguments often presented in the press.[5] Many distinguished economists urged the Fed Chairman to increase the inflation target and, contemporaneously, ease monetary policy to respond to the surge in unemployment. Our results suggest that this policy would indeed be "reckless" and "unwise", as Bernanke recently put it.[6] Finally, a higher inflation target surprisingly diminishes the need to be transparent, because it reduces the difference in the E-stability region between the two polar cases of transparency and opacity.

Moreover, we analyze the speed of convergence of expectations to the REE under E-stability and determinacy. The slower is the speed of adjustment, the more the economy dynamics is far from REE and dominated by the learning dynamics. If instead the convergence speed is fast, then the economic dynamics will be always very close to the REE. This analysis also proves very useful to shed lights on the previous results. An higher inflation target increases the spectral radius of the matrix that defines the T-map under adaptive learning, and that governs the convergence of the learning algorithm. So

[5] E.g., see the recent article by Paul Krugman, 29 April 2012 in the New Yorlk Times (http://www.nytimes.com/2012/04/29/magazine/chairman-bernanke-should-listen-to-professor-bernanke.html?pagewanted=all&_moc.semityn.www) .

[6] *"I guess the question is, does it make sense to actively seek a higher inflation rate in order to achieve a slightly increased pace of reduction in the unemployment rate? The view of the committee is that that would be very reckless."* Fed Chairman Ben S. Bernanke, FOMC Press Conference transcript, 25th of April 2012, http://www.federalreserve.gov/mediacenter/files/FOMCpresconf20120425.pdf.

the higher the inflation target, the lower is the speed of convergence, but also more likely the economy is going to be, ceteris paribus, E-unstable, as found in the previous section. As a result, an higher inflation target unanchors expectations both asymptotically (E-stability) and in the transition phase, because, conditionally on E-stability, it slows down the speed of convergence of expectations to the REE.

The paper is structured as follows. Section 2 presents the model and the methodology employed. Section 3 contains the results both for the zero inflation target case (Section 3.1) and for positive trend inflation (Section 3.2) and some robustness checks (Section 3.3). Section 4 analyzes the speed of convergence of expectations to the REE. Section 5 concludes.

1.1 Related literature

Our paper is strictly linked to the seminal paper by Bullard and Mitra (2002) and two more recent contributions: Eusepi and Preston (2010) and Kobayashi and Muto (2011).

Bullard and Mitra (2002) analyse the determinacy and learnability of simple monetary policy rules in a standard New Keynesian model approximated around the zero inflation steady state. Bullard and Mitra (2007) enrich their previous results introducing monetary policy inertia in the same model and showing how it helps to produce learnability of the rational expectation equilibrium. We are basically following their approach which is based on Evans and Honkapohja (2001). With respect to them, we considers transparency versus opacity and we generalize the model to allow the analysis of the case of positive inflation, based on the model in Ascari and Ropele (2009). This is surely the main contribution of the paper. However, we do also provide analytical results for the standard case of zero steady state inflation.

Based on Preston (2006), we study two possible communication strategies by the central bank. Preston (2006) distinguishes two cases: one where agents know the monetary policy rule, call it the transparency case (TR), and the other where they are forced to infer the interest rate by learning it adaptively, call it the opacity case (OP). We analytically characterize the conditions for E-stability under TR and OP in the case of zero trend inflation. As far as we know, this is the first paper that analyses the difference between OP and TR in an Euler equation learning context. Note that we need to assume agents based their expectation on period $t-1$ information, because there would be no difference between TR and OP if agents' decisions are based on current expectations. The intuition we gained through this analysis proved to be very useful for the case of positive inflation when numerical results are the only option.

It follows that our analysis is linked to Preston (2006) and also to the more recent, and related, contribution by Eusepi and Preston (2010). These papers do not consider the case of positive inflation, as we do. Moreover, they employ what Honkapohja et al. (2011) call the infinite horizon approach due to Preston (2005, 2006). Preston derives this model under arbitrary subjective expectations and finds that the model's equations depend on long-horizon expectations that is, on forecasts into the entire infinite future. Eusepi and Preston (2010) further employ Preston's infinite horizon approach (the only change being assuming decisions are made based on period $t-1$ information) to analyze what happens to E-stability when the Taylor principle holds and the central bank employs a variety of communication strategies. Our paper, beside sharing with Eusepi and Preston (2010) the assumption of lagged expectations, is -like theirs- devoted to disentangle the effects of central bank communication on learnability (and, we add, determinacy) of rational expectations equilibria. However, while they employ the infinite horizon approach, we use the more standard Euler equation approach of Evans and Honkapohja (2001). In the former approach agents are assumed to make forecasts over the infinite future, while in the latter agents forecast only one period ahead. So the two approaches represents the two extreme cases of farsightedness. Honkapohja et al. (2011) show that, in the context of a New Keynesian model, the Euler equation approach in Bullard and Mitra (2002) is anyway consistent with Preston (2005), so that *"both the EE and IH approaches are valid ways to study stability under learning in the New Keynesian setting."* (Honkapohja et al., 2011, p. 13).[7] So our analysis in a zero inflation steady state could be seen as a robustness analysis of the results in Eusepi and Preston

[7] See Honkapohja et al. (2011) for a thorough discussion of the two approaches. See also Evans and Honkapohja (2013).

(2010) in the Euler equation context.[8] As theirs, we are able to obtain analytical results that, though different, have similar implications and intuition. Since the analytics of our model are simpler, however, we do not confine the analysis to the cases where the Taylor priciple holds, as Eusepi and Preston (2010), but we can fully characterize the E-stability regions. Moreover, and most importantly, our analysis departs from theirs by analyzing what happens as trend inflation changes. This, obviously, calls for a different model that allows for positive trend inflation.

In this respect, our paper is also close to a recent contribution by Kobayashi and Muto (2011) that studies expectational stability under trend inflation and we get results consistent with their findings. The analysis in Kobayashi and Muto (2011) borrows a NKPC formulation under trend inflation (see Sbordone, 2007 and Cogley and Sbordone, 2008) and plugs into an otherwise standard New Keynesian model, that is adding an Euler equation and a Taylor rule. This formulation coincides with a simplified version of the model in Ascari and Ropele (2009). Their model, thus, differs from ours because of some simplifying assumptions. The same assumptions (mainly an infinitely elastic labour supply) are made in the analytical part of the paper by Ascari and Ropele (2009). However, both our and Kobayashi and Muto (2011) analysis of the positive inflation case are numerical, so those assumptions are not really needed. This may not be innocuous, because the simplifying assumptions make price dispersion irrelevant for the dynamics of the model.[9] As a consequence, our model has a higher-order system of difference equations and this may affect the results. Furthermore, in contrast with Kobayashi and Muto (2011), we study the effects of central bank's transparency on the anchoring of expectations, by distinguishing between the cases of TR and OP. Moreover, as said above, thanks to our analytical investigation of the case of zero trend inflation, we were able to provide intuition about the effects that trend inflation has on the E-stability regions and on the difference between OP and TR. Further differences between us and

[8] *"The EE and IH approaches to modeling agent's behavior rule are not identical and lead to different detailed learning dynamics. Thus there is in general no guarantee that the convergence conditions for the two dynamics are identical".* (Honkapohja et al., 2011, p. 18).

[9] However, the dynamics of price dispersion is one of the main features of a model with positive trend inflation (with respect to one linearized around zero inflation). It changes the dynamics of the model by adding a backward-looking dynamic equation.

Kobayashi and Muto (2011) are the assumption of lagged expectations, the analysis of the case of inertia in the interest rate rule and of indexation.

Last but not least, none of the above papers study the speed of convergence of expectations to the REE under E-stability and determinacy. Following Ferrero (2007), we characterize the impact of the choice of the inflation target and of the policy response to inflation and output on the speed of convergence of expectation under learning both under TR and OP.

Finally, two other papers employ the Ascari and Ropele (2009) model under learning. In a very insightful paper Branch and Evans (2001) study the dynamics of the model when there is a change in the long-run inflation target, and agents have only imperfect information about the long-run inflation target. They show that imperfect knowledge of the inflation target could generate near-random walk beliefs and unstable dynamics due to self-fulfilling paths. Imperfect information of inflation targets can thus generate instability in inflation rates. A related and very interesting work by Cogley et al. (2010) studies optimal disinflation under learning. When agents have to learn about the new policy rule, then, the optimal disinflation policy is more gradual, and the sacrifice ratio much bigger, than under the case of TR. The optimal disinflation is gradual under OP because the equilibrium law of motion under learning is potentially explosive. However, they find that imperfect information about the policy feedback parameters, rather than about the long-run inflation target, is the crucial source of the explosiveness of the ALM.

2 Model and Methodology

2.1 The Model

The model we use is based on Ascari and Ropele (2009), that extends the basic New Keynesian (NK) model (e.g., Galí 2008, and Woodford, 2003) to allow for positive trend inflation. The details are presented in the Appendix. Log-linearizing the model around a generic positive inflation steady state yields the following equations:

$$\hat{y}_t = E^*_{t-1}\hat{y}_{t+1} - E^*_{t-1}\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) + r^n_t \tag{1}$$

$$\hat{\pi}_t = \beta\bar{\pi}E_{t-1}\pi_{t+1} + \lambda_{\bar{\pi}}E^*_{t-1}\left[(1+\sigma_n)\hat{y}_t + \sigma_n\hat{s}_t\right] + \eta_{\bar{\pi}}E^*_{t-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right] + \lambda_{\bar{\pi}}(1+\sigma_n)u_t \tag{2}$$

$$\hat{\phi}_t = \alpha\beta\bar{\pi}^{(\theta-1)}E^*_{t-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right] \tag{3}$$

$$\hat{s}_t = \xi_{\bar{\pi}}\hat{\pi}_t + \alpha\bar{\pi}^{\theta}\hat{s}_{t-1} \tag{4}$$

$$\hat{\imath}_t = \phi_\pi E^*_{t-1}\hat{\pi}_t + \phi_y E^*_{t-1}\hat{y}_t, \tag{5}$$

where hatted variables denote percentage deviations from steady state, apart from $\hat{y}$, which is the usual output gap term in a NK model defined as deviation from the flexible price output level. The structural parameters and their convolutions ($\lambda_{\bar{\pi}}, \eta_{\bar{\pi}}$ and $\xi_{\bar{\pi}}$) are described in Table 1. r^n_t and u_t are exogenous disturbance terms that follow the processes: $r^n_t = \rho_r r^n_{t-1} + \varepsilon^r_t$ and $u_t = \rho_u u_{t-1} + \varepsilon^u_t$, where ε^r_t and ε^u_t are i.i.d noises and $0 < \rho_r, \rho_u < 1$.

The first equation is the standard Euler Equation in consumption, and r^n_t is the stochastic natural rate of interest. The second and the third equation describe the evolution of inflation in presence of trend inflation, so they are the counterpart of the standard NKPC for the standard zero inflation steady state case, where u_t is a mark-up shock. $\hat{\phi}_t$ is just an auxiliary variable (equals to the present discounted value of future expected marginal revenue) that allows the model to be written in a recursive way. The fourth equation describes the evolution of price dispersion, s. In contrast to the zero inflation steady state case, in presence of positive average inflation price dispersion affects inflation dynamics at first-order approximation and thus has to be taken into account.[10] The fifth equation is the simplest standard contemporaneous Taylor rule.

We deviate from Ascari and Ropele (2009), by following Evans and Honkapohja (2001) and much of the related literature on learning, by assuming that agents have non-rational expectations, that we denote with E^*. Furthermore, we assume that expectations are formed on the basis of period $t-1$ information set (see also Bullard and Mitra, 2002). According to Evans and Honkapohja (2001), this assumption is more nat-

[10] Kobayashi and Muto (2011) do not take into account price dispersion, because they assume a simple proportional relationship between the marginal cost and the output gap. However, as shown in Ascari and Ropele (2009), this is not general and it requires the additional assumption of indivisible labour (i.e., $\sigma_n = 0$).

ural in a learning context, since it avoids simultaneity between expectations and current values of endogenous variables.[11]

Of course, Ascari and Ropele (2009) generalized model of the dynamics of inflation, described by equations (2), (3) and (4), encompasses the standard NKPC. Assuming zero trend inflation $\bar{\pi} = 1$, then $\eta_{\bar{\pi}} = \xi_{\bar{\pi}} = 0$, thus both the auxiliary variable and the measure of relative price dispersion become irrelevant for inflation dynamics. Thus, the above equations turn just into the standard specification of the NK model (where $\kappa = \lambda(1 + \sigma_n)$):

$$\hat{y}_t = E^*_{t-1}\hat{y}_{t+1} - E^*_{t-1}(\hat{\imath}_t - \hat{\pi}_{t+1}) + r^n_t \tag{6}$$

$$\hat{\pi}_t = \beta E^*_{t-1}(\hat{\pi}_{t+1}) + \kappa\hat{y}_t + \kappa u_t. \tag{7}$$

2.2 Methodology

We are interested in analyzing both determinacy and learnability conditions. The determinacy results are obviously the same as in Ascari and Ropele (2009), so we will not comment on those and refer the reader to Ascari and Ropele (2009).

2.2.1 Learnability

When agents do not possess rational expectations, the existence of a determinate equilibrium does not ensure that agents coordinate upon it. As from the seminal contribution of Evans and Honkapohja (2001), we assume agents do not know the true structure of the economy. Rather, they behave as econometricians and learn adaptively, using a recursive least square algorithm based on the data produced by the economy itself. If the REE is learnable, then, the learning dynamics eventually tend toward, and eventually coincide with, the REE. Learnability is an obviously desired feature of monetary policy.

We apply E-stability results outlined in Evans and Honkapohja (2001, section 10.2.1). Agents are assumed to have identical beliefs and to forecast using variables that appear in the minimum state variable (MSV) solution of the system under rational expectations.

[11] *"We have chosen to assume that expectations are conditional on information at time $t-1$. This avoids a simultaneity between expectations and current values of the endogenous variables which may seem more natural in the context of the analysis of learning".* Evans and Honkapohja (2001, p. 229).

Agents' perceived law of motion (PLM) coincides with the system's MSV solution. Given our model, thus, the PLM will not contain any constant term.[12] Agents are assumed to know just the autocorrelation of the shocks but they have to estimate the remaining parameters. Each period, as additional data become available, they re-estimate the coefficients of their model. We then ask whether agents are able to learn the MSV equilibrium of the system (see Appendix for details).

2.2.2 Transparency versus Opacity

In defining OP and TR of monetary policy, we follow closely the work of Preston (2006) and Eusepi and Preston (2010). We assume that the central bank is perfectly credible: the public believes and fully incorporates central bank's announcements. Agents are uncertain about the economy ($\hat{\pi}$ and $\hat{y}$) and about the path of nominal interest rates ($\hat{\imath}$). Communication by the central bank simplifies agents' problem in that it gives them information on how the monetary authority sets interest rates, that is, on the monetary policy strategy. Therefore: (i) under OP, the private sector has to make learning about the economy ($\hat{\pi}$ and $\hat{y}$) and about monetary policy ($\hat{\imath}$); under TR, it needs to forecast just the economy but not the path of nominal interest rates, since the central bank announces its precise reaction function.[13]

In case of TR, we incorporate the reaction function directly in the aggregate demand equation and the agents' problem boils down to forecast inflation and output. This, as we will show, should be of help in anchoring expectations by aligning agents' beliefs with central bank's monetary policy strategy.

[12] Using a PLM with a constant term, our main conclusions do not change. Results are available from the authors upon request.

[13] Alternatively, TR can be defined as in Berardi e Duffy (2007). Under their specification, in the presence of TR the private sector adopts the correct forecast model (it employs a PLM that coincides with the MSV solution, hence without the constant), under OP, instead, they use an overspecified (with a constant) PLM. Incorporating even this specification in our model does not change significantly the results (available from the authors upon request).

3 Results

This Section presents the main results of the paper. We first consider the standard case of a zero inflation target (i.e., zero inflation steady state), for which some analytical results are presented. We then move to the more general and realistic case of a positive inflation target.

3.1 Zero inflation target

The relevant model economy is the standard NK model, as described by the following equations: (5), (6) and (7). This model has been extensively studied in the literature, and Bullard and Mitra (2002) provides us with the seminal contribution regarding learning in this setup. Here, we extend their analysis to the case of TR and OP, as defined above and in Preston (2006). So in what follows, we mainly concentrate on the difference between the TR and OP.

The determinacy conditions are as in Ascari and Ropele (2009):

$$\phi_\pi + \phi_y \frac{1-\beta}{\kappa} > 1 \tag{D1}$$

$$\phi_\pi + \frac{1}{\kappa}\phi_y > \frac{\beta - 1}{\kappa}. \tag{D2}$$

As known (see Woodford 2003, p. 256), (D1) is the "long-run" Taylor principle since $\left(\frac{1-\beta}{\kappa}\right)$ is the long-run multiplier of inflation on output in (7). So (D1) can be interpreted as requiring that the long-run reaction of the nominal interest rate to a permanent change in inflation should be bigger than 1. By the same token, the second condition can be interpreted as requiring that the short-run reaction of the nominal interest rate should be bigger than minus the long-run multiplier of inflation on output.

The first natural question to ask is: *does TR allow a central bank to better anchoring inflation expectations with respect to OP?* The answer is yes, as explained by the following proposition (proof in Appendix).[14]

[14] Our conditions are slightly different from the one in Bullard and Mitra (2002) because our PLM does not have a constant term and we assume lagged expectations. This is also why our conditions

Proposition 1 *The MSV solution is E-stable:*
(i) under TR iff

$$\phi_\pi + \phi_y \frac{1-\beta\rho}{\kappa} > \rho - (1-\rho)\left(\frac{1-\beta\rho}{\kappa}\right) \tag{TR1}$$

$$\phi_y > \rho + \beta\rho - 2 \tag{TR2}$$

(ii) under OP iff (TR1) holds and

$$\phi_y > \frac{1}{(2-\rho)}\left(\Psi(\beta,\rho) + \kappa\phi_\pi\right), \tag{OP}$$

where $\Psi(\beta,\rho) = (\rho + \beta\rho - 2)\left((2-\beta\rho)(2-\rho) - \kappa\rho\right).$

Figure 1 visualizes how the five conditions above define the relevant regions for determinacy and E-stability in both cases of TR and OP in the space (ϕ_π, ϕ_y) implied by Proposition 1. To grasp the main results, it is instructive to confine the analysis to the positive orthant for (ϕ_π, ϕ_y). To facilitate the reader, we highlight the main implications of this proposition, by using a numbered list of results.

Result 1.1 ***Transparency helps anchoring inflation expectations****. Let's* $\phi_\pi, \phi_y \geq 0$*. If the rational expectation equilibrium is determinate (i.e., (D1) holds), then it is always learnable under TR, while this is not true under OP.*

This follows immediately from the fact that for $\phi_y \geq 0$ both (TR2) is always satisfied, and that the long-run Taylor principle (D1) implies (TR1). Note that, if the determinacy conditions hold, then the equilibrium is learnable under TR, but the contrary is not true, in contrast to Bullard and Mitra (2002): E-stability does not imply determinacy.[15] Moreover, while determinacy implies E-stability under TR, this is not true under OP. The learnability region of the parameter space is thus smaller under OP with respect to

would involve the persistence parameter of the shock processes. In what follows, we will have conditions that should hold for both processes, so we should write ρ_i for $i = r, u$. For the sake of brevity we just write ρ without subindex.

[15] Again this is because we do not use the constant term in the PLM (see Section 3.3).

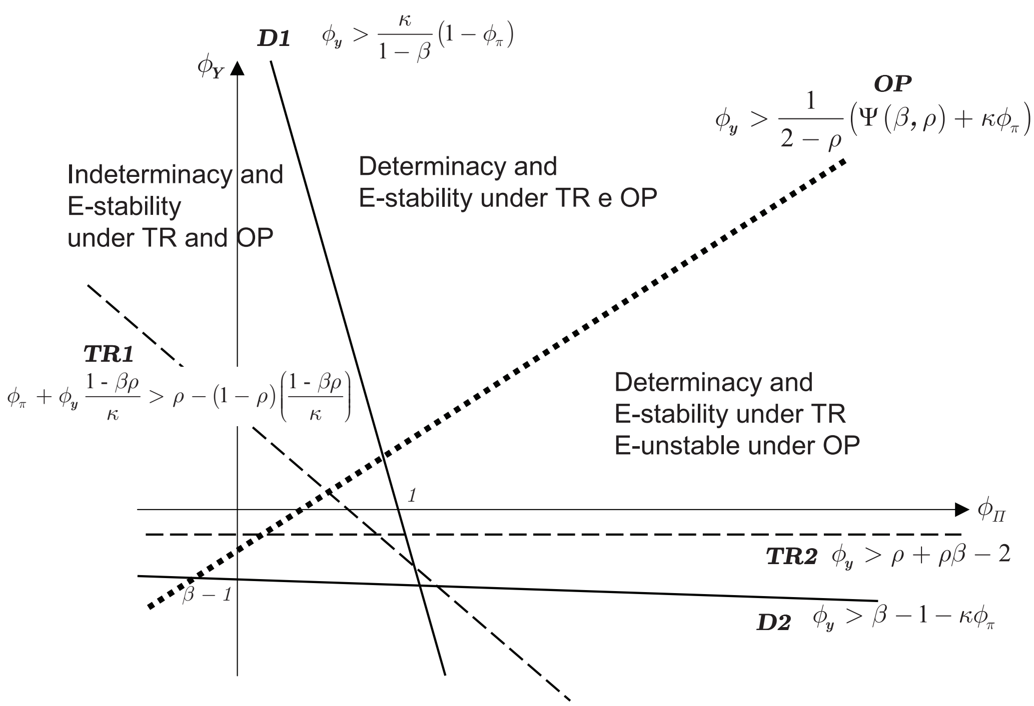


Figure 1. Determinacy and E-stability region under TR and OP from Proposition 1.

TR. This reminds a similar result in Preston (2006). Using his infinite horizon approach, Preston (2006) shows that under OP the Taylor principle is not sufficient for E-stability. However, even when agents form expectations just one-period ahead, requiring them to learn the policy rule is an important source of instability. Hence, the difference between the Bullard and Mitra (2002) and Preston (2006) results are not due to the different assumption regarding the forecast horizon of the expectation process under learning. We have shown that under OP the standard Euler equation approach delivers a similar result as in Preston (2006): the condition for E-stability are more stringent under OP. Besides, Preston (2006) already shows that TR yields the Bullard and Mitra (2002) result that the Taylor principle is necessary and sufficient for E-stability.[16] So it seems that the two approaches deliver similar results. No matter the forecast horizon of the expectation process under learning, TR delivers E-stability if the REE is determinate, while OP generates more instability and requires more stringent conditions.

As Eusepi and Preston (2010) in the infinite horizon case, we were able to obtain an analytical expression for the E-stability condition under OP. This allows us to uncover other important implications. The next question is: *which features of the monetary policy rule make the equilibrium learnable under OP?* For example, does responding more aggressively to inflation help to anchor inflation expectations under OP? Quite surprisingly, the answer is no.

Result 1.2 ***Under OP, monetary policy should respond to the output gap.*** *Let's $\phi_\pi, \phi_y \geq 0$. If the rational expectation equilibrium is determinate (i.e., (D1) and (D2) hold), it is learnable under OP iff $\phi_y > \frac{1}{(2-\rho)}\left(\Psi(\beta,\rho) + \kappa\phi_\pi\right)$.*

For the equilibrium to be learnable under OP, condition (OP) implies a lower bound on ϕ_y that increases with ϕ_π. In other words, an aggressive response to inflation can destabilize expectations under OP, unless it is counteracted by an increase in the response to output. Intuitively, if inflation expectations increase, agents fail to anticipate higher real rates under OP, even if the opaque central bank follows the Taylor Principle. As a result output increases leading to an increase in inflation that validates the

[16] In our case, it is only sufficient, again beacuse of different modelling assumptions about expectations formations.

initial increase in inflation expectations. The intuition for this result in well-explained by Eusepi and Preston (2010, p. 243-244) in an infinite horizon framework. This is due to the fact that policy responds not to current, but to expected variables.[17] A strong response to expected inflation then tends to destabilize the economy, since monetary policy is responding *"too much and too late".* The central bank can stabilize inflation expectations by responding relatively more to expected output, *"which is a more "leading" indicator of inflation".* This result has two main implications. First, OP may be costly, since the optimal policy literature generally suggests that it is suboptimal to respond to the output gap.[18] Second, under OP it is not true that determinacy implies E-stability, as claimed by McCallum (2007). This result echoes similar results in Preston (2006), Bullard and Mitra (2007) and Eusepi and Preston (2010) in the infinite horizon framework.

Moreover, Figure 1 clearly displays an important result: transparency is an essential part of the inflation targeting framework. Under pure inflation targeting, the interest rate rule responds only to inflation. In that case, OP would lead either to E-instability or to indeterminacy, depending if the Taylor principle is satisfied or not. Thus, we can state:

Result 1.3 ***A pure inflation targeting central bank needs to be transparent to anchor inflation expectations.***

Finally, note there is a region of the policy parameter space (ϕ_π, ϕ_y) where despite the rational expectation equilibrium being indeterminate, it is learnable under both TR and OP. And in this particular region, the Taylor principle is not satisfied. In general:

Result 1.4 ***The Taylor principle is not a necessary condition for E-stability neither under TR nor under OP.***

[17] Indeed, if we use contemporaneous expectations, there is no difference between the OP and TR case. See the robustness Section 3.3.

[18] This is generally true for optimal policy in a simple NK model (see Woodford, 2003). Moroever, Schmitt-Grohé and Uribe (2006) shows it in the context of a medium-scale DSGE NK model à la Christiano et al. (2005) or Smets and Wouters (2003).

Having analyzed how the E-stability property depends on the policy parameters, the next obvious question is: *which structural parameters of the economy do affect the different ability of anchoring expectations under TR and under OP?* That is, which features of the economy do make TR more necessary to anchor inflation expectations?

First note that the slope of the NKPC (7), κ, is the key parameter to interpret determinacy. It is immediate to see that determinacy is more likely, the lower the slope of the NKPC (see (D1) and (D2)). Recall that $\kappa = \lambda(1 + \sigma_n)$ and λ depends (inversely) on α. Hence, it follows that the higher the degree of price stickiness (higher α) or the lower the elasticity of the marginal disutility from working (σ_n), then the lower is κ, and the larger is the determinacy region.

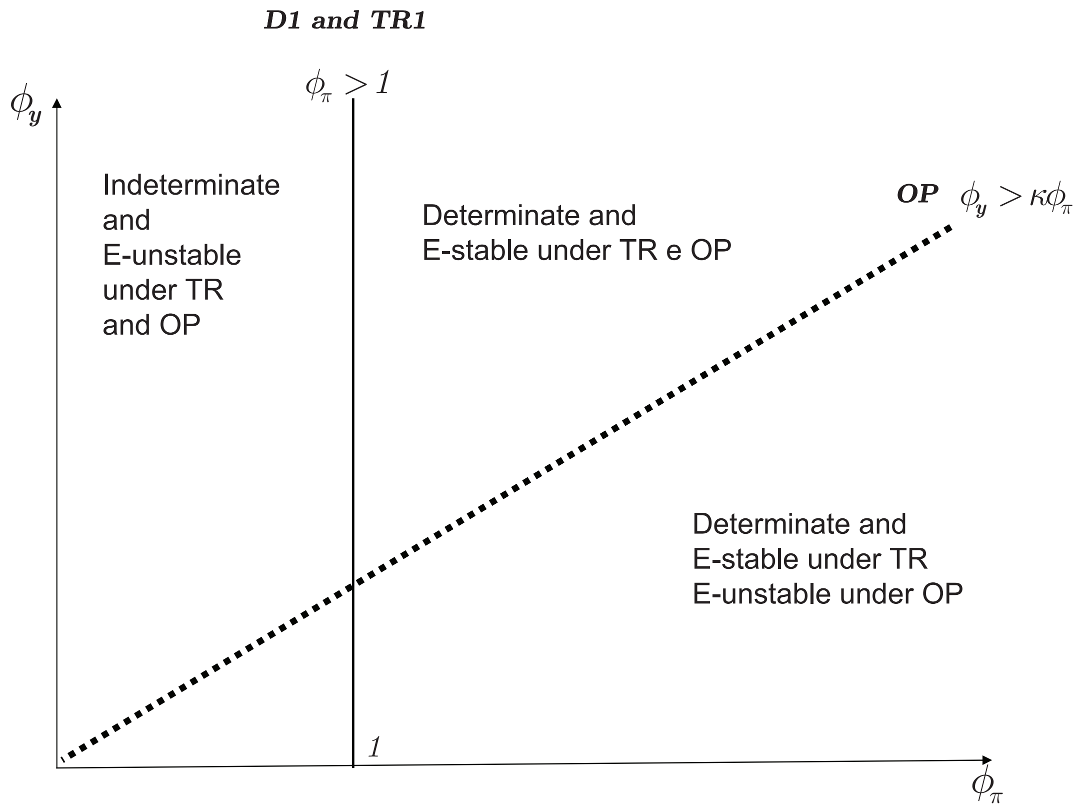


Figure 2. The importance of the slope of the Phillips curve, κ.

Similarly, κ is also the key parameter that affects the E-stability conditions. To clearly see this assume $\beta = \rho = 1$.[19] Then, the determinacy and E-stability conditions under TR collapse to the standard Taylor principle, because (D1) and (TR1) becomes $\phi_\pi > 1$, while (D2) and (TR2) are always satisfied in the positive orthant. The E-stability condition under OP (OP) simply reduces to $\phi_y > \kappa\phi_\pi$. As evident from Figure 2, it follows that the higher the slope of the Phillips curve, the greater the difference between the E-stability regions under TR and OP. Hence:

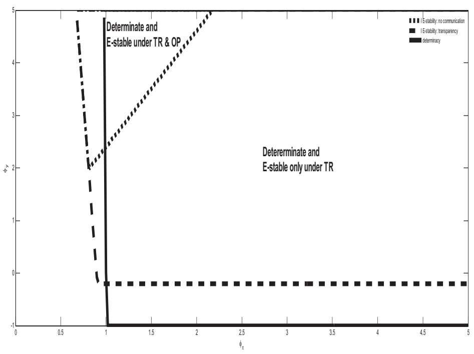
Result 1.5 ***The higher the slope of the Phillips curve, the smaller the E-stability region under OP.*** *That is, the smaller the subset of feasible policies in the parameter space that yields E-stability for an opaque monetary policy.*

The degree of price rigidity is one of the key parameters in determining the slope of the Phillips curve. So one interesting implication is that TR is very important to stabilize expectations for central banks acting in countries with flexible prices (high κ). On the contrary, if the degree of price rigidity is high (a lower κ), then there is not much gain to be transparent, in terms of ability of anchoring expectations, so a central bank may choose to be opaque. Figure 3 shows the difference between the E-stability regions under OP calibrating the degree of price rigidity α respectively to 0.35, as estimated for the US in Christiano et al. (2005), and to 0.91, as estimated for the Euro Area in Smets and Wouters (2003).[20] In the former case, the OP line is steep, and the equilibrium is E-unstable for the values of (ϕ_π, ϕ_y) usually considered in the literature. Only relatively low values of ϕ_π and extreme values of ϕ_y would guarantee an E-stable equilibrium. In the latter case, instead, the OP line is very flat, almost horizontal, and lies below zero.[21] A determinate equilibrium would be E-stable under OP (as well as under TR).

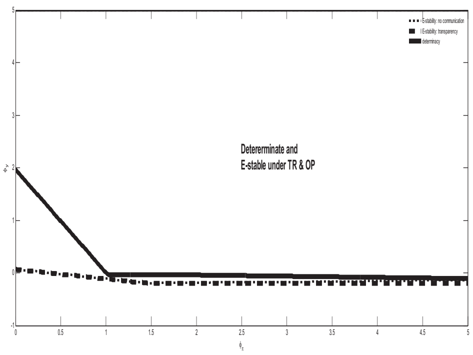
[19] This is just for the sake of exposition without loss of generality. The same argument applies for values $0 < \beta, \rho < 1$.

[20] For the rest, parameters are calibrated with the rather standard values used by Ascari and Ropele (2009): $\sigma_c = \sigma_n = 1$; $\theta = 11$; $\beta = 0.99$; $\rho_{rn} = \rho_u = 0.9$.

[21] Note that κ affects both the slope and the intercept term $\frac{\Psi(\beta,\rho)}{(2-\rho)}$, through $\Psi(\beta, \rho)$. If $\alpha = 0.35$, then $\frac{\Psi(\beta,\rho)}{(2-\rho)} = -0.008$, while if $\alpha = 0.91$, then $\frac{\Psi(\beta,\rho)}{(2-\rho)} = -0.198$.



$\alpha = 0.35$



$\alpha = 0.91$

Figure 3. E-stability regions and price stickiness

It is very important to get the intuition of why this is happening, since this will help also understanding the results in the next section. Recall that an opaque central bank needs to respond to output (and the more so, the more it is aggressive to inflation), because responding to inflation can destabilize expectations, when agents do not know the policy rule and the policy responds to lagged expected values, that is, with a lag. Policy is responding *"too much, too late"*. However, this potentially destabilizing effect under OP depends on the effectiveness of the policy. In a New Keynesian model, a change in the interest rate affects inflation, through output, via the Euler equation. It follows that the effectiveness of monetary policy rests on the link between output and inflation in the NKPC. If the latter is weak, then monetary policy is not very effective. If current output is not an important determinant of current inflation, then, monetary policy does not have destabilizing effects on expectations, because it is not very effective in influencing the dynamics of inflation. Thus, the E-stability region under OP enlarges for higher values of price rigidities (low κ) and, for a given ϕ_π, monetary policy could respond less to output to deliver E-stability. Note that in the simple case of $\beta = \rho = 1$, then κ determines the lower bound of the relative weight $\frac{\phi_y}{\phi_\pi}$ necessary to ensure E-stability under OP. So, the less effective is monetary policy, the more the central bank can be opaque, simply because what it does, it matters less.

For the same reasons, the difference between TR and OP lowers with the slope of the Phillips curve. The less monetary policy is effective, the less knowing or not knowing the path of the interest rate makes a difference. Intuitively, at the limit, if prices are completely rigid and $\kappa = 0$, monetary policy can not influence inflation, there is no advantage in knowing the policy and no difference between TR and OP. On the contrary, the interest rate path is a very valuable information when policy is very effective in affecting inflation.

3.2 Positive inflation

We then move to analyze how the choice of a positive inflation target by a central bank changes the answer to the previous questions. In an inflation targeting framework, it is obvious how pivotal is to assess how the choice of the target influences the ability of the central bank policy of anchoring inflation expectations under both TR and OP.

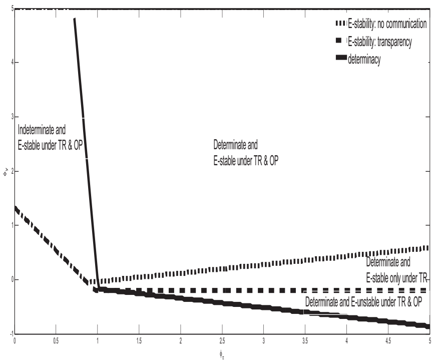
The relevant model is made up by the five equations (1)-(5) described in Section 2.1. The dynamics of the model under positive inflation target, thus, is very different from the one of the simple two equations New Keynesian model of the previous section (see Ascari, 2004 and Ascari and Ropele, 2009). There are two main differences. First, the inflation target directly affects the coefficients of the log-linearized equations. In particular, the higher the inflation target, the more price-setting becomes "forward-looking", because higher trend inflation leads to a smaller coefficient on current output ($\lambda_{\bar{\pi}}$) and a larger coefficient on future expected inflation ($\eta_{\bar{\pi}}$). With high trend inflation the price-resetting firm sets a higher price since it anticipates that trend inflation will erode its relative price in the future. Keeping up with the trending price level becomes a priority for the firm, that will be thus less affected by current marginal costs (see Ascari and Ropele, 2009). Consequently, if the central bank increases the inflation target, the short-run NKPC flattens: the inflation rate becomes less sensitive to variations in current output and more forward-looking. Second, a positive inflation target adds two new endogenous variables: $\hat{\phi}_t$, which is a forward-looking variable, and $\hat{s}_t$, which is a predetermined variable. The dimension of the dynamics of the system is now of fourth order, so we can not have anymore analytical results, and we proceed with numerical simulations.[22] Again, we highlight the main implications of our analysis by listing a number of results.

The first question then is: *does the choice of a higher inflation target undermine the ability of the central bank to anchor inflation expectations?* The answer is yes, very much so.

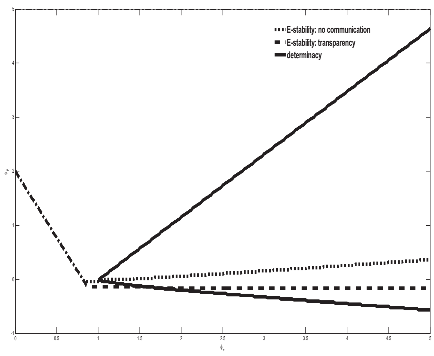
Result 2.1 ***If the central bank fixes an higher inflation target, it is more difficult to anchor inflation expectations under both TR and OP.***

Figure 4 plots the determinacy and E-stability regions both under TR and under OP, for four different values of trend inflation: 0, 2%, 4% and 6%. As known, higher levels of trend inflation shrink the determinacy region. It turns out that higher trend inflation has similar effects also on the conditions for E-stability under both TR and OP. Similarly to the determinacy region, the E-stability region of the policy parameter space (ϕ_π, ϕ_y) is very sensitive to mild variations in the inflation target and shrinks substantially even for moderate levels of inflation targets. Given our calibration and the considered range for (ϕ_π, ϕ_y), when the inflation target is as high as 8% inflation, there is no possibility to anchor inflation expectations and there is no E-stability region under both TR and OP. The intuition rests on the increase in the forward-lookingness in price setting on the part of the firms. The NKPC coefficients are now a function of the inflation target. As trend inflation increases, there is a larger coefficient on future expected inflation and a smaller coefficient on current output. As a result, inflation becomes less sensible to output changes, and thus, monetary policy less effective, making more difficult to stabilize expectations.

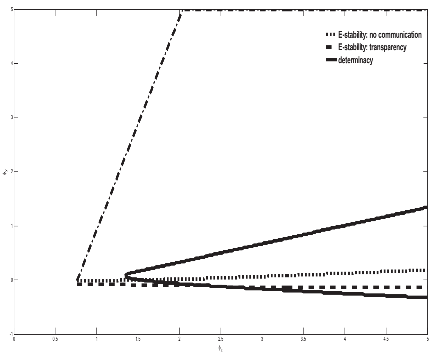
[22] Calibration is as in footnote 20 and $\alpha = 0.75$.



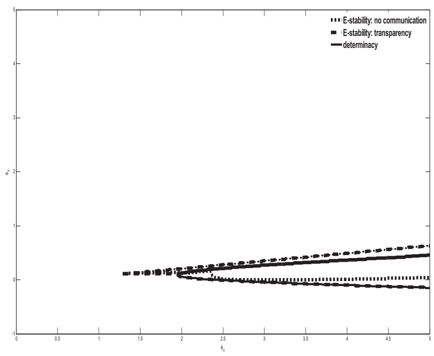
0% trend inflation



2% trend inflation



4% trend inflation



6% trend inflation

Figure 4. E-stability regions and trend inflation

Moreover, changing the relative weights in the Phillips curve, the inflation target affects the slope of the long-run Phillips curve. Roughly speaking, the latter is still given by $\frac{1-\beta}{\kappa}$, but now the coefficients of the NKPC on expected inflation and current output are a function of $\bar{\pi}$, such that now we have $\frac{1-\tilde{\beta}(\bar{\pi})}{\tilde{\kappa}(\bar{\pi})}$, with $\tilde{\beta}(\bar{\pi})$ and $\tilde{\kappa}(\bar{\pi})$ respectively an increasing and a decreasing function of $\bar{\pi}$. Hence, as the target increases, the long-run multiplier of inflation on output switches sign. This is going to affect the (D1) and the (TR1) lines, that depend on the long-run multiplier of inflation on output. The long-run Taylor principle, i.e., (D1), and the (TR1) rotate clockwise, and eventually positively slope in the space (ϕ_π, ϕ_y) (see Ascari and Ropele, 2009 for details).

One main policy implication concerns the recent proposal by Blanchard et al. (2010) to raise the inflation target to have more room of manoeuvre to decrease the real interest rate before hitting the zero lower bound in the case of a crisis. This proposal could hide an important peril: the unanchoring of inflation expectations, as suggested by Fed Chairman Bernanke.

The next obvious questions is *how the inflation target affects the different ability of anchoring expectations under TR and OP? Does a central bank that fixes a higher level of inflation target need to be more transparent?*

Result 2.2 ***If the central bank fixes an higher inflation target, the advantage of being TR in anchoring expectations diminishes.***

Looking at the Figure 4 the line that corresponds to the condition (OP) in a positive inflation targeting framework flattens with trend inflation. Thus the difference between the E-stability regions under TR and OP contracts. The intuition is the same that explains Result 1.5 in the previous section. An higher inflation target flattens the slope of the Phillips curve, so it has a similar effect of a decrease in κ in the previous section. If output becomes less relevant in determining current inflation, then, even knowing the interest rate becomes less important, because the latter affects inflation dynamics only through current output. In other words, Result 1.2 weakens because under OP a strong response to expected inflation is less likely to destabilize expectations, and the central bank can respond relatively less to expected output to stabilize inflation expectations. Moreover, note that looking at this result from a reverse perspective implies that TR is an important component of the inflation targeting approach. Central banks in developed countries moved in the last decades to an inflation targeting framework and greater transaparency with the aim of lowering average inflation. The central idea is that this framework *and* a greater transparency should help coordinating and anchoring inflation expectations. Our result supports this view, because the lower the inflation target, the more TR is important (vs OP) for expectations stabilization.

The next interesting question again regards how an higher inflation targeting shapes the feasible monetary policy rules. *Does a higher inflation target requires monetary* *policy to be more hawkish on inflation and/or on output?*

Result 2.3 ***The higher the inflation target fixed by the central bank, the more monetary policy should respond to inflation under both TR and OP.***

While it is still true that a pure inflation targeting central bank needs to be transparent, the minimum ϕ_π necessary to stabilize expectations increases with the inflation target. As Figure 4 shows, the crossing of the E-stability (as well as for determinacy) conditions under both TR and OP moves to the right.

Result 2.4 ***An high inflation target implies an upper bound for ϕ_y under both TR and OP.***

Moreover, while the lower bound for ϕ_y in case of OP reduces, because condition (OP) flattens with trend inflation (see Result 2.2), in contrast with the zero inflation target case, an higher inflation target actually generates an upper bound for ϕ_y, because of the clockwise rotation of the (TR1), as the long-run relationship between output and inflation becomes negative. Hence, a too strong reaction to expected output gap may destabilize expectations by increasing inflation in the future.

It follows that if, for whatever reasons, the Fed had to adopt a higher inflation target, it would need to be more aggressive on inflation and responds less to output. It would be unwise to suggest a policy that would increase the inflation target and contemporaneously respond less to inflation and output.

To conclude, the level of the inflation target has substantial effects on the E-stability regions and hence on the ability of a central bank to control inflation expectations. The higher the target inflation rate: (i) the more difficult is to anchor expectations, (ii) the less a central bank needs to be transparent; (iii) the more hawkish on inflation a central bank should be.

3.3 Robustness

In this section we investigate the robustness of our results along different dimensions.

Policy Rule. First, we investigate if and how results change when we modify the policy rule. The determinacy properties are basically the same as in Ascari and Ropele (2009). Regarding E-stability, while a forward looking policy rules does not alter the E-stability region with respect to the benchmark case, a backward looking policy rule is quite different because it always returns E-stability under both TR and OP. These results are different from Kobayashi and Muto (2011) because of the different assumptions regarding the learning process (see below).

When one considers the more realistic case of a Taylor rule that includes a lagged interest rate to account for interest rate smoothing by the central bank, then, as the degree of interest rate smoothing increases, the determinacy and the E-stability regions widen for every value of the inflation target considered.[23] This is in line with previous results that show that interest rate inertia enlarges the determinacy region both under zero (e.g., Woodford, 2003) or positive trend inflation (e.g., Ascari and Ropele, 2009), and promotes learnability (Bullard and Mitra, 2007). Moreover, we find that, as trend inflation increases, the E-learnable region shrinks much more slowly (if compared to the baseline case). According to our usual calibration, in the presence of smoothing, the anchoring of expectations becomes now a possibility even for values of inflation target as high as 8%. So we can confirm that inertia do promotes learnability and stability of the REE even for fairly high levels of trend inflation. Moreover, as the inertia parameter approaches the value of one, if one confines the analysis to the positive orthant (ϕ_π, ϕ_y), the difference between TR and OP disappears. With high inertia, in fact, the interest rate today is close to the previous period's one hence there is no need to make learning on it: this lowers the benefit of TR.

In any case, the main message of the paper goes through: higher trend inflation tends to unanchor inflation expectations making learnability more difficult.

Learning assumptions. Second, we also investigate the robustness of our results to our assumptions regarding the specification of learning in our model. If we introduce in the PLM a constant term, as in the pivotal paper by Bullard and Mitra (2002), the main results are largely unaffected. The only change, under a zero inflation target, is the

[23] With the effect larger on the E-stability zone.

coincidence of the "long run" Taylor principle (D1) and the E-stability condition (TR1). As trend inflation increases these two conditions separate. They remain, however, closer to each other if compared to the baseline case.

The hypothesis that changes most our results is the one about expectation formation. Under contemporaneous expectations, the determinacy and E-stability regions in the positive orthant (ϕ_π, ϕ_y) do not change if compared to the baseline case. However, any difference between TR and OP vanishes. In this case there is no central bank's information fruitfully exploitable by the public.

Model structure. Third, we examine the effects of including price indexation. It is well-known that indexation counteracts the effects of trend inflation. We find that this is true both regarding determinacy, as in Ascari and Ropele (2009), and E-stability. We consider the two cases most familiar from the literature: trend inflation and backward-looking indexation (e.g., Christiano et al., 2005). Regarding E-stability, there is no substantial difference between these two cases. The effects of trend inflation are partially offset by indexation, so that as trend inflation increases the E-stability frontiers shifts less with respect to the benchmark case. Partial indexation makes the slope of the Phillips curve less sensitive to trend inflation, because price setters need to a less extent to set very high prices in order to take into account the presence of trend inflation. They are then more sensitive to current marginal costs and economic conditions. As suggested by Result 2.2, this reestablishes the importance of TR.

Finally, we discuss some implications for the degree of price rigidity. More flexibility (lower α) makes both the determinacy and the E-stability frontier close less rapidly compared with the baseline case, because trend inflation matters less the more flexible are the prices. Moreover, recall from Figure 3 that a lower degree of price rigidity implies a larger difference between TR and OP, because the OP line is quite sensitive to the degree of price rigidity. However, α may not be considered a truly structural parameter, and it could decrease with trend inflation (see Levin and Yun, 2007). In other words, firms would change their price more often (i.e., increase price flexibility) as trend inflation increases. As a results, there could be two possible forces acting on the (OP) line as trend inflation changes. On the one hand, higher trend inflation flattens the (OP) line moving it towards the (TR2) line; on the other hand, if trend inflation causes a lower α, higher price flexibility shifts the (OP) line upwards, moving it away from (TR2) and shrinking the E-stability region under OP. Which of the two forces will prevail depends on calibration and on the eventual elasticity of α with respect to trend inflation. If the latter effect prevails, then, Result 2.2 can be overturned and as trend inflation increases there would be a greater need for TR.

Robustness on the other calibration parameters do not qualitatively alter our main results. Decreasing the value of the elasticity of substitution (θ) or increasing the intertemporal elasticity of labour supply (σ_n) makes the difference between TR and OP shrink slower as trend inflation rises. All the above results are available upon request.

4 Speed of convergence

The previous analysis focused on the asymptotical properties of the learning process, by determining the conditions for the E-learnability of the REE. Following Ferrero (2007), we also analyze how monetary policy can affect the transitional properties of the learning process, by studying the speed of convergence of expectations to the REE. The speed of convergence matters because a fast convergence means that the economy, and thus its dynamics, would always be very close to the REE, while a slow convergence implies that economic dynamics would be dominated by the transitional dynamics under learning. As such, the speed of convergence could also well be another criteria to judge a given policy.

To analyze the speed of convergence, it is obviously needed to assume convergence to a unique REE, that is the analysis is conducted in the E-stability *and* determinacy region of the parameter space. The speed of convergence is then determined by the properties of the same ODE employed to value E-stability, that is, the T-mapping from PLM to ALM. In particular, the speed of convergence is determined by the spectral radius (i.e., the largest eigenvalue in absolute value) of the derivative of the T-map (see Ferrero, 2007, and Ferrero and Secchi, 2010). To converge, all the eigenvalues need to be within the unit circle, and the larger the spectral radius, the slower the convergence.

Indeed, when the largest eigenvalue crosses the unit circle, then the economic dynamics cross into the E-unstable region of the parameter space and there is no convergence anymore to the REE. Rather than simply referring to the eigenvalue, we actually offer a measure of the speed of convergence. Given the spectral radius of the derivative of the T-map, one can estimate the number of iterations necessary to reduce the initial error by a certain amount, say one-half, to conform to the common half-life measure. The measure of speed we then plot is simply the inverse of this number of iterations (this should be intended as an asymptotic rate of convergence, see Appendix).

We now study the speed of convergence first in the zero inflation target case and then under positive trend inflation.[24]

4.1 Zero inflation target

Figure 5 displays the iso-speed curves in the case of, respectively, TR and OP. Since the speed of convergence can be calculated just in the E-stability region and this region is smaller under OP, this is reflected into a more acute angle of the iso-speed curves. The two figures convey the same message. Recall from Figure 1 the shape of the region that is both E-stable and determinate in the two cases of OP and TR. When the policy is such that the economy is close either to the E-stability frontier (lower frontier on the right) or the determinacy frontier (upper frontier on the left), then the T-map spectral radius is approaching unity. Thus close to the frontiers the convergence speed is low. On the contrary, when the policy is such that the economy is well within the boundaries then the largest eigenvalue bottoms out and the speed of convergence is high. In other words, to get a higher speed one needs to remain well within the determinacy/E-stability region, while keeping away from its frontiers. Thus, Figure 6 shows that the 3-D graph of the speed of convergence as a function of the policy parameters (ϕ_π, ϕ_y) is like a mountain with a ridge that runs in the middle of the region described by the E-stablility and determinacy frontier. The difference between the two cases of OP and TR then rests on the difference between these two regions, as described in Figure 1. With the help of

[24] Calculations of the speed of convergence are based on simulations obtained under the usual parameter calibration (see footnotes 20 and 22).

Figure 5 and 6 and Table 2 we can answer questions relating to the speed of convergence in the same vein as the previous Sections.

First: does TR allow a central bank to increase the speed of convergence with respect to OP? The TR surface in Figure 6 lies almost always well above the OP one, thus signaling a higher speed of convergence, unless for a limited region of the parameter space where anyway the difference is minor. The maximum speed of convergence under TR is much higher than under OP. So we can conclude that *transparency helps increasing the speed of convergence.*

Second, how does policy affect the speed of convergence? To get the quickest convergence, policy has to move towards the centre of the determinacy/E-stability region, that is, to be on the ridge. Note that the ridge runs across that region, so that to increase the speed of convergence *both* ϕ_π *and* ϕ_y have to increase at the same time both in the case of OP and TR. A stronger reaction both to inflation and to the output gap seems to give more informations to the agents, speeding up the learning process. As in the previous Section, in the OP case it is again the ratio ϕ_y/ϕ_π that matters: the ridge is almost linear and loosely described by a ratio $\phi_y/\phi_\pi \approx 0.5$ (see Figure 5). Moreover, in the TR case, the speed is not decreasing with ϕ_π. For each level of ϕ_y, the speed is first increasing for low values of ϕ_π, till it reaches a roof, where further increases in the reaction of the policy to inflation would not have any effect on the speed of convergence. An interesting particular case is the one of a pure inflation targeting central bank. We have already pointed out the need of transparency by a pure inflation targeting central bank. In this particular case, under TR and $\phi_y = 0$, the speed is constant, that is independent from ϕ_π, and it depends just on the autoregressive component of the exogenous disturbance terms (assuming these are equal). So, *under pure inflation targeting the central bank cannot affect the (low) speed of convergence.*

We are actually able to prove these results in the case of transparency through the following Proposition (proof in Appendix).

Proposition 2 *In the case of TR and zero trend inflation, the speed of convergence:*
(i) does not depend on ϕ_π *iff* $\left(\rho(\beta-1)+\phi_y\right)^2 \leq 4\kappa(\phi_\pi-\rho)$ *and it is strictly increasing otherwise;*

(ii) is decreasing with ϕ_y *iff* $\left(\rho(\beta-1)+\phi_y\right)^2 > 4\kappa(\phi_\pi - \rho)$ *and* $\rho(\beta-1)+\phi_y > 0$ *and* $\phi_\pi > \rho$*, while it is increasing with* ϕ_y *otherwise;*
(iii) does not depend on κ *iff* $\left(\rho(\beta-1)+\phi_y\right)^2 \leq 4\kappa(\phi_\pi - \rho)$*, it is decreasing in* κ *iff* $\left(\rho(\beta-1)+\phi_y\right)^2 > 4\kappa(\phi_\pi - \rho)$ *and* $\phi_\pi < \rho$*, while it is increasing iff* $\left(\rho(\beta-1)+\phi_y\right)^2 > 4\kappa(\phi_\pi - \rho)$ *and* $\phi_\pi > \rho$*.*

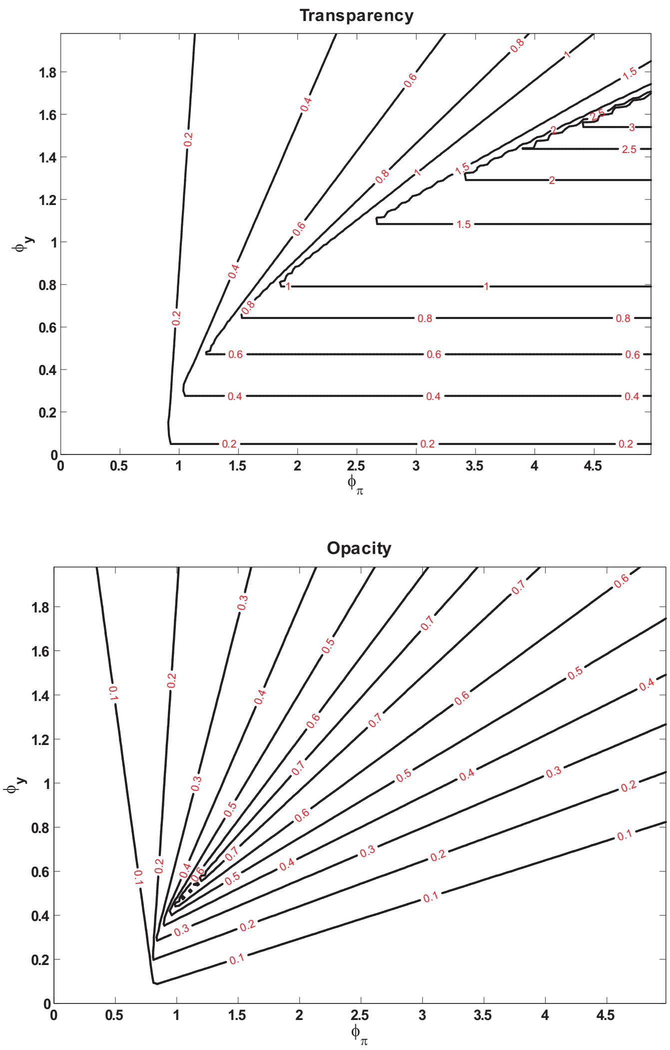


Figure 5. Iso-speed curves

Table 2. Speed of convergence $S = -\log_2(spectral\ radius)$						
trend inflation 0% ϕ_y	0		0.5		1	
ϕ_π	OP	TR	OP	TR	OP	TR
1.5	-	0.15	0.37	0.62	0.47	0.38
2	-	0.15	0.25	0.62	0.71	0.67
3	-	0.15	0.12	0.62	0.43	1.32
trend inflation 2% ϕ_y	0		0.5		1	
ϕ_π	OP	TR	OP	TR	OP	TR
1.5	0	0.12	0.30	0.23	-	-
2	-	0.12	0.42	0.43	0.23	0.20
3	-	0.14	0.22	0.49	0.40	0.38

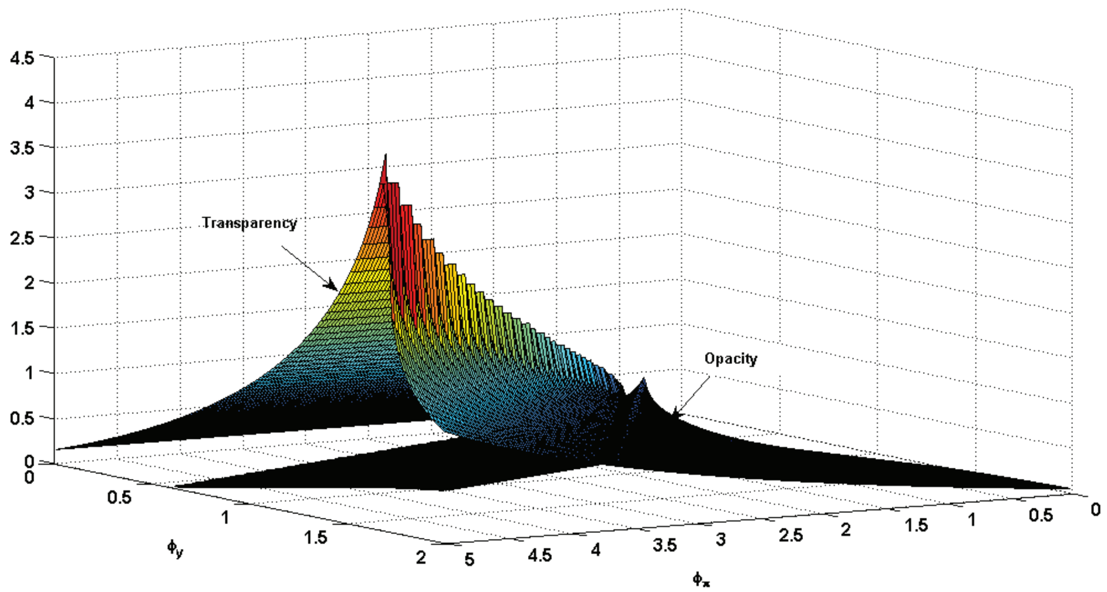


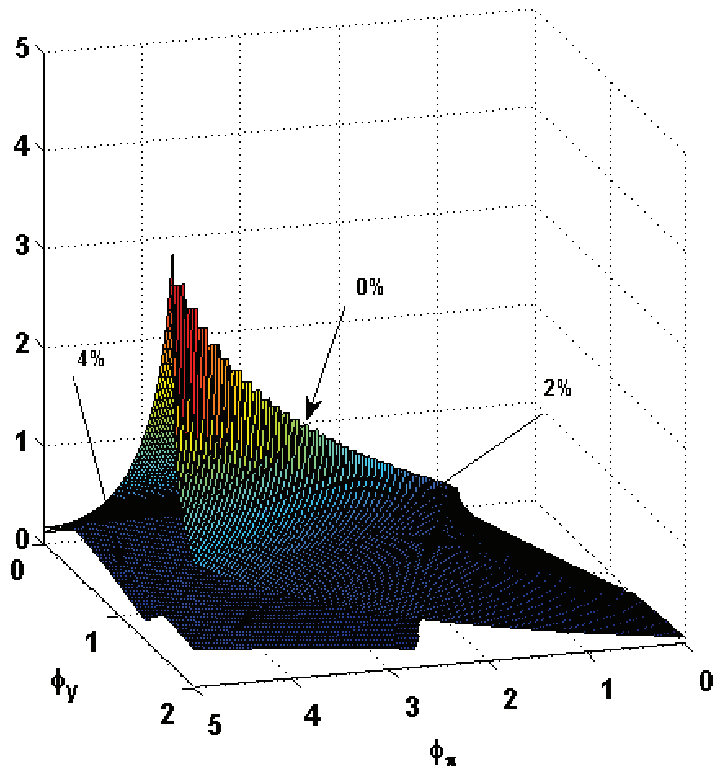
Figure 6. Speed of convergence under opacity and transparency with zero inflation target

4.2 Positive inflation

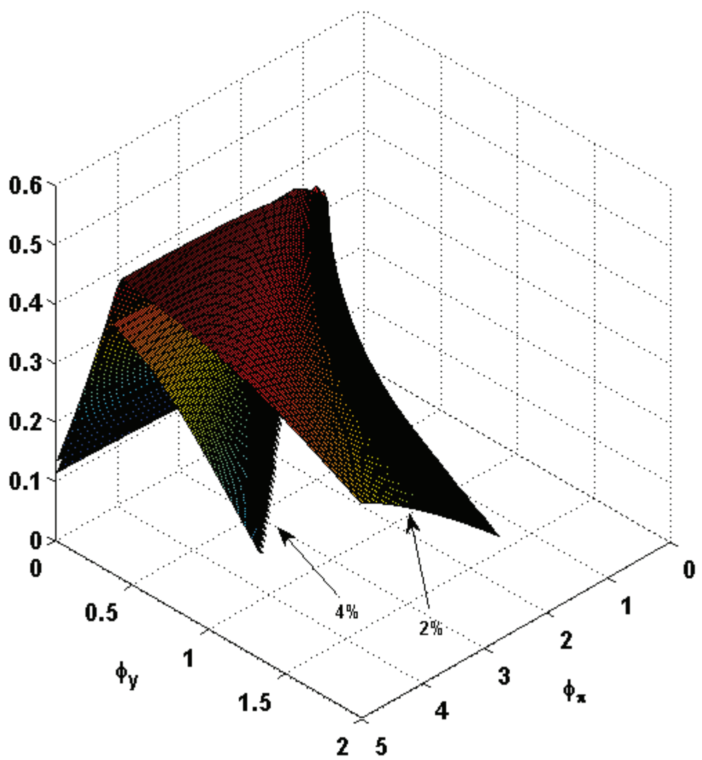
What is the effect of a positive inflation target on the speed of convergence? Figures 7a,b (TR case) and 7c (OP case) clearly show the main result of this section: *an higher inflation target tends to lower the speed of convergence of expectations under learning to the REE both under OP and under TR.* For our parameterization, this is always true in the case of TR, where the difference between a zero and a 2% inflation target is very large.[25] In the latter case the speed is never higher than 0.4, while in the former case it reaches 2.7 and it is higher than 0.4 for most of the policy parameter values. The shape of the surfaces in Figure 7 are qualitatively similar, but trend inflation hammers the ridge down to a great extent. Thus, even a very modest increase in the inflation target has large effects on the speed of convergence of expectations under learning. As such, the values of the convergence speed become close to the OP case. Thus, the advantage of being transparent (the difference between being OP and TR), in terms of the speed of convergence, decreases as trend inflation increases. In the OP case, the speed is already quite low at zero inflation, so the effects of trend inflation are qualitatively similar to the case of TR, but less pronounced. Trend inflation does not change qualitatively the effects of policy parameters on the speed: to increase the speed, policy makers have to increase *both* ϕ_π *and* ϕ_y; under pure inflation targeting and positive trend inflation, the central bank can (barely) affect the (very low) speed of convergence increasing ϕ_π. However, these effects are now much less marked given that the ridge is much lower than in the zero trend inflation case.

This is no surprise, given that: (i) an higher inflation target shrinks the E-stability / determinacy frontier; (ii) the speed of convergence is lower the closer the economy is to these frontiers. An higher inflation target increases the spectral radius of the derivative of the T-map, such that, ceteris paribus, both the speed of convergence of expectations is lower under E-stability, and the learning dynamics are more likely to be E-unstable.

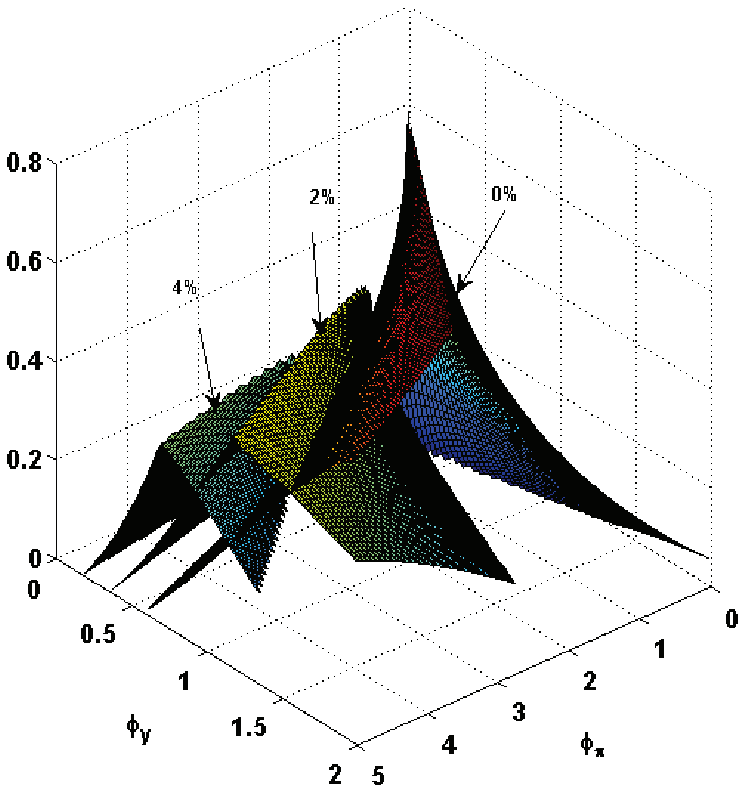
[25] In fact, we add Figure 7b in order to appreciate the surfaces in the case of 2% and 4% inflation target. In Figure 7a, those surfaces can not be visualized because the difference between them and the one relative to the zero trend inflation case is large.



7a: Transparency



7b: Transparency



7c Opacity

Figure 7. Speed of convergence for different values of trend inflation under opacity and transparency

5 Conclusions

This paper proves that a higher inflation target unanchors expectations, as often suggested by Fed Chairman B. Bernanke. We investigate a New Keynesian model that allows for trend inflation under adaptive learning, in the spirit of Evans and Honkapohja (2001). Technically, an higher inflation target increases the spectral radius of the matrix that defines the T-map under adaptive learning, and that governs the convergence of the learning algorithm. Hence, we were able to show that, when a central bank follows a Taylor rule, the higher the inflation target, the smaller the E-stability region and the speed of convergence of the expectations to the rational expectation equilibrium under E-stability. Moreover, the higher the inflation target, the more the policy should be hawkish with respect to inflation to stabilize expectations, while it should not respond too much to output. This result completely dismisses the argument that the Fed should increase the inflation target and, contemporaneously, ease monetary policy to respond to the surge in unemployment. Our results suggest that this policy would indeed be "reckless" and "unwise", as Bernanke recently said.

Moreover, our results confirm the claim that transparency is an essential component of the inflation targeting framework. The paper looks at the distinction between TR and OP. When a central bank is transparent, agents know the policy rule and use it in forming expectations (see Preston, 2006). When a central bank is opaque, instead, agents need to learn also the policy rule. We find that transparency helps anchoring expectations, that is, the E-stability region is wider under transparency than under opacity for all the possible Taylor rules and inflation targets. Moreover, a pure inflation targeting central bank needs to be transparent to anchor inflation expectations. Finally: the more flexible are the prices, the more transparency is valuable; and under opacity, a more aggressive response to inflation could destabilize inflation expectations, while a larger response to output tends to stabilize them.

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Table 1. Parameters and basic symbols

Parameters	Description
β	Subjective discount factor
σ_n	Intertemporal elasticity of labour supply
θ	Dixit-Stiglitz elasticity of substitution
α	Calvo probability not to optimize prices
$\bar{\pi}$	Central Bank's inflation target (or trend inflation)
ϕ_π	Inflation coefficient in the Taylor rule
ϕ_y	Ouput coefficient in the Taylor rule
NKPC Coefficients	
$\lambda_{\bar{\pi}}$	$\frac{\left[1-\alpha\bar{\pi}^{(\theta-1)}\right]\left[1-\alpha\beta\bar{\pi}^{\theta}\right]}{\alpha\bar{\pi}^{(\theta-1)}}$
$\eta_{\bar{\pi}}$	$\beta\left(\bar{\pi}-1\right)\left[1-\alpha\bar{\pi}^{(\theta-1)}\right]$
$\xi_{\bar{\pi}}$	$\frac{\theta\alpha\bar{\pi}^{(\theta-1)}(\bar{\pi}-1)}{1-\alpha\bar{\pi}^{(\theta-1)}}$

A Appendix

A.1 The Model

The model is based on Ascari and Ropele (2009).

Households. Households live forever and their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \chi \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right), \tag{8}$$

where $\beta \in (0,1)$ is the subjective rate of time preference, E_0 is the expectation operator conditional on time $t = 0$ information, C is consumption, N is labour, χ and σ_n are parameters. The period budget constraint is given by:

$$P_t C_t + B_t \leq P_t w_t N_t + (1 + i_{t-1}) B_{t-1} + F_t \tag{9}$$

where P_t is the price of the final good, B_t represents holding of bonds offering a one-period nominal return i_t, w_t is the real wage, and F_t are firms' profits rebated to households. The households maximize (8) subject to the sequence of budget constraints (9), yielding the following first order conditions:

$$w_t = \chi N_t^{\sigma_n} C_t, \tag{10}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \frac{(1+i_t) P_t}{P_{t+1}} \right]. \tag{11}$$

Final Good Producers. In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0,1]$ and a standard CES production function $Y_t = \left[\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}$, with $\theta > 1$. The final good producer demand schedule for intermediate good quantities is: $Y_{i,t} : Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$. The aggregate price index: $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}$.

Intermediate Goods Producers. $Y_{i,t}$ are produced by a continuum of firms indexed by $i \in [0,1]$. The production function for intermediate input firms is: $Y_{i,t} = N_{i,t}$. Intermediate goods producers sets prices according to the usual Calvo mechanism. In each period there is a fixed probability $1-\alpha$ that a firm can re-optimize its nominal price, i.e., $P_{i,t}^{*}$. With probability α, instead, the firm may either keep its nominal price unchanged . The first order condition of the problem is:

$$\frac{P_{i,t}^{*}}{P_t}=\frac{\theta}{\theta-1}\frac{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left(\frac{C_t}{C_{t+j}}\right)\left(\frac{P_t}{P_{t+j}}\right)^{-\theta}Y_{t+j}w_{t+j}}{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left(\frac{C_t}{C_{t+j}}\right)\left(\frac{P_t}{P_{t+j}}\right)^{1-\theta}Y_{t+j}}. \tag{12}$$

The aggregate price level evolves as

$$P_t=\left[\alpha\left(P_{t-1}\right)^{1-\theta}+(1-\alpha)\left(P_{i,t}^{*}\right)^{1-\theta}\right]^{1/(1-\theta)}. \tag{13}$$

As shown by Ascari and Ropele (2009), we will assume that for given parameter values of $0\leqslant\alpha<1$, $0<\beta<1$, $0\leqslant\varepsilon\leqslant1$ and $\theta>1$, the positive level of trend inflation fulfils the restrictions: $1\leqslant\bar{\pi}<\left(\frac{1}{\alpha}\right)^{\frac{1}{\theta-1}}$ and $1\leqslant\bar{\pi}<\left(\frac{1}{\alpha\beta}\right)^{\frac{1}{\theta}}$, to ensure existence of the steady state of the model (see Ascari and Ropele, 2009 for details).

Relative price dispersion. At the level of intermediate firms, it holds true that $\left(P_{i,t}/P_t\right)^{-\theta}Y_t=N_{i,t}$. Aggregating this expression for all the firms i yields $Y_ts_t=N_t$, where $s_t\equiv\int_0^1\left(P_{i,t}/P_t\right)^{-\theta}di$ and $N_t\equiv\int_0^1 N_{i,t}di$. The variable s_t measures the relative price dispersion across intermediate firms. s_t is bounded below at one and it represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism. s_t evolves as

$$s_t=(1-\alpha)\left(\frac{P_{i,t}^{*}}{P_t}\right)^{-\theta}+\alpha\left(\Pi_t\right)^{\theta}s_{t-1}, \tag{14}$$

where $\Pi_t=P_t/P_{t-1}$ denotes the gross inflation rate. The variable s_t directly affects the real wage via the labour supply equation (10): $w_t=\chi Y_t^{\sigma_n}s_t^{\sigma_n}C_t$.

Market clearing conditions. The market clearing conditions in the goods and labour markets are: $Y_t=C_t$; $Y_{i,t}^{s}=Y_{i,t}^{D}=\left(P_{i,t}/P_t\right)^{-\theta}Y_t$, $\forall i$; and $N_t=\int_0^1 N_{i,t}di$.

A.2 Learning

We write our problem as:

$$
\begin{aligned}
y_t &= \beta_0 E^*_{t-1} y_t + \beta_1 E^*_{t-1} y_{t+1} + \delta y_{t-1} + k w_t \qquad (15)\\
w_t &= \varphi w_{t-1} + e_t
\end{aligned}
$$

where y_t is the vector of endogenous variables and w_t are the shocks (exogenous) and e_t white noise. According to EH's notation the MSV solution is written as:

$$
y_t = b y_{t-1} + c w_{t-1} + k e_t, \qquad (16)
$$

where the matrices b, c are to be determined.

Following Evans and Honkapohja (2001) we assume that agents' perceived law of motion (PLM) coincides with the system's MSV solution. In this case, even the PLM will not contain any constant term.[26] Agents are assumed to know just the autocorrelation of the shocks but they have to estimate remaining parameters.

Once the MSV solution is determined one can apply the E-stability criterion in Proposition 10.1 of Evans and Honkapohja (2001). An MSV solution $\bar{b}, \bar{c}$ to (15) is E-stable if:

(i) all the eigenvalues of $DT_b(\bar{b})$ have real parts less than 1.

(ii) all the eigenvalues of $DT_c(\bar{b}, \bar{c})$ have real parts less than 1.

Assuming none of the eigenvalues has real part equal to 1, the solution is not E-stable if any of conditions (i), (ii) do not hold. In other words, we have stabilising expectations or E-stability when expectations converge to REE.

In order to determine the matrices $DT_b(b)$ and $DT_c(b, c)$ one has to proceed with the following steps. Starting from the PLM one can compute the following expectations:

$$
E^*_{t-1} y_t = b y_{t-1} + c w_{t-1} \qquad (17)
$$

[26] Using a PLM with a constant term, our main conclusions do not change. Results are available from the authors upon request.

and

$$E^*_{t-1}y_{t+1} = bE^*_{t-1}y_t + cE^*_{t-1}w_t = b^2 y_{t-1} + (bc + c\varphi)\, w_{t-1}. \tag{18}$$

Substituting these computed expectations into model (15) one obtains the actual law of motion (ALM):

$$y_t = (\beta_1 b^2 + \beta_0 b + \delta)y_{t-1} + (\beta_0 c + \beta_1 bc + \beta_1 c\varphi + \kappa\varphi)w_{t-1} + ke_t \tag{19}$$

The mapping from the PLM to the ALM takes the form

$$T(b, c) = \left(\beta_1 b^2 + \beta_0 b + \delta, \beta_0 c + \beta_1 bc + \beta_1 c\varphi + \kappa\varphi\right). \tag{20}$$

Expectational stability is then determined by the matrix differential equation

$$\frac{d}{d\tau}(b, c) = T(b, c) - (b, c). \tag{21}$$

The fixed points of equation (21) give us the MSV solution $(\bar{b}, \bar{c})$. The derivatives of the T-map are:[27]

$$DT_b(b) = b' \otimes \beta_1 + I \otimes (\beta_0 + \beta_1 b) \tag{22}$$

$$DT_c(b, c) = \varphi' \otimes \beta_1 + I \otimes (\beta_0 + \beta_1 b), \tag{23}$$

where I denotes an identity matrix of conformable size.

A.3 Analytical results

A.3.1 Proposition 1.

With zero tren inflation ($\overline{\pi} = 1$) and $\eta_{\bar{\pi}} = \xi_{\bar{\pi}} = 0$, the model reduces to .

$\hat{\pi}_t = \beta E^*_{t-1}\left(\hat{\pi}_{t+1}\right) + \kappa\hat{y}_t + \kappa u_t$

$\hat{y}_t = E^*_{t-1}\hat{y}_{t+1} - E^*_{t-1}\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) + r^n_t$

[27] See Evans and Honkapohja (2001, p.231-232) for details.

Transparency

$\hat{\pi}_t = \beta E^*_{t-1}(\hat{\pi}_{t+1}) + \kappa\hat{y}_t + \kappa u_t$

$\hat{y}_t = E^*_{t-1}\hat{y}_{t+1} - E^*_{t-1}(\phi_\pi \pi_t + \phi_y y_t) + E^*_{t-1}\hat{\pi}_{t+1} + r^n_t$

the above system can be written as follows:

$$y_t = \beta_0 * E^*_{t-1}y_t + \beta_1 * E^*_{t-1}y_{t+1} + \delta * y_{t-1} + k * w_t \qquad \text{with } w_t = \rho w_{t-1} + e_t \qquad (1A)$$

the MSV solution will be[28]

$PLM \quad y_t = cw_{t-1} + ke_t$

Put the PLM into $(1A)$ and get the ALM:

$y_t = \beta_0 * cw_{t-1} + \beta_1 * c\rho w_{t-1} + k * \rho w_{t-1}$

The T-map will thus be:

$T(c) = (\beta_0 c + \beta_1 c\rho + k\rho)w_{t-1}$

For E-stability we require the eigfenvalues of $DT_c(c) = \beta_0 + \beta_1\rho$ to have real parts less than one.

Thus, in order to have E-stability, the eigenvalues of the matrix $\beta_0 + \beta_1\rho - I$, that is:

$$(DT_c(c) - I) = \begin{vmatrix} \rho\beta - 1 & \kappa \\ \rho - \phi_\pi & \rho - \phi_y - 1 \end{vmatrix}$$

must have negative real parts. This is true if the following conditions hold:[29]

TR1: $\phi_\pi + \phi_y \frac{1-\beta\rho}{\kappa} > \rho - (1-\rho)\left(\frac{1-\beta\rho}{\kappa}\right)$

TR2: $\phi_y > \rho + \beta\rho - 2$

Hence, TR2 is always verified if $\phi_y \geq 0$.

Further, remember that the determinacy conditions (Ascari and Ropele (2009)) are:

D1: $\phi_\pi + \phi_y \frac{1-\beta}{\kappa} > 1$

D2: $\phi_\pi + \frac{1}{\kappa}\phi_y > \frac{\beta-1}{\kappa}$

Note that if, as we assume, $0 < \rho, \beta < 1$ if D1 is verified even TR1 holds.

Opacity

$\hat{\imath}_t = \phi_\pi E^*_{t-1}\hat{\pi}_t + \phi_y E^*_{t-1}\hat{y}_t$

$\hat{\pi}_t = \beta E^*_{t-1}(\hat{\pi}_{t+1}) + \kappa\hat{y}_t + \kappa u_t$

[28] Consider the more general formulation ($y_t = a + by_{t-1} + cw_{t-1} + ke_t$) with $a = 0$, since there is no constant and $by_{t-1} = 0$, since b depends just on the predetermined variable s that, in this case, is absent.

[29] The characteristic polynomial is $X^2 + (\phi_y - \rho - \beta\rho + 2)X - (\kappa(\rho - \phi_\pi) + (\beta\rho - 1)(\phi_y - \rho + 1))$

$\hat{y}_t = E^*_{t-1}\hat{y}_{t+1} - E_{t-1}\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) + r^n_t$

Put this system in (1A) form.You get, as before, that in order to have E-stability, the eigenvalues of the matrix $\beta_0 + \beta_1\rho - I$, that is:

$$(DT_c(c) - I) = \begin{vmatrix} -1 & \phi_\pi & \phi_y \\ 0 & \rho\beta - 1 & \kappa \\ -1 & \rho & \rho - 1 \end{vmatrix}$$

must have negative real parts. This is true if the following conditions hold:[30]

OP1: $3 - \beta\rho - \rho > 0$

OP2: $\phi_\pi + \phi_y \frac{1-\beta\rho}{\kappa} > \rho - (1-\rho)\left(\frac{1-\beta\rho}{\kappa}\right)$

OP3: $(3 - \beta\rho - \rho) * \left(\phi_y - \beta\rho - \lambda\rho + (\rho - 1)(\beta\rho - 2) + 1\right)$

$> \left(\lambda\phi_\pi - \phi_y - \left(\phi_y - \lambda\rho\right)(\beta\rho - 2) + (\rho - 1)(\beta\rho - 1) - \lambda\rho(\beta\rho - 1)\right)$

Note that OP1 is always true, OP2 coincides with TR1 and OP3 (hencefort OP) can be written as:

$$\phi_y > \frac{1}{(2-\rho)}\left(\Psi(\beta,\rho) + \kappa\phi_\pi\right), \tag{OP}$$

where $\Psi(\beta,\rho) = (\rho + \beta\rho - 2)\left((2 - \beta\rho)(2 - \rho) - \kappa\rho\right)$.

A.3.2 Speed of convergence

The spectral radius (i.e., the largest eigenvalue) of the matrix T governs the speed of convergence in an iterative expression as:

$$x^{(k+1)} = Tx^{(k)} + d.$$

Assuming that x^k converges to a fix point $x^* : \lim_{k\to\infty} x^k = x^*$, where $x^* = Tx^* + d$, and defining the error after k iterations equal to $e^{(k)} = x^* - x^{(k)}$, then $\| e^{(k)} \| \leq \| T^k \| \| e^{(0)} \|$, where $\| T^k \|$ gives the reduction in the error after k steps. Consider the geometric mean

[30] The characteristic polynomial is: $X^3 + (3 - \beta\rho - \rho)X^2 + \left(\phi_y - \beta\rho - \lambda\rho + (\rho - 1)(\beta\rho - 2) + 1\right) X + \left(\lambda\phi_\pi - \phi_y - \left(\phi_y - \lambda\rho\right)(\beta\rho - 2) + (\rho - 1)(\beta\rho - 1) - \lambda\rho(\beta\rho - 1)\right)$

of the error reductions

$$\sigma_k = \left(\frac{\| e^{(1)} \|}{\| e^{(0)} \|} \frac{\| e^{(2)} \|}{\| e^{(1)} \|} \cdots \frac{\| e^{(k)} \|}{\| e^{(k-1)} \|} \right)^{1/k} = \left(\frac{\| e^{(k)} \|}{\| e^{(0)} \|} \right)^{1/k} \leq \left(\| T^k \| \right)^{1/k}.$$

But Gelfand's formula shows that the spectral radius of A gives the asymptotic growth rate of the norm:

$$\rho(T) = \lim_{k \to \infty} \left(\| T^k \| \right)^{1/k} = \lim_{k \to \infty} \sigma_k,$$

where $\rho(T)$ is the spectral radius of the matrix. So we can define an asymptotic rate of convergence:

$$S_\infty = -\log_2 \rho(T).$$

Infact to estimate the number of iterations needed for $\frac{\|e^{(k)}\|}{\|e^{(0)}\|} \leq \frac{1}{2}$, that is to half the initial error, we can use $\left(\frac{\|e^{(k)}\|}{\|e^{(0)}\|}\right)^{1/k} \leq \left(\| T^k \| \right)^{1/k} = \rho(T)$, so that

$$\rho(T)^k \leq \frac{1}{2} \Rightarrow k \simeq -\frac{1}{\log_2 \rho(T)} = \frac{1}{S_\infty}.$$

A.3.3 Proposition 2

Under transaparency, $DT_c(c)$:

$$\begin{bmatrix} \rho\beta & \kappa \\ \rho - \phi_\pi & \rho - \phi_y \end{bmatrix}$$

and the spectral radius is given by: $\lambda = \frac{\rho(1+\beta)-\phi_y+\sqrt{\left(\rho(\beta-1)+\phi_y\right)^2+4\kappa(\rho-\phi_\pi)}}{2}$.

Part (i)

1) If $\left(\rho(\beta-1)+\phi_y\right)^2 \leq 4\kappa(\phi_\pi - \rho)$, real part $= \frac{\rho(1+\beta)-\phi_y}{2}$, so it follows, $\frac{\partial\lambda}{\partial\phi_\pi} = 0$, since ϕ_π does not affect the real part of the eigenvalue

2) If the eigenvalue is real it follows immediately that $\frac{\partial\lambda}{\partial\phi_\pi} < 0$

Part (ii)

1) If $\left(\rho(\beta-1)+\phi_y\right)^2 \leq 4\kappa(\phi_\pi - \rho)$, real part $= \frac{\rho(1+\beta)-\phi_y}{2}$, so it follows, $\frac{\partial\lambda}{\partial\phi_y} < 0$

2) If $\left(\rho(\beta-1)+\phi_y\right)^2 > 4\kappa(\phi_\pi - \rho)$:

$\frac{\partial\lambda}{\partial\phi_y} = -1 + \frac{\left(\rho(\beta-1)+\phi_y\right)}{\sqrt{\left(\rho(\beta-1)+\phi_y\right)^2+4\kappa(\rho-\phi_\pi)}}$

so:

if $\rho(\beta-1)+\phi_y<0$, $=>$ $\frac{\partial\lambda}{\partial\phi_y}<0$

if $\rho(\beta-1)+\phi_y>0$, then $\frac{\partial\lambda}{\partial\phi_y}<0$ iff:

$+1-\frac{\left(\rho(\beta-1)+\phi_y\right)}{\sqrt{\left(\rho(\beta-1)+\phi_y\right)^2+4\kappa(\rho-\phi_\pi)}}>0$, which implies:

$4\kappa(\rho-\phi_\pi)>0$

So: if $\rho(\beta-1)+\phi_y>0$, then $\frac{\partial\lambda}{\partial\phi_y}<0\ iff\ \rho>\phi_\pi$

Indeed if $\rho(\beta-1)+\phi_y>0$: $\frac{\overbrace{\left(\rho(\beta-1)+\phi_y\right)}^{\text{positive}}}{\sqrt{\left(\rho(\beta-1)+\phi_y\right)^2+\underbrace{4\kappa(\rho-\phi_\pi)}_{\text{negative / positive}}}}\gtrless 1$, depending on $\rho\lesseqgtr\phi_\pi$

Part (iii) follows immediately from the expression of the spectral radius:

$\lambda=\frac{\rho(1+\beta)-\phi_y+\sqrt{\left(\rho(\beta-1)+\phi_y\right)^2+4\kappa(\rho-\phi_\pi)}}{2}$

then:

1) $\frac{\partial\lambda}{\partial\kappa}=0$, iff $\left(\rho(\beta-1)+\phi_y\right)^2\leq 4\kappa(\phi_\pi-\rho)$

2) $\frac{\partial\lambda}{\partial\kappa}>0$, iff $\left(\rho(\beta-1)+\phi_y\right)^2>4\kappa(\phi_\pi-\rho)$ and $\phi_\pi<\rho$

3) $\frac{\partial\lambda}{\partial\kappa}<0$, iff $\left(\rho(\beta-1)+\phi_y\right)^2>4\kappa(\phi_\pi-\rho)$ and $\phi_\pi>\rho$